

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
ELECTRICAL ENGINEERING DEPARTMENT

EE380 [081]

SEC#

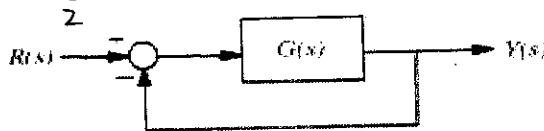
Quiz # 9

Name: Key Solution

ID: _____

Grade: _____

Consider the feedback control system. Sketch the root locus and find the break-away and break-in points. Also find the angle of departures from complex poles and angles of arrivals to the complex zeros.



$$\text{ch. Eqn: } 1 + G(s) = 0$$

$$1 + \frac{K(s^2 + 2s + 2)}{s(s^2 + 4)} = 0$$

$$K = -\frac{s(s^2 + 4)}{s^2 + 2s + 2}$$

where

$$G(s) = \frac{K(s+1+j)(s+1-j)}{s(s+2j)(s-2j)}$$

$$\frac{dK}{ds} = s^4 + 4s^3 + 2s^2 + 8 = 0 \Rightarrow \frac{dK}{ds} = (s+2)(s^3 + 2s^2 - 2s + 4) = 0 \Rightarrow \begin{cases} s = -2 \\ s \approx -3.07 \end{cases}$$

Angle of arrival: θ_a

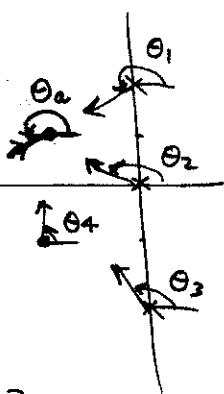
$$[\theta_1 + \theta_2 + \theta_3] - [\theta_4 + \theta_a] = 180$$

$$225 + 135 + [180 - \tan^{-1} 3]$$

$$- [90 + \theta_a] = 180$$

$$\therefore 90 + \theta_a = 360 - \tan^{-1} 3$$

$$\therefore \theta_a = 270 - \tan^{-1} 3 \approx 198.6^\circ$$



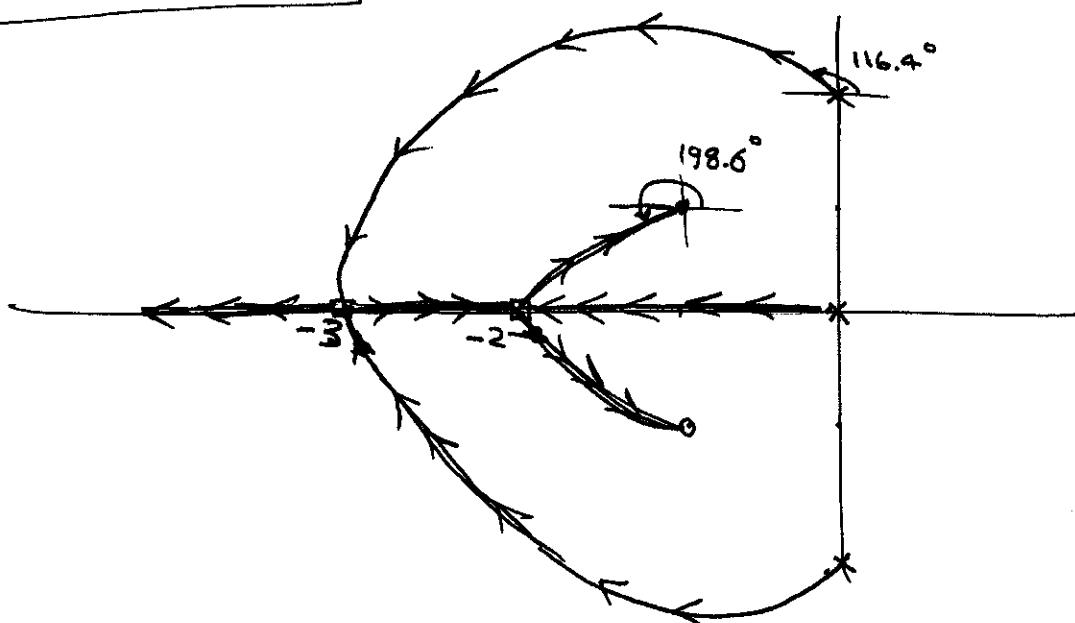
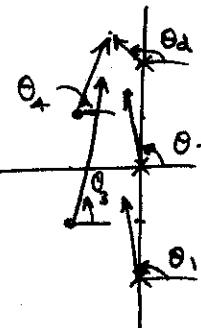
Angle of departure: θ_d

