**KING FAHD UNIVERSITY OF PETROLEUM & MINERALS**

**ELECTRICAL ENGINEERING DEPARTMENT**

**EE 306 – Term 181**

**HW # 1: Three-Phase Circuits**

**Key Solutions**

**Problem # 1:**

Determine the phase sequence of the set of voltages

$$v_{an} = 200\cos(\omega t + 10^\circ)$$

$$v_{bn} = 200\cos(\omega t - 230^\circ), \qquad v_{cn} = 200\cos(\omega t - 110^\circ)$$

**Solution:**

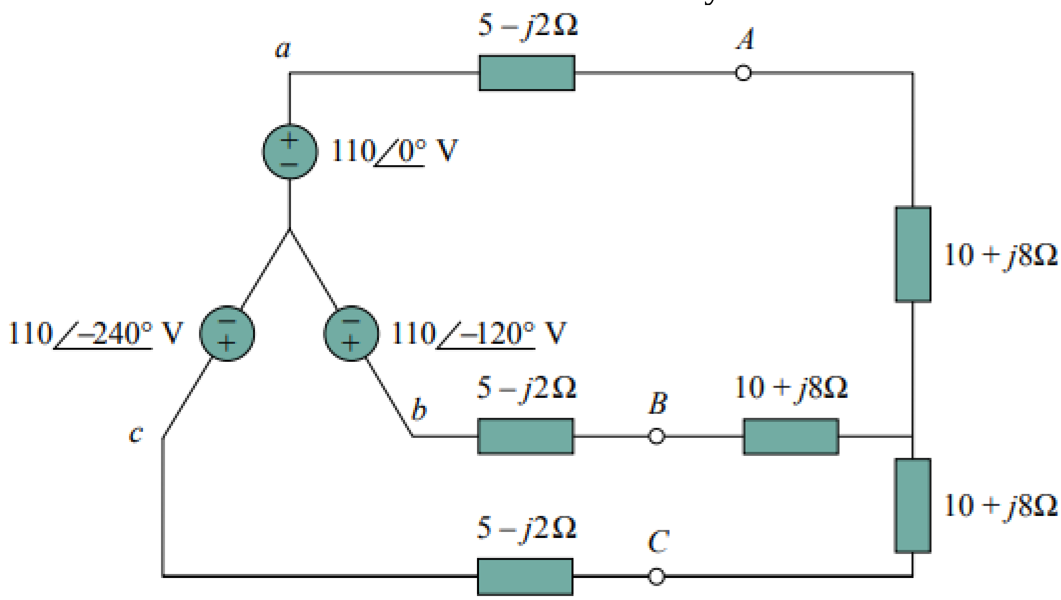
The voltages can be expressed in phasor form as

$$\mathbf{V}_{an} = 200\angle 10^\circ, \qquad \mathbf{V}_{bn} = 200\angle -230^\circ, \qquad \mathbf{V}_{cn} = 200\angle -110^\circ$$

We notice that $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ by 120° and $\mathbf{V}_{cn}$ in turn leads $\mathbf{V}_{bn}$ by 120°. Hence, we have an *acb* sequence.

**Problem # 2:**

Calculate the line currents in the three-wire Y-Y system shown below.



**Solution:**

The three-phase circuit is balanced. This can be replaced by its single-phase equivalent circuit such. $I_a$ from the single-phase analysis as

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.155\angle 21.8^\circ$. Hence,

$$\mathbf{I}_a = \frac{110\angle 0^\circ}{16.155\angle 21.8^\circ} = 6.81\angle -21.8^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 6.81\angle -141.8^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle -240^\circ = 6.81\angle -261.8^\circ \text{ A} = 6.81\angle 98.2^\circ \text{ A}$$

**Problem # 3:**

A balanced *abc*-sequence Y-connected source with $\mathbf{V}_{an} = 100\angle 10^\circ$ V is connected to a Δ-connected balanced load $(8 + j4)$ Ω per phase.

Calculate the phase and line currents at the load side.

**Solution:**

This can be solved in two ways.

