KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

EE-520 (141)

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Key Solutions

Home Work 2 (Due Date: November 5th , 2014)

Q.1) A 4-bus system has the following line and bus data (on the base of 100MVA, 230kV base):

Line-Data

Bus-to-Bus	R (per-unit)	X (per-unit)	Y/2 (per-unit)
1-2	0.01008	0.05040	0.05125
1-3	0.00744	0.03720	0.03875
2-4	0.00744	0.03720	0.03875
3-4	0.01272	0.06360	0.06375

Bus-Data

Bus	Type	P_G (MW)	Q_G (MW)	P_D (MW)	Q_D (MW)	V (per-unit)	Q_{max} (MVAR)
1	Slack	-	-	50	30.99	1.0	-
2	Load	0	0	170	105.35	-	-
3	Load	0	0	200	123.94	-	-
4	Voltage Controlled	318	-	80	49.58	1.02	125+*

* your two-digit serial numbers

a) Ignoring the reactive power limit of bus 4, use Gauss-Seidel method to calculate the first two iterations bus voltages with acceleration factor $\alpha = 1.6$.

b) Considering the reactive power limit of bus 4, use Gauss-Seidel method to calculate ONLY the first iteration bus voltages with acceleration factor $\alpha = 1.6$.

Solution:

Bus admittance matrix

Bus no.	①	②	③	④
①	8.985190 $-j44.835953$	-3.815629 $+j19.078144$	-5.169561 $+j25.847809$	0
②	-3.815629 $+j19.078144$	8.985190 $-j44.835953$	0	-5.169561 $+j25.847809$
③	-5.169561 $+j25.847809$	0	8.193267 $-j40.863838$	-3.023705 $+j15.118528$
④	0	-5.169561 $+j25.847809$	-3.023705 $+j15.118528$	8.193267 $-j40.863838$

Q.1-a)

Iteration # 1

$$V_2^{(1)} = \frac{1}{Y_{22}}\left[\frac{-1.7 + j1.0535}{1.0 + j0.0} - 1.00(-3.815629 + j19.078144) - 1.02(-5.169561 + j25.847809)\right]$$

$$= \frac{1}{Y_{22}}[-1.7 + j1.0535 + 9.088581 - j45.442909]$$

$$= \frac{7.388581 - j44.389409}{8.985190 - j44.835953} = 0.983564 - j0.032316$$

$\boldsymbol{V_{2,acc}^{(1)}}$ **= 1 + 1.6 [(0.983564 –j0.032316) – 1] = 0.973703 – j 0.051706 pu**

$\boldsymbol{V_{3,acc}^{(1)}}$ **= 0.953949 – j 0.066708 pu**

$$Q_4^{(1)} = -\mathrm{Im}\left\{\begin{aligned}&1.02[(-5.169561 + j25.847809)(0.973703 - j0.051706)\\ &\quad + (-3.023705 + j15.118528)(0.953949 - j0.066708)\\ &\quad + (8.193267 - j40.863838)(1.02)]\end{aligned}\right\}$$

$$= -\mathrm{Im}\{1.02[-5.573064 + j40.059396 + (8.193267 - j40.863838)1.02]\}$$

$$= 1.654151 \text{ per unit}$$

$$V_4^{(1)} = \frac{1}{Y_{44}}\left[\frac{P_{4,\text{sch}} - jQ_4^{(1)}}{V_4^{(0)*}} - \left(Y_{42}V_{2,\text{acc}}^{(1)} + Y_{43}V_{3,\text{acc}}^{(1)}\right)\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{2.38 - j1.654151}{1.02 - j0.0} - (-5.573066 + j40.059398)\right]$$

$$= \frac{7.906399 - j41.681115}{8.193267 - j40.863838} = 1.017874 - j0.010604 \text{ per unit}$$

