King Fahd University of Petroleum & Minerals

Electrical Engineering Department
Semester-162
EE 306: Electromechanical Devices

Homework-VI
(Synchronous Machines)

Problem 1:

A 13.8-kV, 50-MVA, 0.9-power-factor-lagging, 60-Hz, four-pole Y-connected synchronous generator has a synchronous reactance of 2.5 Ω and an armature resistance of 0.2 Ω. At 60 Hz, its friction and windage losses are 1 MW, and its core losses are 1.5 MW. The field circuit has a dc voltage of 120 V, and the maximum I_F is 10 A. The current of the field circuit is adjustable over the range from 0 to 10 A. The OCC of this generator is shown in Figure P -1.

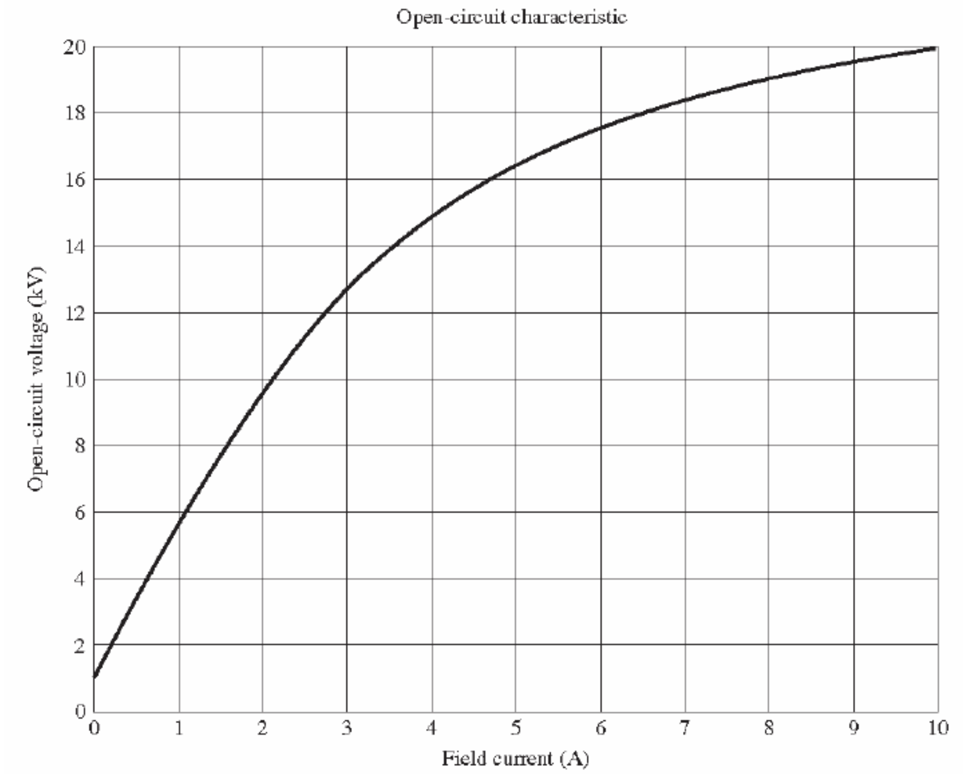


Fig P-1

(a) How much field current is required to make the terminal voltage V_T (or line voltage V_L) equal to 13.8 kV when the generator is running at no load?

(b) What is the internal generated voltage E_A of this machine at rated conditions?

(c) What is the phase voltage V_ϕ of this generator at rated conditions?

(d) How much field current is required to make the terminal voltage V_T equal to 13.8 kV when the generator is running at rated conditions?

(e) Suppose that this generator is running at rated conditions, and then the load is removed without changing the field current. What would the terminal voltage of the generator be?

(f) How much steady-state power and torque must the generator's prime mover be capable of supplying to handle the rated conditions?

Solution:

(a) If the no-load terminal voltage is 13.8 kV, the required field current can be read directly from the open-circuit characteristic. It is 3.50 A.