**Method 1:**

The load impedance is

$$\mathbf{Z}_\Delta = 8 + j4 = 8.944\angle 26.57^\circ \ \Omega$$

If the phase voltage $\mathbf{V}_{an} = 100\angle 10^\circ$, then the line voltage is

$$\mathbf{V}_{ab} = \mathbf{V}_{an}\sqrt{3}\angle 30^\circ = 100\sqrt{3}\angle 10^\circ + 30^\circ = \mathbf{V}_{AB}$$

or

$$\mathbf{V}_{AB} = 173.2\angle 40^\circ \text{ V}$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{173.2\underline{/40^\circ}}{8.944\underline{/26.57^\circ}} = 19.36\underline{/13.43^\circ} \text{ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\underline{/-120^\circ} = 19.36\underline{/-106.57^\circ} \text{ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\underline{/+120^\circ} = 19.36\underline{/133.43^\circ} \text{ A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ} = \sqrt{3}(19.36)\underline{/13.43^\circ - 30^\circ}$$

$$= 33.53\underline{/-16.57^\circ} \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 33.53\underline{/-136.57^\circ} \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ} = 33.53\underline{/103.43^\circ} \text{ A}$$

**Method 2:**

Use Delta-Y transformation

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_\Delta/3} = \frac{100\underline{/10^\circ}}{2.981\underline{/26.57^\circ}} = 33.54\underline{/-16.57^\circ} \text{ A}$$

as above. Other line currents are obtained using the *abc* phase sequence.

**Problem # 4:**

A balanced $\Delta$-connected load having an impedance $20 - j15\ \Omega$ is connected to a $\Delta$-connected, positive-sequence generator having $\mathbf{V}_{ab} = 330\underline{/0^\circ}$ V. Calculate the phase currents of the load and the line currents.

**Solution:**

The load impedance per phase is

$$\mathbf{Z}_\Delta = 20 - j15 = 25\angle -36.87^\circ\ \Omega$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{330\angle 0^\circ}{25\angle -36.87} = 13.2\angle 36.87^\circ\ \text{A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^\circ = 13.2\angle -83.13^\circ\ \text{A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle +120^\circ = 13.2\angle 156.87^\circ\ \text{A}$$

For a delta load, the line current always lags the corresponding phase current by 30° and has a magnitude $\sqrt{3}$ times that of the phase current. Hence, the line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ = (13.2\angle 36.87^\circ)(\sqrt{3}\angle -30^\circ)$$

$$= 22.86\angle 6.87^\circ\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 22.86\angle -113.13^\circ\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ = 22.86\angle 126.87^\circ\ \text{A}$$

**Problem # 5:**

A three-phase motor can be regarded as a balanced Y-load. A three-phase motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Determine the power factor of the motor.