$$V_{4,\text{corr}}^{(1)} = \frac{1.02}{1.017929}(1.017874 - j0.010604)$$

$$= 1.019945 - j0.010625 \text{ per unit}$$

Iteration # 2

$$V_2^{(2)} = \frac{1}{Y_{22}}\left[\frac{P_{2,\text{sch}} - jQ_{2,\text{sch}}}{V_2^{(1)*}} - (Y_{21}V_1^{(1)} + Y_{24}V_{4,\text{acc}}^{(1)})\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{-1.7 + j1.0535}{0.981113 + j0.031518} - \{ -3.815629 + j19.078144\right.$$

$$\left. + (-5.169561 + j25.847809)(1.019922 + j0.012657)\}\right]$$

$$= \frac{7.718854 - j44.247184}{8.985190 - j44.835953}$$

$$= 0.9819338 - j0.0246233$$

$$V_{2,\text{acc}}^{(2)} = 0.981113 - j0.031518 + 1.6\ (0.9819338 - j0.0246233$$
$$- 0.9819338 + j0.0246233)$$

$$= 0.982426 - j0.020486$$

$$V_3^{(2)} = \frac{1}{Y_{33}}\left[\frac{P_{3,\text{sch}} - jQ_{3,\text{sch}}}{V_3^{(1)*}} - (Y_{31}V_1^{(2)} + Y_{34}V_{4,\text{acc}}^{(1)})\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{-2 + j1.2394}{0.966597+j0.040797} - \{ -5.16956 + j25.847809\right.$$

$$\left. + (-3.023705 + j15.118528)(1.019922 + j0.012657)\} \right].$$

$$= \frac{6.433447 - j39.862133}{8.193267 - j40.863838}$$

$$= 0.9681332 - j0.0366761$$

$$V_{3,\text{acc}}^{(2)} = 0.966597 - j0.00.040797 + 1.6\,(0.9681332 - j0.0366761$$
$$- 0.966597 + j0.00.040797)$$

$$= 0.969055 - j0.034195$$

$$Q_4^{(2)} = -\,\mathrm{Im}\left\{ V_4^{(1)*}\left[Y_{42}V_2^{(2)} + Y_{43}V_3^{(2)} + Y_{44}V_4^{(1)}\right]\right\}$$

$$= -\,\mathrm{Im}\{ (1.019922 - j0.012657)$$

$$\times [(-5.16956 + j\,25.847809)(0.982426 - j0.020486)$$
$$+ (-3.023705 + j\,15.118528)(0.969055 - j0.034195)$$
$$+ (8.193267 - j\,40.863837)(1.019922 + j0.012657)]\}$$

$$= -\,\mathrm{Im}\{ 1.911362 - j1.320680\} = 1.320680$$

$$V_4^{(2)} = \frac{1}{Y_{44}}\left[\frac{P_{3,sch} - jQ_4^{(2)}}{V_4^{(1)*}} - (Y_{42}V_2^{(2)} + Y_{43}V_3^{(2)})\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{2.38 - j1.320680}{1.019922 - j0.012657}\right.$$

$$-\{(-5.16956 + j25.847809)(0.982426 - j0.020486)$$

$$\left. + (-3.023705 + j15.118528)(0.969055 - j0.034195)\}\right]$$

$$= \frac{9.311570 - j41.519274}{8.193267 - j40.863838}$$

$$= 1.020695 + j0.023217$$

$$V_{4,corr}^{(2)} = \frac{1.02}{1.020959}(1.020695 + j0.023217)$$

$$= 1.019736 + j0.023195$$

Q.1-b)

The net power injection found at bus ④

$$Q_4 = 1.654151 \text{ per unit} = 165.4151 \text{ Mvar}$$

Considering the reactive load of 49.58 Mvar at the bus, the required reactive power generation is $165.4151 + 49.58 = 214.9951$ Mvar, which exceeds the 125 Mvar limit specified. The bus is now regarded as a load bus, with total reactive power generation of 125 Mvar. So the net injected reactive power in this case is