(b) This generator is Y-connected, so $I_L = I_A$. At rated conditions, the line and phase current in this generator is

$$I_A = I_L = \frac{P}{\sqrt{3}\,V_L} = \frac{50\text{ MVA}}{\sqrt{3}\,(13800\text{ V})} = 2092\text{ A at an angle of } -25.8°$$

The phase voltage of this machine is $V_\phi = V_T / \sqrt{3} = 7967\text{ V}$. The internal generated voltage of the machine is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A \mathbf{I}_A + jX_S \mathbf{I}_A$$
$$\mathbf{E}_A = 7967\angle 0° + (0.20\ \Omega)(2092\angle -25.8°\text{ A}) + j(2.5\ \Omega)(2092\angle -25.8°\text{ A})$$
$$\mathbf{E}_A = 11544\angle 23.1°\text{ V}$$

(c) The phase voltage of the machine at rated conditions is $V_\phi = 7967\text{ V}$

From the OCC, the required field current is 10 A.

(d) The equivalent open-circuit terminal voltage corresponding to an E_A of 11544 volts is

$$V_{T,\text{oc}} = \sqrt{3}\,(11544\text{ V}) = 20\text{ kV}$$

From the OCC, the required field current is 10 A.

(e) If the load is removed without changing the field current, $V_\phi = E_A = 11544\text{ V}$. The corresponding terminal voltage would be 20 kV.

(f) The input power to this generator is equal to the output power plus losses. The rated output power is

$$P_{\text{OUT}} = (50\text{ MVA})(0.9) = 45\text{ MW}$$

$$P_{\text{CU}} = 3I_A^{\,2}R_A = 3(2092\text{ A})^2(0.2\ \Omega) = 2.6\text{ MW}$$

$$P_{\text{F\&W}} = 1\text{ MW}$$

$$P_{\text{core}} = 1.5\text{ MW}$$

$$P_{\text{stray}} = (\text{assumed } 0)$$

$$P_{\text{IN}} = P_{\text{OUT}} + P_{\text{CU}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{stray}} = 50.1\text{ MW}$$

Therefore the prime mover must be capable of supplying 50.1 MW. Since the generator is a four-pole 60 Hz machine, to must be turning at 1800 r/min. The required torque is

$$\tau_{\text{APP}} = \frac{P_{\text{IN}}}{\omega_m} = \frac{50.1\text{ MW}}{(1800\text{ r/min})\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ r}}\right)} = 265{,}800\text{ N}\cdot\text{m}$$

Problem-2:

Assume that the field current of the generator in Problem 1 is adjusted to achieve rated voltage (13.8 kV) at full load conditions in each of the questions below.

(a) What is the efficiency of the generator at rated load?

(b) What is the voltage regulation of the generator if it is loaded to rated kilovoltamperes with 0.9-PF-lagging loads?

(c) What is the voltage regulation of the generator if it is loaded to rated kilovoltamperes with 0.9-PF-leading loads?

(d) What is the voltage regulation of the generator if it is loaded to rated kilovoltamperes with unity-power-factor loads?

SOLUTION

(a) This generator is Y-connected, so $I_L = I_A$. At rated conditions, the line and phase current in this generator is

$$I_A = I_L = \frac{P}{\sqrt{3}\,V_L} = \frac{50\text{ MVA}}{\sqrt{3}\,(13800\text{ V})} = 2092\text{ A at an angle of } -25.8°$$

The phase voltage of this machine is $V_\phi = V_T / \sqrt{3} = 7967\text{ V}$. The internal generated voltage of the machine is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = 7967\angle 0° + (0.20\ \Omega)(251\angle -36.87°\text{ A}) + j(2.5\ \Omega)(2092\angle -25.8°\text{ A})$$

$$\mathbf{E}_A = 11544\angle 23.1°\text{ V}$$

The input power to this generator is equal to the output power plus losses. The rated output power is

$$P_{\text{OUT}} = (50\text{ MVA})(0.9) = 45\text{ MW}$$

$$P_{\text{CU}} = 3I_A^{\,2}R_A = 3(2092\text{ A})^2(0.2\ \Omega) = 2.6\text{ MW}$$

$$P_{\text{F\&W}} = 1\text{ MW}$$

$$P_{\text{core}} = 1.5\text{ MW}$$

$$P_{\text{stray}} = (\text{assumed } 0)$$

$$P_{\text{IN}} = P_{\text{OUT}} + P_{\text{CU}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{stray}} = 50.1\text{ MW}$$

