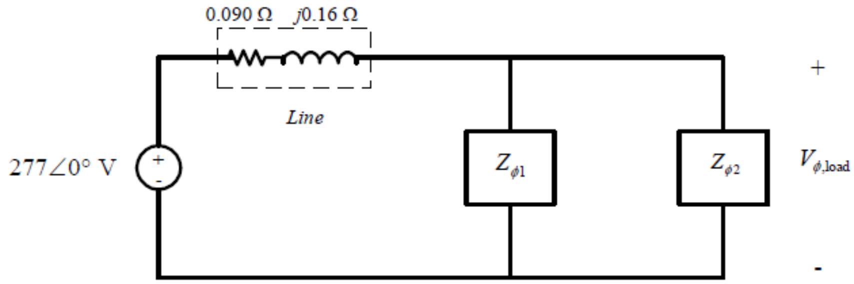
EE 306: Solution of Home Work #1

Problem 1 Solution:

SOLUTION To solve this problem, first convert the delta-connected load 2 to an equivalent wye (by dividing the impedance by 3), and get the per-phase equivalent circuit.



$Z_{\phi 1} = 2.5\angle 36.87° \ \Omega$

$Z_{\phi 2} = 1.67\angle -20° \ \Omega$

(a) The phase voltage of the equivalent Y-loads can be found by nodal analysis.

$$\frac{\mathbf{V}_{\phi,\text{load}} - 277\angle 0° \text{ V}}{0.09 + j0.16 \ \Omega} + \frac{\mathbf{V}_{\phi,\text{load}}}{2.5\angle 36.87° \ \Omega} + \frac{\mathbf{V}_{\phi,\text{load}}}{1.67\angle -20° \ \Omega} = 0$$

$$(5.443\angle -60.6°)\left(\mathbf{V}_{\phi,\text{load}} - 277\angle 0° \text{ V}\right) + (0.4\angle -36.87°)\mathbf{V}_{\phi,\text{load}} + (0.6\angle 20°)\mathbf{V}_{\phi,\text{load}} = 0$$

$$(5.955\angle -53.34°)\ \mathbf{V}_{\phi,\text{load}} = 1508\angle -60.6°$$

$$\mathbf{V}_{\phi,\text{load}} = 253.2\angle -7.3° \text{ V}$$

Therefore, the line voltage at the loads is $V_L \sqrt{3}\ V_\phi = 439 \text{ V}$.

(b) The voltage drop in the transmission lines is

$$\Delta\mathbf{V}_{\text{line}} = \mathbf{V}_{\phi,\text{gen}} - \mathbf{V}_{\phi,\text{load}} = 277\angle 0° \text{ V} - 253.2\angle -7.3° = 41.3\angle 52° \text{ V}$$

(c) The real and reactive power of each load is

$$P_1 = 3\frac{V_\phi^{\,2}}{Z}\cos\theta = 3\frac{(253.2 \text{ V})^2}{2.5 \ \Omega}\cos\ 36.87° = 61.6 \text{ kW}$$

$$Q_1 = 3\frac{V_\phi^{\,2}}{Z}\sin\theta = 3\frac{(253.2 \text{ V})^2}{2.5 \ \Omega}\sin 36.87° = 46.2 \text{ kvar}$$

$$P_2 = 3\frac{V_\phi^{\,2}}{Z}\cos\theta = 3\frac{(253.2 \text{ V})^2}{1.67 \ \Omega}\cos\ (-20°) = 108.4 \text{ kW}$$

$$Q_2 = 3\frac{V_\phi^{\,2}}{Z}\sin\theta = 3\frac{(253.2 \text{ V})^2}{1.67 \ \Omega}\sin\ (-20°) = -39.5 \text{ kvar}$$

(d) The line current is

$$\mathbf{I}_{\text{line}} = \frac{\Delta\mathbf{V}_{\text{line}}}{Z_{\text{line}}} = \frac{41.3\angle 52° \text{ V}}{0.09 + j0.16 \ \Omega} = 225\angle -8.6° \text{A}$$

