KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

EE 306 – Term 181

HW # 5: Synchronous Machines

Key Solutions

Problem # 1:

During a short-circuit test, a Y-connected synchronous generator produces 100 A of short-circuit armature current per phase at a field current of 2.5 A. At the same field current, the open-circuit line voltage is measured to be 440 V.

(a) Calculate the saturated synchronous reactance under these conditions.

(b) If the armature resistance is 0.3 Ω per phase, and the generator supplies 60 A to a purely resistive Y-connected load of 3 Ω per phase at this field current setting, determine the voltage regulation under these load conditions.

SOLUTION

(a) The saturated synchronous reactance at a field current of 2.5 A can be found from the information supplied in the problem. The open circuit line voltage at I_F = 2.5 A is 440 V, and the short-circuit current is 100 A. Since this generator is Y-connected, the corresponding phase voltage is $V_\phi = 440\text{ V}/\sqrt{3} = 254\text{ V}$ and the armature current is $I_A = 100\text{ A}$. Therefore, the *saturated* synchronous reactance is

$$X_S = \frac{254\text{ V}}{100\text{ A}} = 2.54\ \Omega$$

(b) Assume that the desired line voltage is 440 V, which means that the phase voltage $\mathbf{V}_\phi = 254\angle 0°\text{ V}$. The armature current is $\mathbf{I}_A = 60\angle 0°\text{ A}$, so the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 254\angle 0° + (0.30\ \Omega)(60\angle 0°\text{ A}) + j(2.54\ \Omega)(60\angle 0°\text{ A})$$
$$\mathbf{E}_A = 312\angle 29.3°\text{ V}$$

This is also the phase voltage at no load conditions. The corresponding line voltage at no load conditions would be $V_{L,\text{nl}} = (312\text{ V})\left(\sqrt{3}\right) = 540\text{ V}$. The voltage regulation is

$$\text{VR} = \frac{V_{T,\text{nl}} - V_{T,\text{fl}}}{V_{T,\text{fl}}} \times 100\% = \frac{540 - 440}{440} \times 100\% = 22.7\%$$

Problem # 2:

The internal generated voltage E_A of a **2-pole, Δ-connected, 60 Hz,** three phase synchronous generator is 14.4 kV, and the terminal voltage V_T is 12.8 kV. The synchronous reactance of this machine is 4 Ω, and the armature resistance can be ignored.

(a) If the torque angle of the generator δ = 18°, how much power is being supplied by this generator at the current time?

(b) What is the power factor of the generator at this time?

(c) Sketch the phasor diagram under these circumstances.

(d) Ignoring losses in this generator, what torque must be applied to its shaft by the prime mover at these conditions?

SOLUTION

(a) If resistance is ignored, the output power from this generator is given by

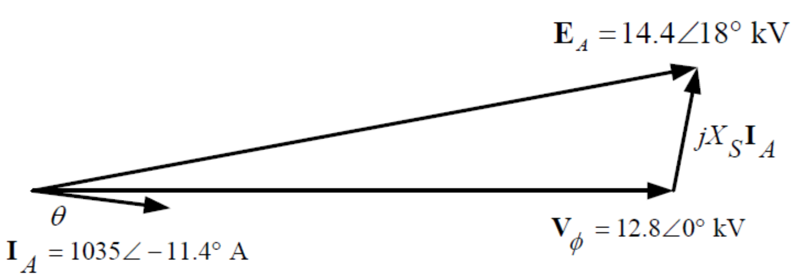
$$P = \frac{3V_\phi E_A}{X_S}\sin\delta = \frac{3(12.8\text{ kV})(14.4\text{ kV})}{4\ \Omega}\sin 18^\circ = 42.7\text{ MW}$$

(b) The phase current flowing in this generator can be calculated from

$$\mathbf{I}_A = \frac{14.4\angle 18^\circ\text{ kV} - 12.8\angle 0^\circ\text{ kV}}{j4\ \Omega} = 1135\angle -11.4^\circ\text{ A}$$

Therefore the impedance angle $\theta = 11.4^\circ$, and the power factor is $\cos(11.4^\circ) = 0.98\text{ lagging}$.

(c) The phasor diagram is



(d) The induced torque is given by the equation

$$P_{\text{conv}} = \tau_{\text{ind}}\omega_m$$

With no losses,

$$\tau_{\text{app}} = \tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{42.7\text{ MW}}{2\pi(60\text{ hz})} = 113{,}300\text{ N}\bullet\text{m}$$

Problem # 3:

A 480-V, 60 Hz, 400-hp 0.8-PF-leading eight-pole Δ-connected synchronous motor has a synchronous reactance of 0.6 Ω and negligible armature resistance. Ignore its friction, windage, and core losses for the purposes of this problem. Assume that $|\mathbf{E}_A|$ is directly proportional to the field current I_F (in other words, assume that the motor operates in the linear part of the magnetization curve), and that $|\mathbf{E}_A|$ = 480 V when I_F = 4 A.

(a) What is the speed of this motor?

(b) If this motor is initially supplying 400 hp at 0.8 PF lagging, what are the magnitudes and angles of $\mathbf{E}_A$ and $\mathbf{I}_A$?

(c) How much torque is this motor producing? What is the torque angle δ? How near is this value to the maximum possible induced torque of the motor for this field current setting?

(d) If $|\mathbf{E}_A|$ is increased by 30 percent, what is the new magnitude of the armature current? What is the motor's new power factor?

