KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

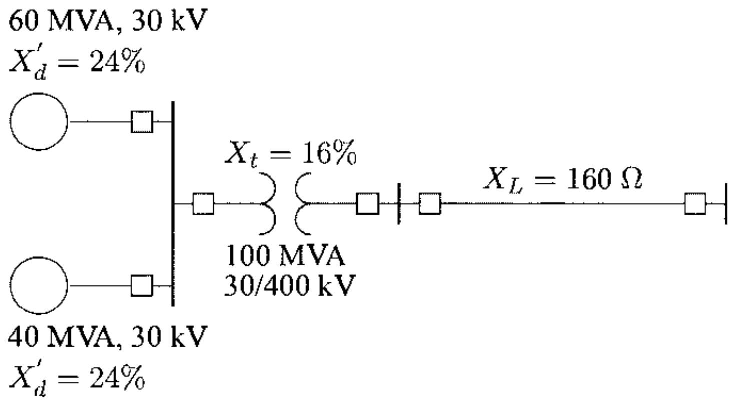
EE-520 (141)

Dr. Ibrahim O. Habiballah

Key Solutions

Home Work 3 (Due Date: December 17, 2014)

Q.1) The system shown below is initially on no load with generators operating at their rated voltage with their emfs in phase. The rating of the generators and the transformers and their respective percent reactances are marked on the diagram. All resistances are neglected. The line impedance is $j160$ (+ your two-digit serial no.) Ohm. A three-phase balanced fault occurs at the receiving end of the transmission line. Determine the short circuit current and the short-circuit MVA.



Solution:

The base impedance for line is

$$Z_B = \frac{(400)^2}{100} = 1,600 \ \ \Omega$$

and the base current is

$$I_B = \frac{100,000}{\sqrt{3}(400)} = 144.3375 \ \ \text{A}$$

The reactances on a common 100 MVA base are

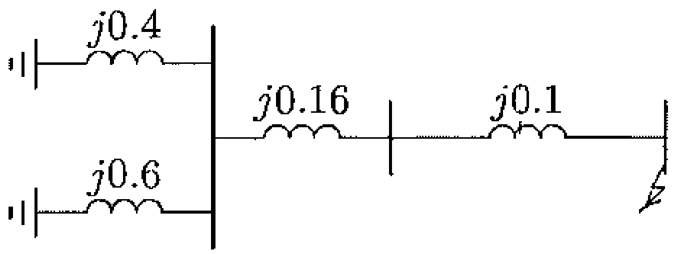
$$X'_{dg1} = \frac{100}{60}(0.24) = 0.4 \ \ \text{pu}$$

$$X'_{dg2} = \frac{100}{40}(0.24) = 0.6 \ \ \text{pu}$$

$$X_t = \frac{100}{100}(0.16) = 0.16 \ \ \text{pu}$$

$$X_{line} = \frac{160}{1600} = 0.1 \ \ \text{pu}$$

The impedance diagram is



Impedance to the point of fault is

$$X = j\frac{(0.4)(0.6)}{0.4+0.6} + j0.16 + j0.1 = j0.5 \text{ pu}$$

The fault current is

$$\begin{aligned} I_f &= \frac{1}{j0.5} = 2\angle{-90^\circ} \text{ pu} \\ &= (144.3375)(2\angle{-90^\circ}) = 288.675\angle{-90^\circ} \text{ A} \end{aligned}$$

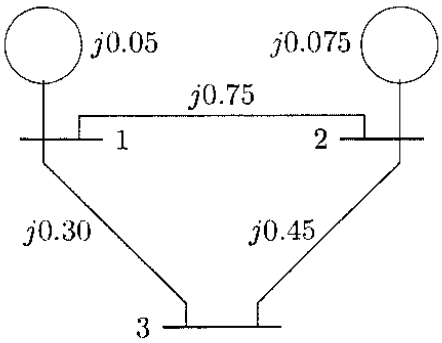
The Short-circuit MVA is

$$\text{SCMVA} = \sqrt{3}(400)(288.675)(10^{-3}) = 200 \text{ MVA}$$

Q.2) The one-line diagram of a simple three-bus power system is shown below. Each generator is represented by an emf behind the subtransient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 3 through a fault impedance of $Z_f = j0.19$ (+ 0.your two-digit serial no.) per unit.

a. Use circuit analysis to determine the bus voltages and line currents during fault.