Therefore, the loses in the transmission line are

$$P_{\text{line}} = 3I_{\text{line}}^{\ 2}R_{\text{line}} = 3\ (225 \text{ A})^2 (0.09 \ \Omega) = 13.7 \text{ kW}$$

$$Q_{\text{line}} = 3I_{\text{line}}^{\ 2}X_{\text{line}} = 3\ (225 \text{ A})^2 (0.16 \ \Omega) = 24.3 \text{ kvar}$$

(e) The real and reactive power supplied by the generator is

$$P_{\text{gen}} = P_{\text{line}} + P_1 + P_2 = 13.7\text{ kW} + 61.6\text{ kW} + 108.4\text{ kW} = 183.7\text{ kW}$$

$$Q_{\text{gen}} = Q_{\text{line}} + Q_1 + Q_2 = 24.3\text{ kvar} + 46.2\text{ kvar} - 39.5\text{ kvar} = 31\text{ kvar}$$

The power factor of the generator is

$$\text{PF} = \cos\left[\tan^{-1}\frac{Q_{\text{gen}}}{P_{\text{gen}}}\right] = \cos\left[\tan^{-1}\frac{31\text{ kvar}}{183.7\text{ kW}}\right] = 0.986\text{ lagging}$$

Problem 2 Solution:

The per-phase equivalent circuit is first constructed. For load 1, a Y-connected balanced load, the per-phase impedance is 150 + j50 *Ω*. For load 2, a Δ-connected balanced load, the per-phase impedance is the equivalent Y-connected load which is $Z_{\Delta}/3$ = 300 + j200 *Ω*. We will represent load 3 in terms of the complex power it absorbs per-phase. This is given by

$$S_{3/\phi} = \frac{95040}{3}(0.6 - j0.8)$$
$$= 19{,}008 - j25{,}344VA$$

The voltage across the per-phase equivalents of these loads has been specified as 4800 V, which is the line-to-neutral voltage at the load end.

The per-phase equivalent circuit is shown below in Figure 3

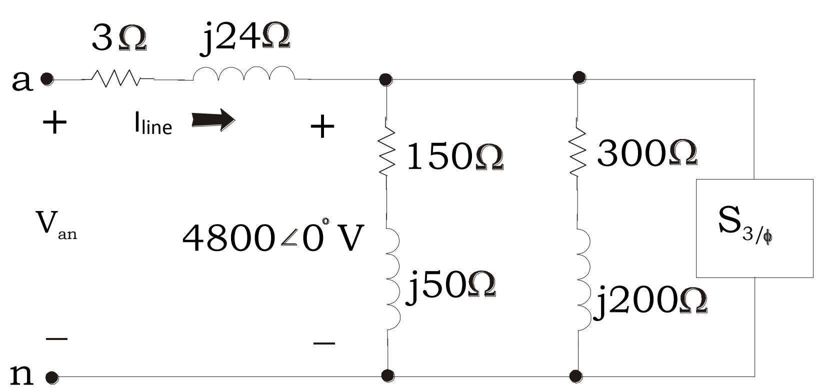


Figure 3 Per-Phase Equivalent Circuit

$$\mathbf{I}_{\ell} = \frac{4800}{150 + j50} + \frac{4800}{300 + j200} + \frac{19{,}008 + j25{,}344}{4800}$$
$$= 28.8 - j9.6 + 11.0769 - j7.3846 + 3.96 + j5.28$$
$$= 43.8369 - j11.7046A(rms)$$
$$= 45.3725\angle -14.949^{\circ}\ A(rms)$$

In the above step, the total current is obtained by summing the individual currents through the three loads. For loads 1&2, we use the expression $\mathbf{I} = \dfrac{\mathbf{V}}{Z}$, and for load 3 the current is determined using $\mathbf{I} = \left(\dfrac{S}{\mathbf{V}}\right)^*$.