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{45\text{ MW}}{50.1\text{ MW}} \times 100\% = 89.8\%$$

(b) If the generator is loaded to rated MVA with lagging loads, the phase voltage is $\mathbf{V}_\phi = 7967\angle 0°\text{ V}$ and the internal generated voltage is $\mathbf{E}_A = 11544\angle 23.1°\text{ V}$. Therefore, the phase voltage at no-load would be $\mathbf{V}_\phi = 11544\angle 0°\text{ V}$. The voltage regulation would be:

$$\text{VR} = \frac{11544 - 7967}{7967} \times 100\% = 44.9\%$$

(c) If the generator is loaded to rated kVA with leading loads, the phase voltage is $\mathbf{V}_\phi = 7967\angle 0°$ V and the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 7967\angle 0° + (0.20\ \Omega)(2092\angle 25.8°\ \text{A}) + j(2.5\ \Omega)(2092\angle 25.8°\ \text{A})$$
$$\mathbf{E}_A = 7793\angle 38.8°\ \text{V}$$

The voltage regulation would be:

$$\text{VR} = \frac{7793 - 7967}{7967} \times 100\% = -2.2\%$$

(d) If the generator is loaded to rated kVA at unity power factor, the phase voltage is $\mathbf{V}_\phi = 7967\angle 0°$ V and the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 7967\angle 0° + (0.20\ \Omega)(2092\angle 0°\ \text{A}) + j(2.5\ \Omega)(2092\angle 0°\ \text{A})$$
$$\mathbf{E}_A = 9883\angle 32°\ \text{V}$$

The voltage regulation would be:

$$\text{VR} = \frac{9883 - 7967}{7967} \times 100\% = 24\%$$

Problem-3:

The internal generated voltage E_A of a **2-pole, Δ-connected, 60 Hz,** three phase synchronous generator is 14.4 kV, and the terminal voltage V_T is 12.8 kV. The synchronous reactance of this machine is 4 Ω, and the armature resistance can be ignored.

(a) If the torque angle of the generator $\delta = 18°$, how much power is being supplied by this generator at the current time?

(b) What is the power factor of the generator at this time?

(c) Sketch the phasor diagram under these circumstances.

(d) Ignoring losses in this generator, what torque must be applied to its shaft by the prime mover at these conditions?

SOLUTION

(a) If resistance is ignored, the output power from this generator is given by

$$P = \frac{3V_\phi E_A}{X_S}\sin\delta = \frac{3(12.8\text{ kV})(14.4\text{ kV})}{4\ \Omega}\sin 18° = 42.7\text{ MW}$$

(b) The phase current flowing in this generator can be calculated from

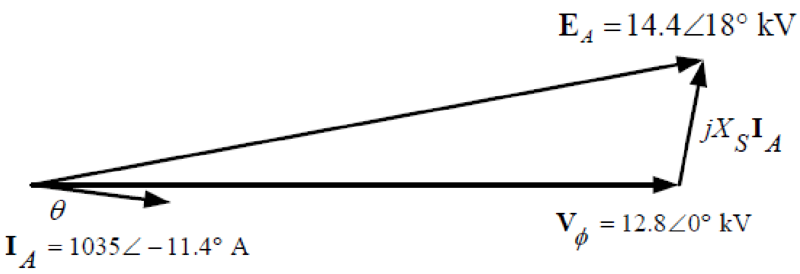
$$\mathbf{E}_A = \mathbf{V}_\phi + jX_S\mathbf{I}_A$$

$$\mathbf{I}_A = \frac{\mathbf{E}_A - \mathbf{V}_\phi}{jX_S}$$

$$\mathbf{I}_A = \frac{14.4\angle 18°\text{ kV} - 12.8\angle 0°\text{ kV}}{j4\ \Omega} = 1135\angle -11.4°\text{ A}$$

Therefore the impedance angle $\theta = 11.4°$, and the power factor is $\cos(11.4°) = 0.98$ lagging.