**Solution:**

The apparent power is

$$S = \sqrt{3}V_L I_L = \sqrt{3}(220)(18.2) = 6935.13\ \text{VA}$$

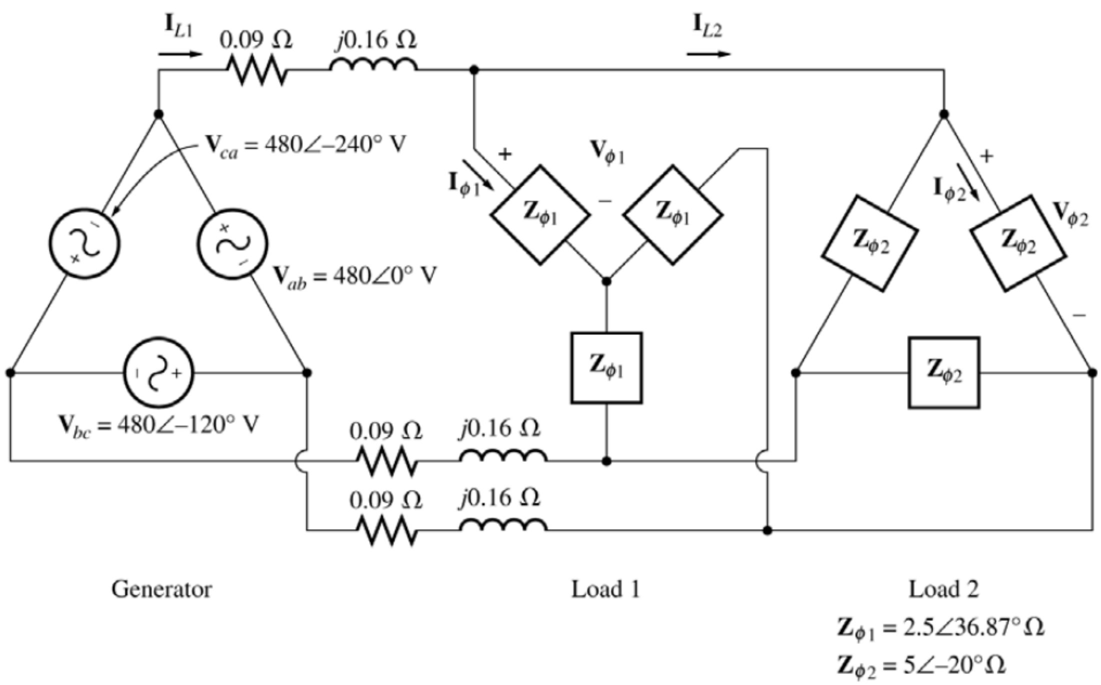
Since the real power is

$$P = S\cos\theta = 5600\ \text{W}$$

$$\text{pf} = \cos\theta = \frac{P}{S} = \frac{5600}{6935.13} = 0.8075 \quad \text{lagging}$$

**Problem # 6:**

Consider the three-phase circuit below




*(a)* What is the line voltage of the two loads?

*(b)* What is the voltage drop on the transmission lines?

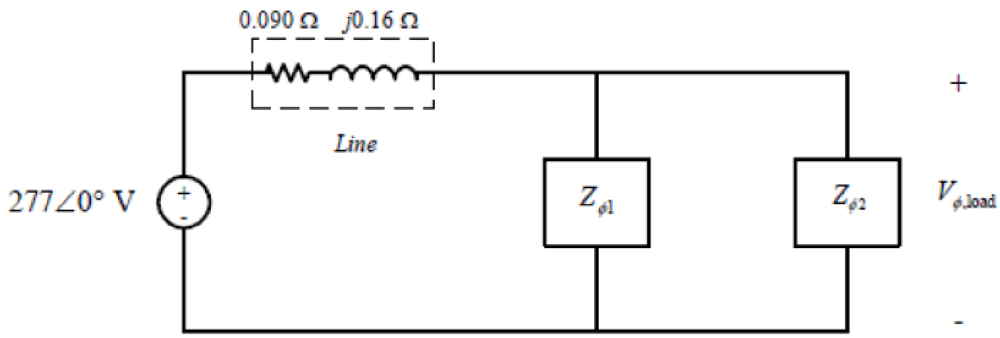
*(c)* Find the real and reactive powers supplied to each load.

*(d)* Find the real and reactive power losses in the transmission line.

*(e)* Find the real power, reactive power, and power factor supplied by the generator.

**Solution:**

To solve this problem, first convert the delta-connected load 2 to an equivalent wye (by dividing the impedance by 3), and get the per-phase equivalent circuit.




*(a)* The phase voltage of the equivalent Y-loads can be found by nodal analysis.

$$\frac{\mathbf{V}_{\phi,\text{load}} - 277\angle 0°\text{ V}}{0.09 + j0.16\ \Omega} + \frac{\mathbf{V}_{\phi,\text{load}}}{2.5\angle 36.87°\ \Omega} + \frac{\mathbf{V}_{\phi,\text{load}}}{1.67\angle -20°\ \Omega} = 0$$

$$(5.443\angle -60.6°)\ \left(\mathbf{V}_{\phi,\text{load}} - 277\angle 0°\text{ V}\right) + (0.4\angle -36.87°)\mathbf{V}_{\phi,\text{load}} + (0.6\angle 20°)\mathbf{V}_{\phi,\text{load}} = 0$$

$$(5.955\angle -53.34°)\ \mathbf{V}_{\phi,\text{load}} = 1508\angle -60.6°$$

$$\mathbf{V}_{\phi,\text{load}} = 253.2\angle -7.3°\text{ V}$$

Therefore, the line voltage at the loads is $V_L\sqrt{3}\ V_\phi = 439\text{ V}$.