$$125 - 49.58 = 75.42 \text{ Mvar} = 0.7521 \text{ per unit}$$

V_4 is now calculated as

$$\begin{aligned} V_4^{(1)} &= \frac{1}{Y_{44}}\left[\frac{P_{4,sch} - jQ_4^{(1)}}{V_4^{(0)*}} - \left(Y_{42}V_{2,acc}^{(1)} + Y_{43}V_{3,acc}^{(1)}\right)\right] \\ &= \frac{1}{8.193267 - j40.863838}\left[\frac{2.38 - j0.7542}{1.02} - (-5.573064 + j40.05939)\right] \\ &= 0.997117 - j0.006442 \text{ per unit} \end{aligned}$$

and using an acceleration factor of 1.6 yields

$$V_{4,acc}^{(1)} = 1.02 + 1.6\,(0.997117 - j0.06442 - 1.02) = 0.983387 - j0.0103073 \text{ per unit}$$

Q.2) A 3-bus system has the following line and bus data (on the base of 100MVA, 230kV base):

Line-Data

Bus-to-Bus	**R (per-unit)**	**X (per-unit)**
1-2	0.02	0.04
1-3	0.01	0.03
2-3	0.0125	0.025

Bus-Data

Bus	**Type**	**P_G (MW)**	**Q_G (MW)**	**P_D (MW)**	**Q_D (MW)**	**V (per-unit)**
1	Slack	-	-	0	0	1.05
2	Load	0	0	400	250	-
3	Voltage Controlled	200+*	-	0	0	1.04

* your two-digit serial numbers

a) Use Newton-Raphson (Polar-Form) method to calculate P_1 , Q_1 , and Q_3 (considering mismatch voltage tolerance of $\boldsymbol{\varepsilon = 2.5 \times 10^{-4}}$ for both magnitudes and phase-angles, and maximum number of iteration **5**).

b) Use Fast-Decoupled method to calculate P_1 , Q_1 , and Q_3 (considering mismatch voltage tolerance of $\boldsymbol{\varepsilon = 2.5 \times 10^{-4}}$ for both magnitudes and phase-angles, and maximum number of iteration **15**).

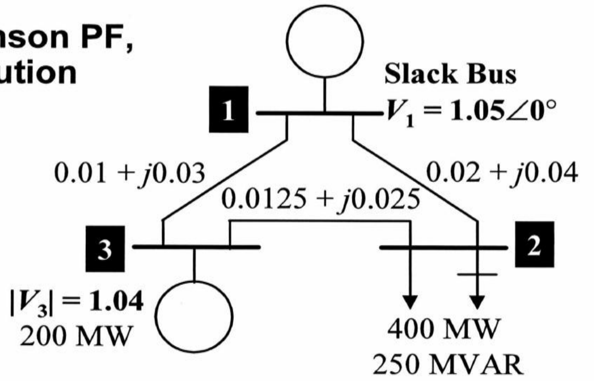
Important Notes:

- In all parts, show all steps clearly.
- If you are writing any programming routines, attach it with the solutions.
- Using ready-made Load-Flow programs (such as ETAP, Power-World Simulator, Power Tool Box in MATLAB, ….etc) are NOT allowed.

Solution:

Q.2-a)

- **Using the Newton-Raphson PF, find the power flow solution**



$$y_{12} = 10 - j20\ pu$$
$$y_{13} = 10 - j30\ pu$$
$$y_{23} = 16 - j32\ pu$$
$$S_2^{sch} = -\frac{400 + j250}{100} = -4.0 - j2.5\ pu$$
$$P_3^{sch} = \frac{200}{100} = 2.0\ pu$$

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$
$$= \begin{bmatrix} 53.9\angle -1.90 & 22.4\angle 2.03 & 31.6\angle 1.89 \\ 22.4\angle 2.03 & 58.1\angle -1.11 & 35.8\angle 2.03 \\ 31.6\angle 1.89 & 35.8\angle 2.03 & 67.2\angle -1.17 \end{bmatrix}$$ **angles are in radians**