$$[\theta_1 + \theta_2 + \theta_d] - [\theta_2 + \theta_4] = 180^\circ$$

$$[90 + 90 + \theta_d] - [\tan^{-1} 3 + 45] = 180^\circ$$

$$\therefore \theta_d = \tan^{-1} 3 + 45$$

$$\theta_d = 71.4 + 45 = 116.4^\circ$$



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SEC# 4

Quiz # 9

Name: _____ ID: _____ Grade: _____

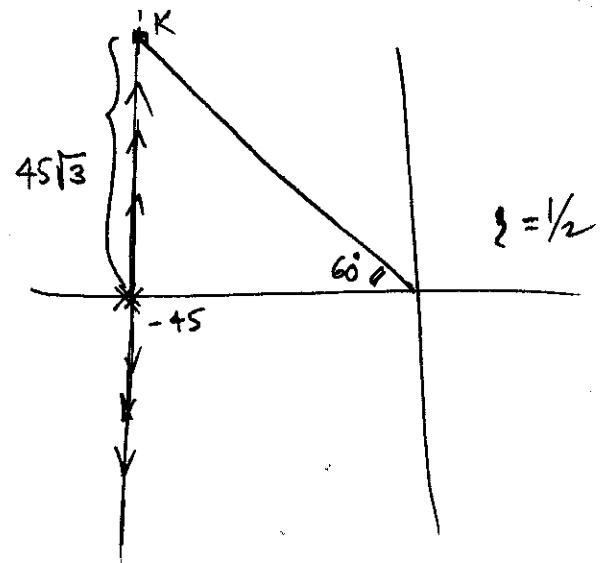
A mechanical system has the closed-loop transfer function given by

$$T(s) = \frac{K}{(s + 45)^2 + K}$$

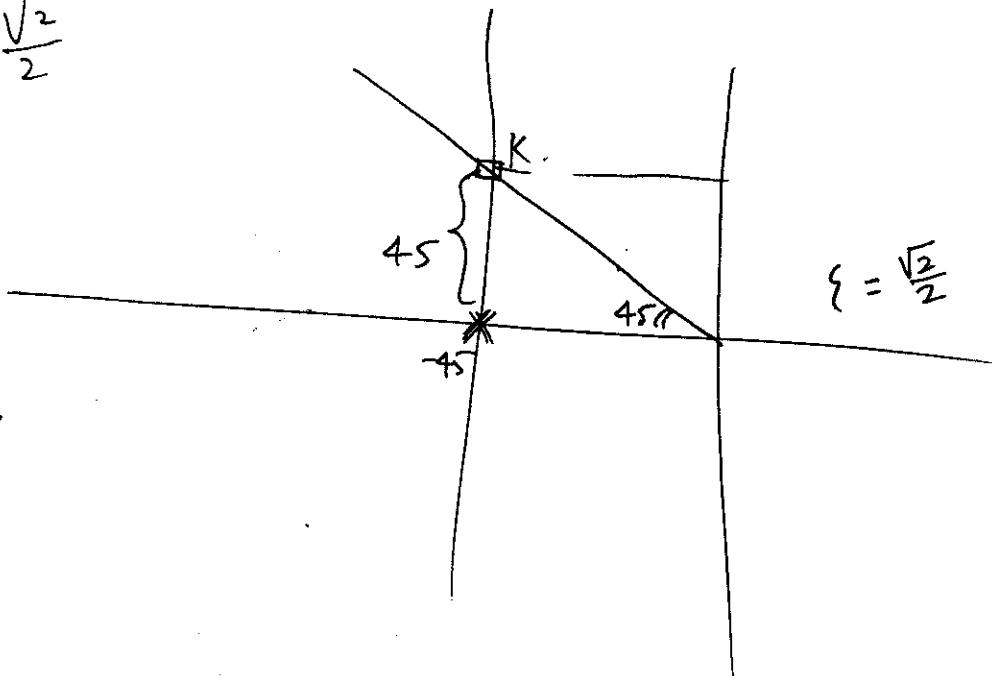
Using the root-locus method, determine the value of the gain K so that the closed-loop system has a damping factor $\zeta = \frac{1}{2}$.

$$1 + \frac{K}{(s + 45)^2} = 0$$

$$\begin{aligned} |K| &= (45\sqrt{3})(45\sqrt{3}) \\ &= 2025 \times 3 = 6075 \end{aligned}$$



$$\text{For } \zeta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$



$$\begin{aligned} K &= 45 \times 45 \\ &= 2025 \end{aligned}$$