*(b)* The voltage drop in the transmission lines is

$$\Delta\mathbf{V}_{\text{line}} = \mathbf{V}_{\phi,\text{gen}} - \mathbf{V}_{\phi,\text{load}} = 277\angle 0°\text{ V} - 253.2\angle -7.3° = 41.3\angle 52°\text{ V}$$

*(c)* The real and reactive power of each load is

$$P_1 = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(253.2\text{ V})^2}{2.5\ \Omega}\cos\ 36.87° = 61.6\text{ kW}$$

$$Q_1 = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(253.2\text{ V})^2}{2.5\ \Omega}\sin 36.87° = 46.2\text{ kvar}$$

$$P_2 = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(253.2\text{ V})^2}{1.67\ \Omega}\cos\ (-20°) = 108.4\text{ kW}$$

$$Q_2 = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(253.2\text{ V})^2}{1.67\ \Omega}\sin\ (-20°) = -39.5\text{ kvar}$$

*(d)* The line current is

$$\mathbf{I}_{\text{line}} = \frac{\Delta \mathbf{V}_{\text{line}}}{Z_{\text{line}}} = \frac{41.3\angle 52^\circ \text{ V}}{0.09 + j0.16\,\Omega} = 225\angle -8.6^\circ \text{A}$$

Therefore, the loses in the transmission line are

$$P_{\text{line}} = 3I_{\text{line}}^{\ 2}R_{\text{line}} = 3\ (225 \text{ A})^2 (0.09\ \Omega) = 13.7 \text{ kW}$$

$$Q_{\text{line}} = 3I_{\text{line}}^{\ 2}X_{\text{line}} = 3\ (225 \text{ A})^2 (0.16\ \Omega) = 24.3 \text{ kvar}$$

*(e)* The real and reactive power supplied by the generator is

$$P_{\text{gen}} = P_{\text{line}} + P_1 + P_2 = 13.7 \text{ kW} + 61.6 \text{ kW} \ + \ 108.4 \text{ kW} = 183.7 \text{ kW}$$

$$Q_{\text{gen}} = Q_{\text{line}} + Q_1 + Q_2 = 24.3 \text{ kvar} + 46.2 \text{ kvar} \ - \ 39.5 \text{ kvar} = 31 \text{ kvar}$$

The power factor of the generator is

$$\text{PF} = \cos\left[\tan^{-1}\frac{Q_{\text{gen}}}{P_{\text{gen}}}\right] = \cos\left[\tan^{-1}\frac{31 \text{ kvar}}{183.7 \text{ kW}}\right] = 0.986 \text{ lagging}$$

## Problem # 7:

A single phase electrical load draws 10 MW at 0.6 power factor lagging.

a. Find the real and reactive power absorbed by the load
b. Draw the power triangle.
c. Determine the kVAR of a capacitor to be connected across the load to raise the power factor to 0.95.

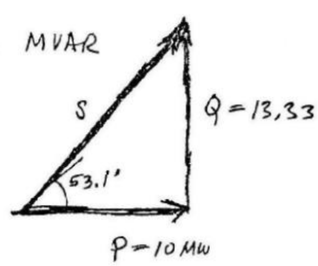
**Solution:**

$P = 10$ MW, PF = 0,6 lagging

$\theta = \cos^{-1} 0.6 = 53.1°$

(a) $P = 10$ MW

$Q = P \tan\theta = 10 \tan 53.1° = 13.33$ MVAR




(c) $\theta_{new} = \cos^{-1} 0.95 = 18.2°$

$Q_{new} = P \tan\theta_{new} = 10 \tan 18.2°$

$= 3.29$ MVAR

$= Q_{old} + Q_{cap}$

$Q_{cap} = 3.29 - 13.33 = -10$ MVAR $= -10{,}000$ KVAR