$$P_2 = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2|^2|Y_{22}|\cos(\theta_{22}) + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$
$$P_3 = |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) + |V_3|^2|Y_{33}|\cos(\theta_{33})$$
$$Q_2 = -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2|^2|Y_{22}|\sin(\theta_{22}) - |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\bar{x} = \begin{bmatrix} \bar{\delta}_2 \\ \bar{\delta}_3 \\ \bar{V}_2 \end{bmatrix} \qquad f(\bar{x}) = \begin{bmatrix} P_2(\bar{\delta}_2, \bar{\delta}_3, \bar{V}_2) \\ P_3(\bar{\delta}_2, \bar{\delta}_3, \bar{V}_2) \\ Q_2(\bar{\delta}_2, \bar{\delta}_3, \bar{V}_2) \end{bmatrix}$$
$$= \begin{bmatrix} |\bar{V}_2||1.05||22.3|\cos(2.03 - \bar{\delta}_{21}) + |\bar{V}_2|^2|58.1|\cos(-1.11) + |\bar{V}_2||1.04||35.8|\cos(2.03 - \bar{\delta}_2 + \bar{\delta}_3) \\ |V_3||1.05||31.6|\cos(1.89 - \bar{\delta}_3) + |1.04||\bar{V}_2||35.8|\cos(2.03 - \bar{\delta}_3 + \bar{\delta}_2) + |1.04|^2|67.2|\cos(-1.17) \\ -|\bar{V}_2||1.05||22.3|\sin(2.03 - \bar{\delta}_2) - |\bar{V}_2|^2|58.1|\sin(-1.11) - |\bar{V}_2||1.04||35.8|\sin(2.03 - \bar{\delta}_2 + \bar{\delta}_3) \end{bmatrix}$$

$$\Delta c = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = c - f(\bar{x}) = \begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} - \begin{bmatrix} P_2(\bar{\delta}_2, \bar{\delta}_3, \bar{V}_2) \\ P_3(\bar{\delta}_2, \bar{\delta}_3, \bar{V}_2) \\ Q_2(\bar{\delta}_2, \bar{\delta}_3, \bar{V}_2) \end{bmatrix} = \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} - \begin{bmatrix} P_2(\bar{\delta}_2, \bar{\delta}_3, \bar{V}_2) \\ P_3(\bar{\delta}_2, \bar{\delta}_3, \bar{V}_2) \\ Q_2(\bar{\delta}_2, \bar{\delta}_3, \bar{V}_2) \end{bmatrix}$$

$$\frac{\partial P_2}{\partial \delta_2} = \sum_{j=1,\, j\neq 2}^{3} |V_2||V_j||Y_{2j}|\sin(\theta_{2j} - \delta_2 + \delta_j)$$
$$= |V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2) + |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$
$$= |V_2||1.05||22.4|\sin(2.03 - \delta_2) + |V_2||1.04||35.8|\sin(2.03 - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3) = -|V_2||1.04||35.8|\sin(2.03 - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_2|} = 2|V_2||Y_{22}|\cos(\theta_{22}) + \sum_{j=1, j\neq 2}^{3} |V_j||Y_{2j}|\cos(\theta_{2j} - \delta_2 + \delta_j)$$
$$= 2|V_2||Y_{22}|\cos(\theta_{22}) + |V_2||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$
$$= 2|V_2||58.1|\cos(2.03) + |1.05||22.4|\cos(2.03 - \delta_2)$$
$$+ |1.04||35.8|\cos(2.03 - \delta_2 + \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = -|V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2) = -|1.04||V_2||35.8|\sin(2.03 - \delta_2 + \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_3} = \sum_{j=1, j\neq 3}^{3} |V_3||V_j||Y_{3j}|\sin(\theta_{3j} - \delta_3 + \delta_j)$$
$$= |V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2)$$
$$= |1.04||1.05||31.6|\sin(1.89 - \delta_3) + |1.04||V_2||35.8|\sin(2.03 - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_2|} = |V_3||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) = |1.04||35.8|\cos(2.03 - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial \delta_2} = \sum_{j=1, j\neq 2}^{3} |V_2||V_j||Y_{2j}|\cos(\theta_{2j} - \delta_2 + \delta_j)$$
$$= |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$
$$= |V_2||1.05||22.4|\cos(2.03 - \delta_2) + |V_2||1.04||35.8|\cos(2.03 - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3) = -|V_2||1.04||35.8|\cos(2.03 - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -2|V_2||Y_{22}|\sin(\theta_{22}) - \sum_{j=1, j\neq 2}^{3} |V_j||Y_{2j}|\sin(\theta_{2j} - \delta_2 + \delta_j)$$
$$= -2|V_2||Y_{22}|\sin(\theta_{22}) - |V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$
$$= -2|V_2||58.1|\sin(-1.11) - |1.05||22.4|\sin(2.03 - \delta_2)$$
$$- |1.04||35.8|\sin(2.03 - \delta_2 + \delta_3)$$