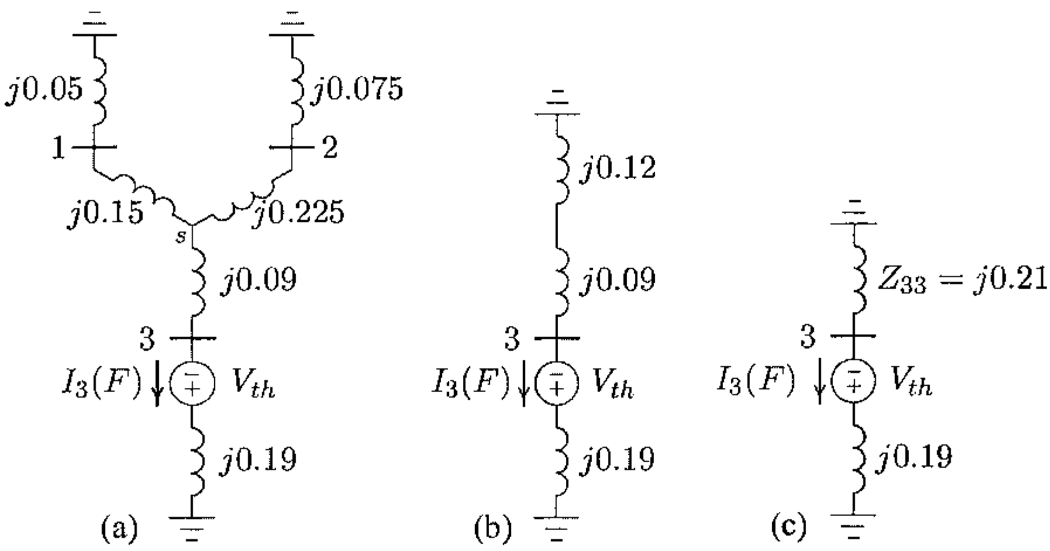
b. Use the bus impedance matrix to calculate the bus voltages and line currents during fault.



Solution:

a.

Convert the Δ formed by buses 123 to an equivalent Y as shown



$$Z_{1s} = \frac{(j0.3)(j0.75)}{j1.5} = j0.15$$

$$Z_{2s} = \frac{(j0.75)(j0.45)}{j1.5} = j0.225$$

$$Z_{3s} = \frac{(j0.3)(j0.45)}{j1.5} = j0.09$$

Combining the parallel branches, Thévenin's impedance is

$$Z_{33} = \frac{(j0.2)(j0.3)}{j0.2 + j0.3} + j0.09$$
$$= j0.12 + j0.09 = j0.21$$

From Figure (c), the fault current is

$$I_3(F) = \frac{V_3(F)}{Z_{33} + Z_f} = \frac{1.0}{j0.21 + j0.19} = -j2.5 \quad \text{pu}$$

With reference to Figure (a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.3}{j0.2 + j0.3} I_3(F) = -j1.5 \quad \text{pu}$$
$$I_{G2} = \frac{j0.2}{j0.2 + j0.3} I_3(F) = -j1.0 \quad \text{pu}$$

For the bus voltage changes from Figure 72(a), we get

$$\Delta V_1 = 0 - (j0.05)(-j1.5) = -0.075 \;\text{ pu}$$
$$\Delta V_2 = 0 - (j0.075)(-j1) = -0.075 \;\text{ pu}$$
$$\Delta V_3 = (j0.19)(-j2.5) - 1.0 = -0.525 \;\text{ pu}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, i.e.,

$$V_1(F) = V_1(0) + \Delta V_1 = 1.0 - 0.075 = 0.925 \;\text{ pu}$$
$$V_2(F) = V_2(0) + \Delta V_2 = 1.0 - 0.075 = 0.925 \;\text{ pu}$$
$$V_3(F) = V_3(0) + \Delta V_3 = 1.0 - 0.525 = 0.475 \;\text{ pu}$$

The short circuit-currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.925 - 0.925}{j0.75} = 0 \quad \text{pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.925 - 0.475}{j0.3} = -j1.5 \quad \text{pu}$$
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.925 - 0.475}{j0.45} = -j1.0 \quad \text{pu}$$

b.

