KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

EE 306 – Term 172

HW # 1: Three-Phase Circuits
ST Classes Due: February 4th ; MW Classes February 5th , 2018

Key Solution

Problem # 1: (1-point)

Given the number $A_1 = 5\angle 30^o$ (in polar form) and $A_2 = -3 + j4$ (in rectangular form). Calculate the following, given the answers in both rectangular and polar forms:

a. $A_1 + A_2$
b. $A_1 * A_2$
c. $A_1/(A_2)^*$

Solution:

$$A_1 = 5\angle 30^\circ = 4.33 + j2.50$$

$$A_2 = -3 + j4 = 5\angle 126.9^\circ$$

(a) $A_1 + A_2 = 1.33 + j6.5 = 6.63\angle 78.4^\circ$

(b) $A_1 A_2 = 25\angle 156.9^\circ = -23 + j9.8$

(c) $A_1/A_2^* = 1\angle 156.9^\circ = -0.92 + j0.39$

Problem # 2: (1-point)

A load with an impedance of $Z = 25\angle 53.1^{o}\,\Omega$ is fed from a single-phase source of 220V.

a. Find the resistance and reactance of the load.
b. Find the real (active) and imaginary (reactive) power of the load.
c. Find the power factor of the load, and state whether it is lagging or leading.

Solution:

(a) $Z = 15 + j20$

$R = 15\ \Omega$

$X = 20\ \Omega$

(b) $I = \dfrac{V}{Z} = \dfrac{220\angle 0^\circ}{25\angle 53.1^\circ} = 8.8\angle -53.1^\circ$

(b) $S = VI^* = (220\angle 0^\circ)(8.8\angle -53.1^\circ)^* = 1936\angle 53.1^\circ = 1162 + j\,1549$ VA

$P = 1162$ W

$Q = 1549$ VAR

(c) $PF = \cot 53.1^\circ = 0.6$ lagging

Problem # 3: (2-points)

A delta connected load has per-phase impedance of $45\angle 60°\ \Omega$ is fed a 208-V 3-phase substation through a 3-phase feeder. The per-phase impedance of the feeder is (1.2 + j 1.6) Ω. Calculate the line to line voltage at the load terminals.

Solution:

$$Z_\Delta = 45\angle 60°\ \Omega \quad ; \quad Z_{fdr} = 1.2 + j1.6\ \Omega$$

$$Z_Y = \frac{1}{3} Z_\Delta = 15\angle 60° = 7.5 + j13\ \Omega$$

$$Z_T = Z_{fdr} + Z_Y = 7.5 + j13 + 1.2 + j1.6 = 8.7 + j14.6\ \Omega$$

$$I = \frac{V_{ph}}{Z_T} = \frac{(208/\sqrt{3})\angle 0°}{8.7 + j14.6} = 7.06\angle -59.2°\ A$$

$$V_{LOAD} = V_{ph} - Z_{fdr} I = 120\angle 0° - (1.2 + j1.6)(7.06\angle -59.2°)$$

$$= 120 + j0 - (14.04 - j1.493) = 106\angle 0.81°\ V_{LN}$$

$$= 183.6\angle 30.81°\ V_{LL}$$

Problem # 4: (2-points)

A 345-kV, 3-phase transmission line delivers 500 MVA, 0.866 power factor lagging, to a 3-phase star-connected load.

a. Find the line and phase currents drawn by the load.
b. Find the per-phase impedance of the load in polar form.
c. Find the total active and reactive power of the load.

Solution:

ⓐ $I_{ph} = \frac{500{,}000}{\sqrt{3}\,(345)} \angle -\cos^{-1} 0.866 = 836.74 \angle -30^\circ \ A$

$V_{ph} = \frac{345}{\sqrt{3}} \angle 0^\circ = 199.2 \angle 0^\circ \ V$

$I_L = I_{ph} = 836.74 \angle -30^\circ \ A$

ⓑ $Z_{Y,ph} = \frac{199.2 \angle 0^\circ}{836.74 \angle -30^\circ} = 238 \angle 30^\circ \ \Omega$

ⓒ $P_T = 433 \ MW \ ; \ Q_T = 250 \ MVAR$

Problem # 5: (2-points)

A 3-phase motor draws 40 kVA at 0.65 power factor lagging from a 230-V source. A capacitor bank (i.e., 3-phase capictors) is connected across (i.e., in parallel) the motor terminals to make the compined power factor 0.95 lagging.

a. Find the required KVAR rating of the capacitor bank.
b. Find the line current before and after the capacitors are added.

Solution:

$$S_M = 40\ kVA\ ,\ V_M = 230\ V\ ,\ PF_M = 0.65\ \text{lagging}$$

$$P_M = 40\,(0.65) = 26\ kW$$

$$\theta_M = \cos^{-1} 0.65 = 49.46°$$

ⓐ $$Q_M = P_M \tan\theta_M = 26 \tan 49.46° = 30.4\ kVAR$$

$$PF_{new} = 0.95$$

$$\theta_{new} = \cos^{-1} 0.95 = 18.19°$$

$$Q_{new} = P_M \tan\theta_{new} = 8.54\ kVAR = Q_M + Q_C$$

$$\therefore\ Q_C = Q_{new} - Q_M = 8.54 - 30.4 = -21.86\ kVAR$$

ⓑ $$I_{before} = \frac{40{,}000}{\sqrt{3}\,(230)} \angle -\cos^{-1} 0.65 = 100 \angle -49.46°$$

$$I_{after} = \frac{26{,}000}{\sqrt{3}\,(230)(0.95)} \angle -\cos^{-1} 0.95 = 68.7 \angle -18.19°$$

Problem # 6: (2-points)

A balanced 3-phase, 173-V, 60-Hz source supplies the two following loads:

- A Δ-connected load with a phase impedance of (18+*j*24) Ω,
- A Y-connected load with a phase impedance of 10∠53.13° Ω.

Find:

a. The power factor of the entire load.
b. The total line current supplied.
c. The total real, reactive, and apparent powers.

Solution:

Convert Δ to Y $$Z_y = \frac{18 + j\,24}{3} = 6 + j\,8$$

Parallel combination of the 2 loads (per phase)

$$Z_T = \frac{(6 = j\,8)(10\angle 53.1^0)}{6 + j\,8 + 10\angle 53.1^0} = 5\angle 53.1^0$$

a. Power factor= $\cos(53.1^0)$ = 0.6 lag

b. $$I_L = I_{ph} = \frac{173/\sqrt{3}\angle 0^0}{5\angle 53.1^0} = 20\angle -53.1^0 A$$

c.

$$P_T = \sqrt{3}x\,173x\,20x\,0.6 = 3.596kW$$

$$Q_T = \sqrt{3}x\,173x\,20x\,0.8 = 4.794kVAR$$

$$|S_T| = \sqrt{3}x\,173x\,20 = 5.993kVA$$