$$\bar{x}^{[k+1]} = \bar{x}^{[k]} + J^{-1} \cdot \Delta c^{[k]}$$

$$= \begin{bmatrix} \bar{\delta}_2 \\ \bar{\delta}_3 \\ \bar{V}_2 \end{bmatrix}^{[k+1]} = \begin{bmatrix} \bar{\delta}_2 \\ \bar{\delta}_3 \\ \bar{V}_2 \end{bmatrix}^{[k]} + \begin{bmatrix} \partial P_2/\partial\delta_2 & \partial P_2/\partial\delta_3 & \partial P_2/\partial V_2 \\ \partial P_3/\partial\delta_2 & \partial P_3/\partial\delta_3 & \partial P_3/\partial V_2 \\ \partial Q_2/\partial\delta_2 & \partial Q_2/\partial\delta_3 & \partial Q_2/\partial V_2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix}^{[k]}$$

$$\bar{x}^{[0]} = \begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{bmatrix} \quad \Delta c^{[0]} = \begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} - \begin{bmatrix} P_2^{[0]} \\ P_3^{[0]} \\ Q_2^{[0]} \end{bmatrix} = \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} - \begin{bmatrix} -1.14 \\ 0.562 \\ -2.28 \end{bmatrix} = \begin{bmatrix} -2.86 \\ 1.438 \\ -0.22 \end{bmatrix}$$

$$\Delta x^{[0]} = J^{-1}\Delta c^{[0]}$$

$$\Delta x^{[0]} = \begin{bmatrix} \Delta\delta_2^{[0]} \\ \Delta\delta_3^{[0]} \\ \Delta\left|V_2^{[0]}\right| \end{bmatrix} = \begin{bmatrix} 54.28 & -33.28 & 24.86 \\ -33.28 & 66.04 & -16.64 \\ -27.14 & 16.64 & 49.72 \end{bmatrix}^{-1} \begin{bmatrix} -2.86 \\ 1.438 \\ -0.22 \end{bmatrix} = \begin{bmatrix} -0.04526 \\ -0.00772 \\ -0.02655 \end{bmatrix}$$

$$\bar{x}^{[1]} = \begin{bmatrix} \delta_2^{[1]} \\ \delta_3^{[1]} \\ \left|V_2^{[1]}\right| \end{bmatrix} = \begin{bmatrix} 0.0 + (-0.04526) \\ 0.0 + (-0.00772) \\ 1.0 + (-0.02655) \end{bmatrix} = \begin{bmatrix} -0.04526 \\ -0.00772 \\ 0.9734 \end{bmatrix}$$

$$\bar{x}^{[1]} = \begin{bmatrix} -0.04526 \\ -0.00772 \\ 0.9734 \end{bmatrix} \quad \Delta c^{[1]} = \begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} - \begin{bmatrix} P_2^{[1]} \\ P_3^{[1]} \\ Q_2^{[1]} \end{bmatrix} = \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} - \begin{bmatrix} -3.901 \\ 1.978 \\ -2.449 \end{bmatrix} = \begin{bmatrix} -0.099 \\ 0.0217 \\ -0.051 \end{bmatrix}$$