The per unit bus impedance matrix for the power system is

$$Z_{bus} = j\begin{bmatrix} 0.0450 & 0.0075 & 0.0300 \\ 0.0075 & 0.06375 & 0.0300 \\ 0.0300 & 0.0300 & 0.2100 \end{bmatrix}$$

For a fault at bus 3 with fault impedance $Z_f = j0.19$ per unit, the fault current is

$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f} = \frac{1.0}{j0.21 + j0.19} = -j2.5 \quad \text{pu}$$

bus voltages during the fault are

$$V_1(F) = V_1(0) - Z_{13}I_3(F) = 1.0 - (j0.03)(-j2.5) = 0.925 \ \text{ pu}$$
$$V_2(F) = V_2(0) - Z_{23}I_3(F) = 1.0 - (j0.03)(-j2.5) = 0.925 \ \text{ pu}$$
$$V_3(F) = V_3(0) - Z_{33}I_3(F) = 1.0 - (j0.21)(-j2.5) = 0.475 \ \text{ pu}$$

the short circuit currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.925 - 0.925}{j0.75} = 0 \ \text{ pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.925 - 0.475}{j0.3} = -j1.5 \ \text{ pu}$$
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.925 - 0.475}{j0.45} = -j1.0 \ \text{ pu}$$

Q.3) The operator a is defined as $a = 1 < 120^{o}$, drive the formulation for the following relationship:

a. The positive-sequence of phase voltage V_{an} in terms of the positive-sequence of line voltage V_{bc}.

b. The negative-sequence of phase voltage V_{an} in terms of the negative-sequence of line voltage V_{bc}.

Solution:

a.

$$\begin{aligned} V_{bc}^1 = V_{bn}^1 - V_{cn}^1 = a^2 V_{an}^1 - a V_{an}^1 &= (a^2 - a) V_{an}^1 \\ &= (-0.5 - j0.866 + 0.5 - j0.866) V_{an}^1 \\ &= \sqrt{3} \angle -90^\circ V_{an}^1 \end{aligned}$$

or

$$V_{an}^1 = \frac{1}{\sqrt{3}} V_{bc}^1 \angle 90^\circ$$

b.

$$\begin{aligned} V_{bc}^2 = V_{bn}^2 - V_{cn}^2 = a V_{an}^2 - a^2 V_{an}^2 &= (a - a^2) V_{an}^2 \\ &= (-0.5 + j0.866 + 0.5 + j0.866) V_{an}^2 \\ &= \sqrt{3} \angle 90^\circ V_{an}^2 \end{aligned}$$

or

$$V_{an}^2 = \frac{1}{\sqrt{3}} V_{bc}^2 \angle -90^\circ$$

Q.4) The line-to-line voltages in an unbalanced three-phase supply are $V_{ab} = 1000 < 0^o$, $V_{bc} = 866 <150^o$, and V_{ca} = 500 (+ your two-digit serial no.) $< 120^o$. Determine the phase voltages *Van*, *Vbn*, and *Vcn*.

Solution:

The symmetrical components of the line voltages are

$$VL^{012} = \begin{matrix} 0.0000 & 30.0000 \\ 763.7626 & -10.8934 \\ 288.6751 & 30.0000 \end{matrix}$$

The symmetrical components of the phase voltages are

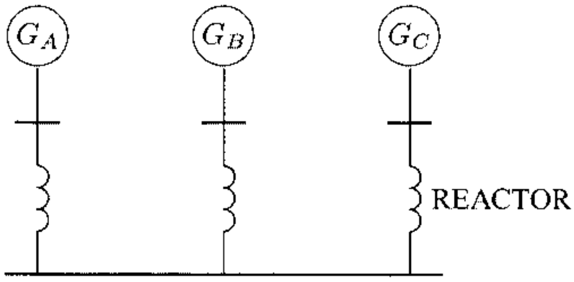
$$Va^{012} = \begin{matrix} 0 & 0 \\ 440.9586 & -40.8934 \\ 166.6667 & 60.0000 \end{matrix}$$

The phase voltages are

$$V^{abc} = \begin{matrix} 440.9586 & -19.1066 \\ 600.9252 & -166.1021 \\ 333.3333 & 60.0000 \end{matrix}$$

Q.5) Three 15-MVA, 30-kV synchronous generators A, B, and C are connected via three reactors to a common bus bar, as shown below. The neutrals of generators A and B are solidly grounded, and the neutral of generator C is grounded through a reactor of 2.0 (+ 0.your two-digit serial no.) Ohm. The generator data and the reactance of the reactors are tabulated below. Neglect prefault currents and assume generators are operating at their rated voltage. Determine the fault current for the following:

a. A bolted line-to-ground fault occurs on phase *a* of the common bus bar.
b. A bolted line-to-line fault occurs on between phase *b* and phase *c* of the common bus bar.
c. A bolted double line-to-ground fault occurs on phases *b* and *c* of the common bus bar.



