KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

EE-520 (131)

Dr. Ibrahim O. Habiballah

Key Solutions

Home Work 3 (Due Date: December 16, 2013)

Q.1) An unloaded synchronous generator is connected to a three-phase transformer. The generator is rated 100MVA, 18 kV, $X'' = 0.19$ pu. The transformer is rated 100 MVA, 240-Wye / 18-Delta kV, X = (10 + your two-digit serial no.)% . A three-phase short circuit occurs on the high-voltage side of the transformer.

a.Calculate the fault current on the high-voltage side of the transformer in ampers.

b.Calculate the fault current on the low-voltage side of the transformer in ampers.

Solution:

$$I'' = \frac{1.0}{j\,(0.19 + 0.10)} = -j3.448 \text{ per unit}$$

$$\text{Base } I_{\text{HV}} = \frac{100,000}{\sqrt{3} \times 240} = 240.6 \text{ A}$$

$$\text{Base } I_{\text{LV}} = \frac{100,000}{\sqrt{3} \times 18} = 3207.5 \text{ A}$$

(a) $3.448 \times 240.6 = 829.5$ A

(b) $3.448 \times 3207.5 = 11,060$ A

Q.2) The currents flowing in the lines toward a balanced load connected in delta are

$I_a = 100 \angle 0^\circ$ A ; $I_b = 141.4 \angle 225^\circ$ A ; $I_c = 100 \angle 90^\circ$ A ;

Calculate I_{ab}.

Solution:

$$I_a^{(1)} = \frac{1}{3}\left(100 + 141.4\angle 345^\circ + 100\angle 330^\circ \right)$$
$$= 107.7 - j28.9 = 111.5\angle -15^\circ \text{ A}$$

$$I_a^{(2)} = \frac{1}{3}\left(100 + 141.4\angle 105^\circ + 100\angle 210^\circ \right)$$
$$= -7.73 + j28.9 = 29.9\angle 105^\circ \text{ A}$$

$$I_a^{(0)} = \frac{1}{3}\left(100 - 100 - j100 + j100 \right)$$
$$= 0 \quad \text{(since zero-sequence cannot flow into the } \Delta\text{)}.$$

and,

$$I_{ab}^{(1)} = \frac{111.5}{\sqrt{3}} \angle -15^\circ + 30^\circ = 64.4\angle 15^\circ = 62.2 + j16.66$$

$$I_{ab}^{(2)} = \frac{29.9}{\sqrt{3}} \angle 105^\circ - 30^\circ = 17.26\angle 75^\circ = 4.47 + j16.67$$

$$I_{ab} = 66.67 + j33.33 = 74.5\angle 26.6^\circ \text{ A}$$

Q.3) A wye-connected synchronous generator has the following sequence reactances: X_1 = 0.22 pu, X_2 = 0.36 pu, and X_0 = 0.09 pu. The neutral point of the generator is grounded through a reactance of 0.(09 + your two-digit serial no.) pu. The generator is running on load with rated terminal voltage when it suffers an unbalanced fault. Find the terminal voltages of the machine, the voltage of the neutral point, and the type of fault if

a. The line currents out of the machine are $I_a = 0\angle 0^\circ$ pu; $I_b = 3.75\angle 150^\circ$ pu; $I_c = 3.75\angle 30^\circ$ pu; all with respect to phase voltage.

b. The line currents out of the machine are $I_a = 0$ pu; $I_b = -2.986$ pu; $I_c = 2.986$ pu; all with respect to phase voltage.