$$\Delta x^{[1]} = \begin{bmatrix} 51.72 & -31.77 & 21.30 \\ -32.98 & 65.66 & -15.38 \\ -28.54 & 17.40 & 48.10 \end{bmatrix}^{-1} \begin{bmatrix} -0.099 \\ 0.0217 \\ -0.051 \end{bmatrix} = \begin{bmatrix} -0.001795 \\ -0.000985 \\ -0.001767 \end{bmatrix}$$

$$\bar{x}^{[2]} = \begin{bmatrix} \delta_2^{[2]} \\ \delta_3^{[2]} \\ \left|V_2^{[2]}\right| \end{bmatrix} = \begin{bmatrix} -0.04526 + (-0.001795) \\ -0.00772 + (-0.000985) \\ 0.9734 + (-0.001767) \end{bmatrix} = \begin{bmatrix} -0.04706 \\ -0.00870 \\ 0.9717 \end{bmatrix}$$

$$\bar{x}^{[2]} = \begin{bmatrix} -0.04706 \\ -0.00870 \\ 0.9717 \end{bmatrix} \quad \Delta c^{[2]} = \begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} - \begin{bmatrix} P_2^{[1]} \\ P_3^{[1]} \\ Q_2^{[1]} \end{bmatrix} = \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} - \begin{bmatrix} -3.999 \\ 1.999 \\ -2.499 \end{bmatrix} = \begin{bmatrix} -0.0002 \\ 0.00004 \\ -0.0001 \end{bmatrix}$$

$$\Delta x^{[2]} = \begin{bmatrix} 51.60 & -31.69 & 21.14 \\ -32.93 & 65.60 & -15.35 \\ -28.55 & 17.40 & 47.95 \end{bmatrix}^{-1} \begin{bmatrix} -0.000216 \\ 0.000038 \\ -0.000143 \end{bmatrix} = \begin{bmatrix} -0.000038 \\ -0.000002 \\ -0.000004 \end{bmatrix}$$

$$\bar{x}^{[3]} = \begin{bmatrix} \delta_2^{[3]} \\ \delta_3^{[3]} \\ |V_2^{[3]}| \end{bmatrix} = \begin{bmatrix} -0.04706 + (-0.000038) \\ -0.00870 + (-0.000002) \\ 0.9717 + (-0.000004) \end{bmatrix} = \begin{bmatrix} -0.04706 \\ -0.008705 \\ 0.97168 \end{bmatrix}$$

$$\bar{x}^{[3]} = \begin{bmatrix} -0.04706 \\ -0.008705 \\ 0.97168 \end{bmatrix} \quad \Delta c^{[2]} = \begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} - \begin{bmatrix} P_2^{[1]} \\ P_3^{[1]} \\ Q_2^{[1]} \end{bmatrix} = \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} - \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \end{bmatrix}$$

$$\varepsilon_{\max} = 2.5 \times 10^{-4}$$

$$P_1 = |V_1|^2 |Y_{11}| \cos(\theta_{11}) + |V_1||V_2||Y_{12}| \cos(\theta_{12} - \delta_1 + \delta_2) + |V_1||V_3||Y_{13}| \cos(\theta_{13} - \delta_1 + \delta_3)$$

$$Q_1 = -|V_1|^2 |Y_{11}| \sin(\theta_{11}) - |V_1||V_2||Y_{12}| \sin(\theta_{12} - \delta_1 + \delta_2) - |V_1||V_3||Y_{13}| \sin(\theta_{13} - \delta_1 + \delta_3)$$

$$Q_3 = -|V_3||V_1||Y_{31}| \sin(\theta_{31} - \delta_3 + \delta_1) - |V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2) - |V_3|^2 |Y_{33}| \sin(\theta_{33})$$

$$P_1 = 2.1842\,pu$$

$$Q_1 = 1.4085\,pu$$

$$Q_3 = 1.4617\,pu$$

Q.2-d)

$$B' = \begin{bmatrix} -52 & 32 \\ 32 & -62 \end{bmatrix}$$

$$[B']^{-1} = \begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix}$$

$$B'' = [-52]$$

$$[B'']^{-1} = [-0.019231]$$

Initial values: $$V^{[0]} = \begin{bmatrix} 1.05\angle 0^\circ \\ 1.00\angle 0^\circ \\ 1.00\angle 0^\circ \end{bmatrix}$$

