KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

EE-520 (141)

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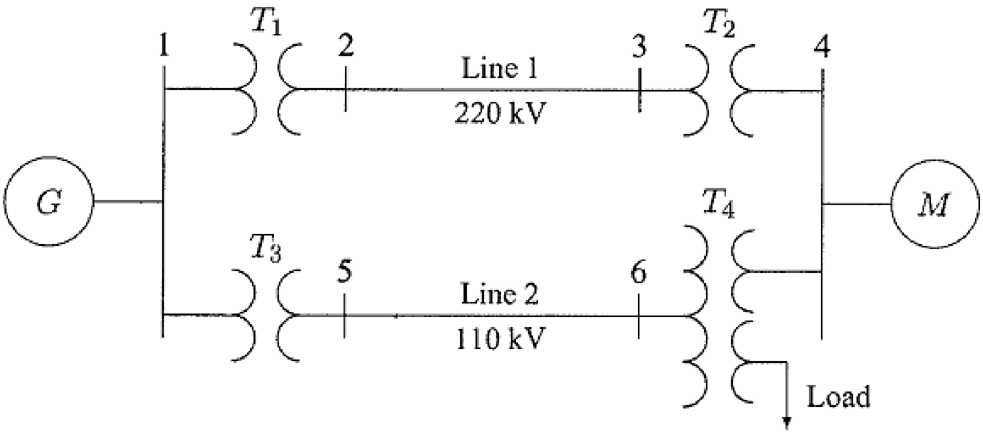
Home Work 1 (Key-Solution)

Consider the one-line diagram shown bellow,



The three-phase power and line-line ratings are given below:
***G*:** 80 MVA, 22 kV, X = 24%.
T_1: 50 MVA, 22/220 kV, X = 10%.
T_2: 40 MVA, 220/22 kV, X = 6.** %.
T_3: 40 MVA 22/110 kV X = 6.4%.
Line 1: 220 kV X = 121 Ohm.
Line 2: 110 kV X = 42.35 Ohm.
***M*:** 68.85 MVA, 20 kV, X = 22.5%.
Load: 10 Mvar, 4 kV, delta-connected capacitors.

The three-phase ratings of the **three- winding transformer T_4** are
Primary: Y-connected, 40MVA, 110 kV.
Secondary: Y-connected, 40 MVA, 22 kV.
Tertiary: delta-connected, 15 MVA, 4 kV.
The per-phase measured reactances at the terminal of a winding with the second one short-circuited and the third open-circuited are
Primary-Secondary Impedance Z_{PS} = 9.6%, 40 MVA, 110 kV / 22 kV.
Primary-Tertiary Impedance Z_{PT} = 7.2%, 40 MVA, 110 kV / 4 kV.
Secondary-Tertiary Impedance Z_{ST} = 12%, 40 MVA, 22 kV / 4 kV.

**** YOUR TWO-DIGIT SERIAL NUMBER**

Solution:

Q1. Obtain the T-circuit equivalent impedances of the three-winding transformer to the common MVA base. Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 22 kV as the voltage base for generator.

The base voltage V_{B1} on the LV side of T_1 is 22 kV. Hence the base on its HV side is

V_{B2} = 22 (220/22) = 220 kV

This fixes the base on the HV side of T_2 at V_{B3} = 220 kV, and on its LV side at

V_{B4} = 220 (22/220) = 22 kV

Similarly, the voltage base at buses 5 and 6 are

V_{B5} = V_{B6} =22 (110/22) = 110 kV

Voltage base for the tertiary side of T_4 is

V_{BT} = 110 (4/110) = 4 kV

The per-unit impedances on a 100 MVA base are:

***G*:** X = 0.24 (100/80) = 0.3 pu

T_1: X = 0.10 (100/50) = 0.2 pu

T_2: X = 0.06 (100/40) = 0.15 pu

T_3: X = 0.064 (100/40) = 0.16 pu

The motor reactance is expressed on its nameplate rating of 68.85 MVA, and 20 kV. However, the base voltage at bus 4 for the motor is 22 kV, therefore