(c) The phasor diagram is



(d) The induced torque is given by the equation

$$P_{\text{conv}} = \tau_{\text{ind}}\omega_m$$

With no losses,

$$\tau_{\text{app}} = \tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{42.7\text{ MW}}{2\pi(60\text{ hz})} = 113{,}300\text{ N}\cdot\text{m}$$

Problem-4:

A 230-V, 50 Hz, two-pole synchronous motor draws 40 A from the line at unity power factor and full load. Assuming that the motor is lossless, answer the following questions:

(a) What is the output torque of this motor?

(b) What must be done to change the power factor to 0.85 leading? Explain your answer, using phasor diagrams.

(c) What will the magnitude of the line current be if the power factor is adjusted to 0.85 leading?

SOLUTION

(a) If this motor is assumed lossless, then the input power is equal to the output power. The input power to this motor is

$$P_{\mathrm{IN}} = \sqrt{3} V_T I_L \cos\theta = \sqrt{3}(230\text{ V})(40\text{ A})(1.0) = 15.93\text{ kW}$$

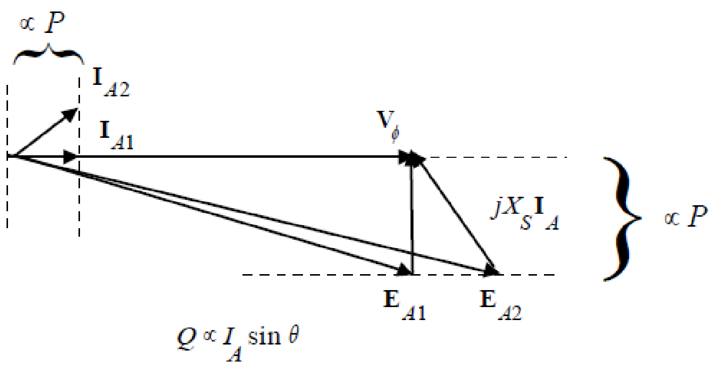
The rotational speed of the motor is

$$n_m = \frac{120 f_{se}}{P} = \frac{120(50\text{ Hz})}{4} = 1500\text{ r/min}$$

The output torque would be

$$\tau_{\mathrm{LOAD}} = \frac{P_{\mathrm{OUT}}}{\omega_m} = \frac{15.93\text{ kW}}{(1500\text{ r/min})\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)} = 101.4\text{ N}\cdot\text{m}$$

(b) To change the motor's power factor to 0.8 leading, its field current must be increased. Since the power supplied to the load is independent of the field current level, an increase in field current increases $|\mathbf{E}_A|$ while keeping the distance $E_A \sin\delta$ constant. This increase in E_A changes the angle of the current $\mathbf{I}_A$, eventually causing it to reach a power factor of 0.8 leading.



(c) The magnitude of the line current will be

$$I_L = \frac{P}{\sqrt{3}\, V_T\, \mathrm{PF}} = \frac{15.93\text{ kW}}{\sqrt{3}(230\text{ V})(0.8)} = 50.0\text{ A}$$

Problem-5:

A synchronous machine has a synchronous reactance of 1.0 Ω per phase and an armature resistance of 0.1 Ω per phase. If $\mathbf{E}_A = 460\angle\text{-}10°$ V and $\mathbf{V}_\phi = 480\angle 0°$ V, is this machine a motor or a generator? How much power *P* is this machine consuming from or supplying to the electrical system? How much reactive power *Q* is this machine consuming from or supplying to the electrical system?

SOLUTION This machine is a motor, *consuming* power from the power system, because $\mathbf{E}_A$ is lagging $\mathbf{V}_\phi$. It is also *consuming* reactive power, because $E_A \cos\delta < V_\phi$. The current flowing in this machine is

$$\mathbf{I}_A = \frac{\mathbf{V}_\phi - \mathbf{E}_A}{R_A + jX_S} = \frac{480\angle 0°\text{ V} - 460\angle -10°\text{ V}}{0.1 + j1.0\ \Omega} = 83.9\angle -13°\text{ A}$$

Therefore the real power consumed by this motor is

$$P = 3V_\phi I_A \cos\theta = 3(480\text{ V})(89.3\text{ A})\cos(13°) = 125.3\text{ kW}$$

and the reactive power consumed by this motor is

$$Q = 3V_\phi I_A \sin\theta = 3(480\text{ V})(89.3\text{ A})\sin(13°) = 28.9\text{ kVAR}$$