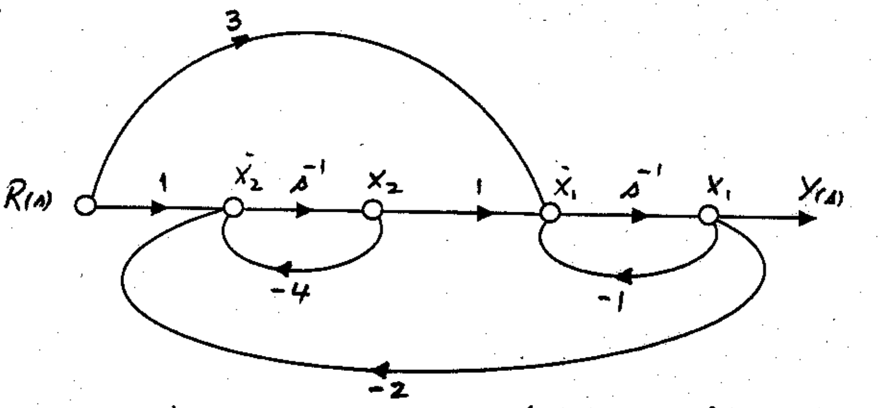
**KING FAHD UNIVERSITY OF PETROLEUM & MINERALS**
**ELECTRICAL ENGINEERING DEPARTMENT**

EE380 [081] Sec# 3 Quiz # 4
Name: Key ID: Grade:

1. Find the state space equations.
2. Find the transfer function Y(s)/R(s) using state matrices.
3. Find the transfer function Y(s)/R(s) using "SFG" or Mason's Rule.
4. Find $\Phi(t)$ the state transition matrix.




1) $\dot{x}_1 = -x_1 + x_2 + 3r$

$\dot{x}_2 = -2x_1 - 4x_2 + r$

$\underline{\dot{x}} = \begin{pmatrix} -1 & 1 \\ -2 & -4 \end{pmatrix} \underline{x} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} r$

$Y = x_1 = (1 \quad 0)\, \underline{X}$

2) $T(s) = C(sI - A)^{-1} B = (1 \quad 0) \begin{bmatrix} s+1 & -1 \\ 2 & s+4 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = (1 \quad 0) \begin{pmatrix} s+4 & 1 \\ -2 & s+1 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \frac{1}{\Delta}$

$\Delta(s) = (s+1)(s+4) + 2 = s^2 + 5s + 6 = (s+2)(s+3)$

$\therefore T(s) = \frac{1}{\Delta(s)} (s+4 \quad 1) \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \frac{3(s+4)+1}{\Delta(s)} = \frac{3s + 13}{s^2 + 5s + 6} \quad (*)$

3) BY SFG: $P_1 = s^{-2}$, $\Delta_1 = 1$ ; $P_2 = 3s^{-1}$, $\Delta_2 = 1 + 4s^{-1}$

$L_1 = -4s^{-1}$, $L_2 = -s^{-1}$, $L_3 = -2s^{-2}$

$\therefore \Delta = 1 - (L_1 + L_2 + L_3) + L_1 L_2$ ($L_1$ & $L_2$ are nontouching)

$\therefore \frac{Y(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{s^{-2} + 3s^{-1}(1 + 4s^{-1})}{1 - (-4s^{-1} - s^{-1} - 2s^{-2}) + (-4s^{-1})(-s^{-1})}$

$= \frac{13s^{-2} + 3s^{-1}}{1 + 5s^{-1} + 6s^{-2}} = \frac{3s + 13}{s^2 + 5s + 6} \quad (**)$

4) $\Phi(s) = (sI - A)^{-1} = \begin{pmatrix} \frac{s+4}{(s+2)(s+3)} & \frac{1}{(s+2)(s+3)} \\ \frac{-2}{(s+2)(s+3)} & \frac{s+1}{(s+2)(s+3)} \end{pmatrix} = \begin{pmatrix} \frac{+2}{s+2} + \frac{-1}{s+3} & \frac{1}{s+2} - \frac{1}{s+3} \\ \frac{-2}{s+2} + \frac{2}{s+3} & \frac{-1}{s+2} + \frac{2}{s+3} \end{pmatrix}$

$\therefore \Phi(t) = \begin{bmatrix} 2e^{-2t} - e^{-3t} & e^{-2t} - e^{-3t} \\ -2e^{-2t} + 2e^{-3t} & -e^{-2t} + 2e^{-3t} \end{bmatrix}$