First iteration:

$$\bar{y} = \begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} = \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} \quad \bar{x}^{[k]} = \begin{bmatrix} \delta_2^{[k]} \\ \delta_3^{[k]} \\ V_2^{[k]} \end{bmatrix} \quad \bar{x}^{[0]} = \begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{bmatrix}$$

$$f(\bar{x}) = \begin{bmatrix} P_{inj\,2}(\bar{x}) \\ P_{inj\,3}(\bar{x}) \\ Q_{inj\,2}(\bar{x}) \end{bmatrix} \quad P_{inj\,i} = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \quad Q_{inj\,i} = -\sum_{i=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

$$f(\bar{x}) = \begin{bmatrix} |V_2|^2|Y_{22}|\cos(\theta_{22}) + |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3) \\ |V_3|^2|Y_{33}|\cos(\theta_{33}) + |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) \\ -|V_2|^2|Y_{22}|\sin(\theta_{22}) - |V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3) \end{bmatrix}$$

$$= \begin{bmatrix} |V_2|^2|58.1|\cos(-1.11) + |V_2||1.05||22.4|\cos(2.03 - \delta_2) + |V_2||1.04||35.8|\cos(2.03 - \delta_2 + \delta_3) \\ |1.04|^2|67.2|\cos(-1.17) + |1.04||1.05||31.6|\cos(1.89 - \delta_3) + |1.04||V_2||35.8|\cos(2.03 - \delta_3 + \delta_2) \\ -|V_2|^2|58.1|\sin(-1.11) - |V_2||1.05||22.4|\sin(2.03 - \delta_2) - |V_2||1.04||35.8|\sin(2.03 - \delta_2 + \delta_3) \end{bmatrix}$$

$$\Delta y^{[0]} = \begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} - \begin{bmatrix} P_2^{[0]} \\ P_3^{[0]} \\ Q_2^{[0]} \end{bmatrix} = \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} - \begin{bmatrix} -1.14 \\ 0.562 \\ -2.28 \end{bmatrix} = \begin{bmatrix} -2.86 \\ 1.438 \\ -0.22 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2^{[0]} \\ \Delta\delta_3^{[0]} \end{bmatrix} = \begin{bmatrix} 0.028182 & 0.014545 \\ 0.014545 & 0.023636 \end{bmatrix} \begin{bmatrix} -2.86/1.0 \\ 1.438/1.04 \end{bmatrix} = \begin{bmatrix} -0.06048 \\ -0.00891 \end{bmatrix}$$

$$\left[\Delta\left|V_2^{[0]}\right|\right] = [0.019231][-0.22/1.0] = [-0.004231]$$

$$\delta_2^{[1]} = 0.0 + (-0.06048) = -0.06048$$
$$\delta_3^{[1]} = 0.0 + (-0.00891) = -0.00891$$
$$\left|V_2^{[1]}\right| = 1.0 + (-0.004231) = 0.995769$$

Remaining iterations:

Iter	δ2	δ3	|V2|	ΔP2	ΔP3	ΔQ2
1	-0.060482	-0.008909	0.995769	-2.860000	1.438400	-0.220000
2	-0.056496	-0.007952	0.965274	0.175895	-0.070951	-1.579042
3	-0.044194	-0.008690	0.965711	0.640309	-0.457039	0.021948
4	-0.044802	-0.008986	0.972985	-0.021395	0.001195	0.365249
5	-0.047665	-0.008713	0.973116	-0.153368	0.112899	0.006657
6	-0.047614	-0.008645	0.971414	0.000520	0.002610	-0.086136
7	-0.046936	-0.008702	0.971333	0.035980	-0.026190	-0.004067
8	-0.046928	-0.008720	0.971732	0.000948	-0.001411	0.020119
9	-0.047087	-0.008707	0.971762	-0.008442	0.006133	0.001558
10	-0.047094	-0.008702	0.971669	-0.000470	0.000510	-0.004688