In the distribution line,

$$P_{loss} = 3\left|I_{eff}\right|^2 R = 3(45.3725)^2\ (3) = 18{,}528.04\,W$$

$$Q_{loss} = 3\left|I_{eff}\right|^2 X = 3(45.3725)^2\ (24) = 148{,}224.34\,VAR$$

In each load,

$$P_1 = 3\left|28.8 - j9.6\right|^2 (150) = 414{,}720\,W$$

$$Q_1 = 3\left|28.8 - j9.6\right|^2 (50) = 138{,}240\,VAR\,(abs)$$

$$P_2 = 3\left|11.0769 - j7.3846\right|^2 (300) = 159{,}507.02\,W$$

$$Q_2 = 3\left|11.0769 - j7.3846\right|^2 (200) = 106{,}338.02\,VAR\,(abs)$$

$$P_3 = 95{,}040\ (0.6) = 57{,}024W$$

$$Q_3 = -95{,}040\,(0.8) = -76{,}032VAR$$

$$S_{total} = 636{,}251 + j168{,}546\ VA\ (load\ end)$$

Check :

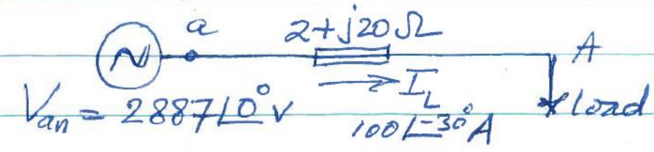
$$S_{total} = 3(4800)(43.8369 + j11.74046) = 631{,}251 + j168{,}546\,VA$$

$$S_{sending} = 631{,}251 + j168{,}546 + 18{,}528.04 + j148{,}224.34VA$$

$$= 649{,}779.04 + j316{,}770.34VA$$

$$\%P = \frac{631{,}251}{649{,}779.04} \times 100 = 97.148$$

Problem 3 Solution:



1. $Pf_{source} = \cos(\theta_v - \theta_I) = \cos(0 + 30°) = 0.866$ lagging
2. $V_{AN} = \bar{V}_{an} - Z_L I_L = 2887\angle 0 - (2+j20)\,100\angle -30°$

$V_{AN} = 2887 - (2+j20)(86.6 - j50)$

$V_{AN} = 2887 - (1173.2 + j1632) = 1713.8 - j1632$

$V_{AN} = 2366.54 \angle -43.6°$ V

3. $Pf_{load} = \cos(\theta_v - \theta_I) = \cos(-43.6° + 30) = 0.972$ leading

a. leading

b. Capacitive

4. $P = 3|\bar{V}_{AN}||\bar{I}_L| \cos\theta_{load} = 3 \times 2366.54 \times 100 \times 0.972$

$Q = 3\, V_{AN} |I_L| \sin\theta_{load} = 3 \times 2366.54 \times 100 \times -0235$

$P = 690083.064$ W

$Q = -166942$ VAR

Problem 4 Solution:

(a) Assume $\bar{V}_{an} = \frac{480}{\sqrt{3}} \angle 0^\circ$ V $\quad \bar{V}_{ab} = 480 \angle 30^\circ$ V

$Z_Y = 4 \angle 36.87^\circ \, \Omega \qquad Z_\Delta = 10 \angle 30^\circ \, \Omega$

$$S_Y = 3 \frac{V_\phi^2}{Z_Y^*} = \frac{V_L^2}{Z_Y^*} = \frac{(480)^2}{4 \angle -36.87} = 57600 \angle 36.87^\circ$$

$$S_Y = 46080 + j\,34560 \text{ VA}$$

$$S_\Delta = 3 \frac{V_L^2}{Z_\Delta^*} = 3 \frac{480^2}{10 \angle -30^\circ} = 69120 \angle 30^\circ = 59859.7 + j\,34560 \text{ VA}$$

$$\therefore \; S_T = P_T + j\,Q_T = 105939.7 + j\,69120 \text{ VA}$$

$$= 126494.25 \angle 33.12^\circ \text{ VA}$$

$$I_T = \frac{S}{\sqrt{3}\,V_L} = \frac{126494.25}{\sqrt{3}\,480} = 152.14 \text{ A}$$

(b) $$S_{Cap} = -j \frac{(480)^2}{5} = Q_c = -j\,46080 \text{ VAR}$$

$$S_{T_{new}} = 105939.7 + j\,23040 = 108416.15 \angle 12.27^\circ \text{ VA}$$

(c) $$I = \frac{S_{T_{new}}}{\sqrt{3}\,V_L} = \frac{108416.15}{\sqrt{3}\,480} = 130.4 \text{ A}$$

The current is reduced since part of the reactive power is compensated by the capacitor bank.