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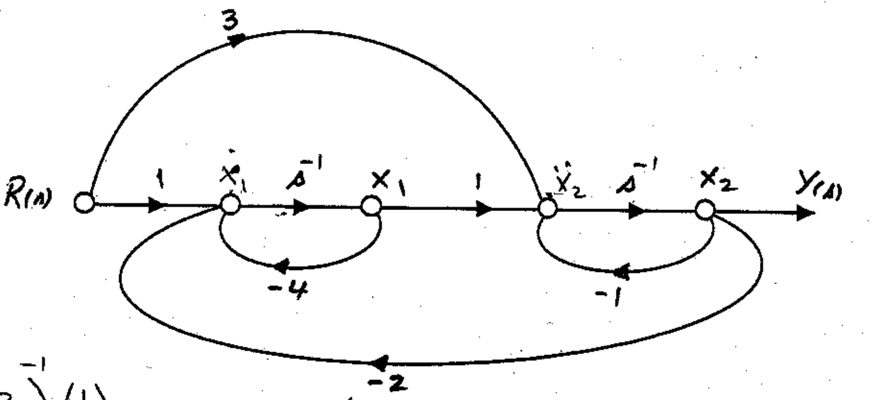


① $\dot{x}_1 = -4x_1 - 2x_2 + r$

$\dot{x}_2 = x_1 - x_2 + 3r$

$Y = x_2$

$\therefore \underline{\dot{X}} = \begin{pmatrix} -4 & -2 \\ 1 & -1 \end{pmatrix} \underline{X} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} r$

$Y = (0 \quad 1)\, \underline{X}$

② $T(s) = C(sI - A)^{-1}B = (0 \quad 1) \begin{pmatrix} s+4 & 2 \\ -1 & s+1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = (0 \quad 1) \frac{1}{\Delta(s)} \begin{pmatrix} s+1 & -2 \\ 1 & s+4 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = (1 \quad s+4) \begin{pmatrix} 1 \\ 3 \end{pmatrix} \frac{1}{\Delta(s)}$

$= \frac{1 + 3(s+4)}{(s+1)(s+4) + 2} = \frac{3s + 13}{s^2 + 5s + 6} \quad (*)$

③ BY SFG: $P_1 = s^{-2}$, $\Delta_1 = 1$ ; $P_2 = 3s^{-1}$, $\Delta_2 = 1 + 4s^{-1}$

Loops: $L_1 = -4s^{-1}$, $L_2 = -s^{-1}$, $L_3 = -2s^{-2}$

$\Delta = 1 - (L_1 + L_2 + L_3) + L_1 L_2 = 1 - (-4s^{-1} - s^{-1} - 2s^{-2}) + (-4s^{-1})(-s^{-1})$

$= 1 + 5s^{-1} + 6s^{-2}$ ($L_1$ & $L_2$) are nontouching loops

$\therefore \frac{Y(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{s^{-2} + 3s^{-1}(1 + 4s^{-1})}{1 + 5s^{-1} + 6s^{-2}} = \frac{3s^{-1} + 13s^{-2}}{1 + 5s^{-1} + 6s^{-2}}$

$= \frac{3s + 13}{s^2 + 5s + 6} \quad (**)$ [ Same as (*) ]

$\Phi(s) = (sI - A)^{-1} = \begin{pmatrix} s+1 & -2 \\ 1 & s+4 \end{pmatrix} \frac{1}{s^2 + 5s + 6} = \begin{bmatrix} \frac{s+1}{(s+2)(s+3)} & \frac{-2}{(s+2)(s+3)} \\ \frac{1}{(s+2)(s+3)} & \frac{s+4}{(s+2)(s+3)} \end{bmatrix}$

$= \begin{bmatrix} \frac{-1}{s+2} + \frac{2}{s+3} & \frac{-2}{s+2} + \frac{2}{s+3} \\ \frac{1}{s+2} - \frac{1}{s+3} & \frac{2}{s+2} - \frac{1}{s+3} \end{bmatrix}$

$\therefore \Phi(t) = \begin{bmatrix} -e^{-2t} + 2e^{-3t} & -2e^{-2t} + 2e^{-3t} \\ e^{-2t} - e^{-3t} & 2e^{-2t} - e^{-3t} \end{bmatrix}$