Item	X^1	X^2	X^0
G_A	0.25 pu	0.155 pu	0.056 pu
G_B	0.20 pu	0.155 pu	0.056 pu
G_C	0.20 pu	0.155 pu	0.060 pu
Reactor	6.0 Ω	6.0 Ω	6.0 Ω

Solution:

a.

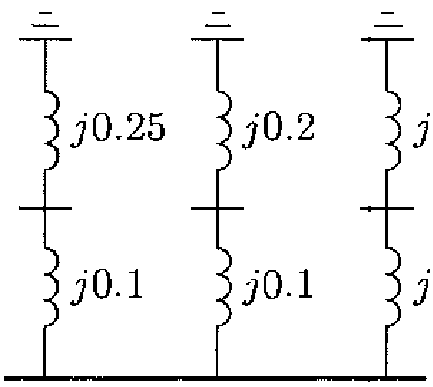
The generator base impedance is

$$Z_B = \frac{(30)^2}{15} = 60 \ \Omega$$

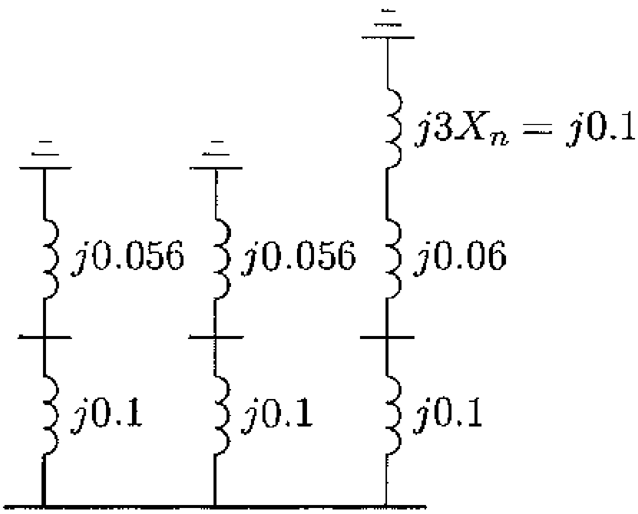
The reactor per-unit reactance, and the per-unit generator C neutral reactor are

$$X_R = \frac{6}{60} = 0.1 \ \text{pu}$$
$$X_n = \frac{2}{60} = 0.3333 \ \text{pu}$$

The positive-sequence impedance network is shown in Figure (a), and the zero-sequence impedance network is shown in Figure (b).



(a) Positive sequence network



(b) Zero sequence network

The positive-sequence impedance is

$$\frac{1}{X^1} = \frac{1}{0.35} + \frac{1}{0.3} + \frac{1}{0.3} \quad \text{or} \quad X^1 = 0.105$$

The negative-sequence impedance network is the same as the positive-sequence impedance network, except for the value of the generator negative-sequence reactance. Therefore, the negative-sequence impedance is

$$\frac{1}{X^2} = \frac{1}{0.255} + \frac{1}{0.255} + \frac{1}{0.255} \quad \text{or} \quad X^2 = 0.085$$

The zero-sequence impedance is

$$\frac{1}{X^0} = \frac{1}{0.156} + \frac{1}{0.156} + \frac{1}{0.26} \quad \text{or} \quad X^0 = 0.06$$

The line-to-ground fault current in phase a is

$$I_a = 3I_a^0 = \frac{3(1)}{j(0.105 + 0.085 + 0.06)} = 12\angle{-90^\circ} \ \text{pu}$$

b.

The positive-sequence fault current in phase a is

$$I_a^1 = \frac{1}{Z^1 + Z^2} = \frac{1}{j(0.105 + 0.085)} = -j5.26316 \ \text{pu}$$

The fault current is

$$I_b = -j\sqrt{3}I_a^1 = -9.116 \ \text{pu}$$

c.

The positive- and zero-sequence fault currents in phase a are

$$\begin{aligned} I_a^1 &= \frac{1}{j0.105 + j\left(\frac{(0.085)(0.06)}{0.085+0.06}\right)} = -j7.13407 \ \text{pu} \\ I_a^0 &= -\frac{1-(j0.105)(-j7.13407)}{j0.06} = j4.182 \ \text{pu} \end{aligned}$$

The fault current is

$$I_f = 3I_a^0 = 12.546\angle 90^\circ$$