Solution:

(a)

$$Z_1 = j0.22 \text{ p.u.}, \quad Z_2 = 0.36 \text{ p.u.},$$
$$Z_0 = Z_{g0} + 3Z_n = j0.09 + 3\times j0.09 = 0.36 \text{ p.u.}$$

$$I_a = 0,\ I_b = 3.75\underline{/0^\circ} \text{ p.u., and } I_c = 3.75\underline{/0^\circ} \text{ p.u.}$$

$$\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ 3.75\underline{/150^\circ} \\ 3.75\underline{/30^\circ} \end{bmatrix} = \begin{bmatrix} j1.25 \\ -j2.5 \\ j1.25 \end{bmatrix}$$

Hence,

$$V_a^{(0)} = -I_a^{(0)}Z_0 = -j1.25 \times j0.36 = 0.45 \text{ p.u.}$$

$$V_a^{(1)} = E_{an} - I_a^{(1)}Z_1 = 1\underline{/0^\circ} - (-j2.5 \times j0.22)$$
$$= 0.45 \text{ p.u.}$$

$$V_a^{(2)} = -I_a^{(2)}Z_2 = -j1.25 \times j0.36 = 0.45 \text{ p.u.}$$

and,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} 0.45\underline{/0^\circ} \\ 0.45\underline{/0^\circ} \\ 0.45\underline{/0^\circ} \end{bmatrix} = \begin{bmatrix} 1.35\underline{/0^\circ} \\ 0 \\ 0 \end{bmatrix} \text{ p.u.}$$

$$
\begin{aligned}
V_n &= -3I_a^{(0)} \times j0.09 \text{ p.u.} \\
&= -3 \times j1.25 \times j0.09 \text{ p.u.} \\
&= 0.3375 \text{ p.u.}
\end{aligned}
$$

since $V_b = V_c = 0$, it is a double-line-to-ground fault.

(*b*)

$$
\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ -2.986 \\ 2.986 \end{bmatrix} = \begin{bmatrix} 0 \\ -j1.724 \\ j1.724 \end{bmatrix}
$$

$$
V_a^{(0)} = -I_a^{(0)} Z_0 = 0
$$

$$
\begin{aligned}
V_a^{(1)} &= E_{an} - I_a^{(1)} Z_1 = 1\angle 0^\circ - (-j1.724)(j0.22) \\
&= 0.621 \text{ p.u.}
\end{aligned}
$$

$$
\begin{aligned}
V_a^{(2)} &= -I_a^{(2)} Z_2 = -(-j1.724)(j0.36) \\
&= 0.621 \text{ p.u.}
\end{aligned}
$$

$$
\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} 0 \\ 0.621 \\ 0.621 \end{bmatrix} = \begin{bmatrix} 1.242\angle 0^\circ \\ -0.621\angle 0^\circ \\ -0.621\angle 0^\circ \end{bmatrix} \text{ p.u.}
$$

Since $I_a^{(0)} = 0$, $V_n = 0$.

Since $V_b = V_c$, it is a line-to-line fault.

Q.4) An unloaded Y-connected solidly grounded synchronous generator is rated 500 MVA, 22 kV. Its reactances are $X'' = X_2 = 0.(15 + \text{your two-digit serial no.})$ pu, and $X_0 = 0.05$ pu. Find the proper inductive reactance in ohms to be inserted in the neutral of the machine in order to limit the subtarnsient line current ratio of a single line to ground fault to a three-phase fault to one.

Solution:

The subtransient line current due to a three-phase fault is

$I_a = 1.0 / j\ 0.15 = -j\ 6.667$ pu

Let x be the inductive reactance in per-unit to be inserted in the neutral of the machine.

The subtransient line current due to a single line to ground fault is

$$I_a = 3I_a^{(1)} = \frac{3}{j(0.15 + 0.15 + 0.05 + 3x)}$$

For a three-phase fault, $I_a = 1/j0.15 = -j6.667$ per unit. Equating the values for I_a, we have

$$\begin{aligned} 3 &= -j^2(0.35 + 3x)(6.667) \\ x &= 0.0333 \text{ per unit} \\ \text{Base } Z &= \frac{(22)^2}{500} = 0.968\ \Omega \\ x &= 0.0333 \times 0.968 = 0.3226\ \Omega \end{aligned}$$