SOLUTION

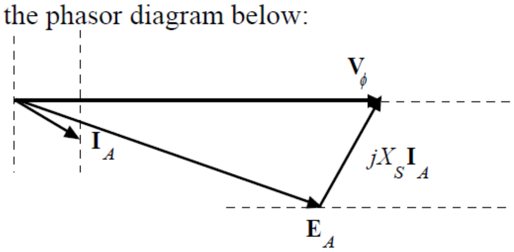
(a) The speed of this motor is given by

$$n_m = \frac{120 f_{se}}{P} = \frac{120(60\text{ Hz})}{8} = 900\text{ r/min}$$

(b) If losses are being ignored, the output power is equal to the input power, so the input power will be

$$P_{\text{IN}} = (400\text{ hp})(746\text{ W/hp}) = 298.4\text{ kW}$$

This situation is shown in the phasor diagram below:



The line current flow under these circumstances is

$$I_L = \frac{P}{\sqrt{3}\ V_T\ \text{PF}} = \frac{298.4\text{ kW}}{\sqrt{3}(480\text{ V})(0.8)} = 449\text{ A}$$

Because the motor is Δ-connected, the corresponding phase current is $I_A = 449/\sqrt{3} = 259\text{ A}$. The angle of the current is $-\cos^{-1}(0.80) = -36.87°$, so $\mathbf{I}_A = 259\angle -36.87°\text{ A}$. The internal generated voltage $\mathbf{E}_A$ is

$$\mathbf{E}_A = \mathbf{V}_\phi - jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = (480\angle 0°\text{ V}) - j(0.6\ \Omega)(259\angle -36.87°\text{ A}) = 406\angle -17.8°\text{ V}$$

(c) This motor has 6 poles and an electrical frequency of 60 Hz, so its rotation speed is n_m = 1200 r/min. The induced torque is

$$\tau_{\text{ind}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{298.4 \text{ kW}}{(900 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)} = 3166 \text{ N}\cdot\text{m}$$

The maximum possible induced torque for the motor at this field setting is the maximum possible power divided by ω_m

$$\tau_{\text{ind,max}} = \frac{3V_\phi E_A}{\omega_m X_S} = \frac{3(480 \text{ V})(406 \text{ V})}{(900 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)(0.6\ \Omega)} = 10{,}340 \text{ N}\cdot\text{m}$$

The current operating torque is about 1/3 of the maximum possible torque.

(d) If the magnitude of the internal generated voltage E_A is increased by 30%, the new torque angle can be found from the fact that $E_A \sin\delta \propto P = \text{constant}$.

$$E_{A2} = 1.30\ E_{A1} = 1.30(406 \text{ V}) = 487.2 \text{ V}$$

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_1\right) = \sin^{-1}\left(\frac{406 \text{ V}}{487.2 \text{ V}}\sin(-17.8°)\right) = -14.8°$$

The new armature current is

$$\mathbf{I}_{A2} = \frac{\mathbf{V}_\phi - \mathbf{E}_{A2}}{jX_S} = \frac{480\angle 0° \text{ V} - 487.2\angle -14.8° \text{ V}}{j0.6\ \Omega} = 208\angle -4.1° \text{ A}$$

The magnitude of the armature current is 208 A, and the power factor is cos (-24.1°) = 0.913 lagging.

Revised calaculation:

E_{A2} = 527.8 V ; Delta$_2$ = -13.6^o ; I_{A2} = 213.3 (Theta = 15^o)

Power factor is cos(15) = 0.966 leading

Problem # 4:

A 230-V, 50 Hz, two-pole synchronous motor draws 40 A from the line at unity power factor and full load. Assuming that the motor is lossless, answer the following questions:

(a) What is the output torque of this motor? Express the answer both in newton-meters

(b) What must be done to change the power factor to 0.85 leading? Explain your answer, using phasor diagrams.

(c) What will the magnitude of the line current be if the power factor is adjusted to 0.8 leading?

SOLUTION

(a) If this motor is assumed lossless, then the input power is equal to the output power. The input power to this motor is

$$P_{\text{IN}} = \sqrt{3} V_T I_L \cos\theta = \sqrt{3}(230 \text{ V})(40 \text{ A})(1.0) = 15.93 \text{ kW}$$

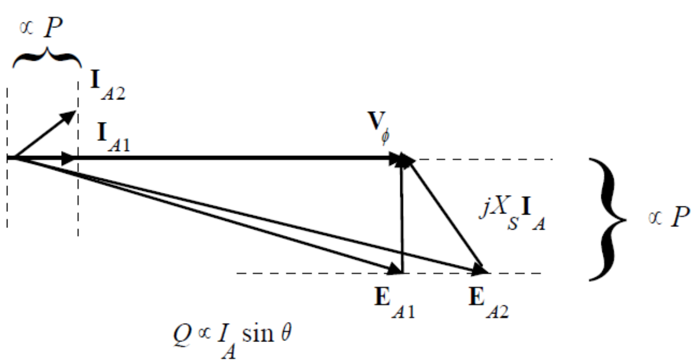
The rotational speed of the motor is

$$n_m = \frac{120 f_{se}}{P} = \frac{120(50 \text{ Hz})}{4} = 1500 \text{ r/min}$$

The output torque would be

$$\tau_{\text{LOAD}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{15.93 \text{ kW}}{(1500 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)} = 101.4 \text{ N}\cdot\text{m}$$

(b) To change the motor's power factor to 0.8 leading, its field current must be increased. Since the power supplied to the load is independent of the field current level, an increase in field current increases $|\mathbf{E}_A|$ while keeping the distance $E_A \sin\delta$ constant. This increase in E_A changes the angle of the current $\mathbf{I}_A$, eventually causing it to reach a power factor of 0.8 leading.



(c) The magnitude of the line current will be

$$I_L = \frac{P}{\sqrt{3}\, V_T \text{ PF}} = \frac{15.93 \text{ kW}}{\sqrt{3}(230 \text{ V})(0.8)} = 50.0 \text{ A}$$