***M*:** $X = 0.225\ (100/68.85)\ (20/22)^2 = 0.27$ pu

Impedance bases for lines 1 and 2 are

Z_{B2}: $X = (220)^2/100 = 484$ Ohm

Z_{B5}: $X = (110)^2/100 = 121$ Ohm

Line 1 and 2 per-unit reactances are

Line 1: X = 121 / 484 = 0.25 pu

Line 2: X = 42.35 / 121 = 0.35 pu

The load impedance in ohms is

$$Z_L = (4)^2 / j10 = - j1.6 \text{ Ohm}$$

Therefore, the load impedance in per unit is

$$Z_L = - j1.6/0.16 = - j\ 10 \text{ pu}$$

The three-winding impedances on a 100 MVA base are

$$Z_{PS} = 0.096\ (100/40) = 0.24 \text{ pu}$$

$$Z_{PT} = 0.072\ (100/40) = 0.18 \text{ pu}$$

$$Z_{ST} = 0.12\ (100/40) = 0.30 \text{ pu}$$

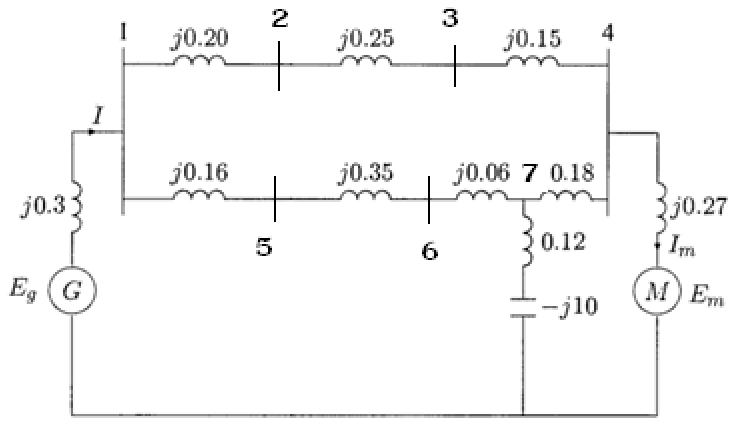
The equivalent T-circuit impedances are

$$Z_P = (1/2) * (j0.24+j0.18-j0.30) = j0.06 \text{ pu}$$

$$Z_S = (1/2) * (j0.24+j0.30-j0.18) = j0.18 \text{ pu}$$

$$Z_T = (1/2) * (j0.18+j0.30-j0.24) = j0.12 \text{ pu}$$

The per unit equivalent circuit is



Q2. Use the Chained Data Structure Method (row fashion approach; STO, IC, NX, and NFIRST) for storing and modifying the following:

a) Bus admittance matrix referenced to bus "0" using the building algorithm. Include all machines and loads impedances.
b) Bus impedance matrix referenced to bus "0" using the building algorithm. Include all machines and loads impedances.

a) Bus Admittance Matrix including all machines and loads impedances

j

-14.583	5.0	0	0	6.25	0	0
5.0	-9.0	4.0	0	0	0	0
0	4.0	-10.667	6.667	0	0	0
0	0	6.667	-15.927	0	0	5.556
6.25	0	0	0	-9.107	2.857	0
0	0	0	0	2.857	-19.524	16.667
0	0	0	5.556	0	16.667	-22.121

Use the Chained Data Structure Method for storing Symmetric Y_{BUS} (5 Points)

	STO	IC	NX	NFIRST
1	-14.583	1	1	1
2	5.0	2	1	4
3	6.25	5	0	6
4	-9.0	2	1	8
5	4.0	3	0	10
6	-10.667	3	1	12
7	6.667	4	0	14
8	-15.927	4	1	
9	5.556	7	0	
10	-9.107	5	1	
11	2.857	6	0	
12	-19.524	6	1	
13	16.667	7	0	
14	-22.121	7	0	

b) Bus Impedance Matrix (25 Points)

Consider the addition of the radial line between Bus 1 and the reference

$Z_{BUS} =$ j

0.3

Consider the addition of the radial line between Bus 1 and Bus 2

$Z_{BUS} =$ j

0.3	0.3
0.3	0.5

Consider the addition of the radial line between Bus 2 and Bus 3

$Z_{BUS} =$ j

0.3	0.3	0.3
0.3	0.5	0.5
0.3	0.5	0.75

Consider the addition of the radial line between Bus 3 and Bus 4

$Z_{BUS} =$ j

0.3	0.3	0.3	0.3
0.3	0.5	0.5	0.5
0.3	0.5	0.75	0.75
0.3	0.5	0.75	0.9

Consider the addition of the radial line between Bus 1 and Bus 5

$Z_{BUS} =$ j

0.3	0.3	0.3	0.3	0.3
0.3	0.5	0.5	0.5	0.3
0.3	0.5	0.75	0.75	0.3
0.3	0.5	0.75	0.9	0.3
0.3	0.3	0.3	0.3	0.46

Consider the addition of the radial line between Bus 5 and Bus 6

$Z_{BUS} =$ j

0.3	0.3	0.3	0.3	0.3	0.3
0.3	0.5	0.5	0.5	0.3	0.3
0.3	0.5	0.75	0.75	0.3	0.3
0.3	0.5	0.75	0.9	0.3	0.3
0.3	0.3	0.3	0.3	0.46	0.46
0.3	0.3	0.3	0.3	0.46	0.81

Consider the addition of the radial line between Bus 6 and Bus 7

$Z_{BUS} = j$

0.3	0.3	0.3	0.3	0.3	0.3	0.3
0.3	0.5	0.5	0.5	0.3	0.3	0.3
0.3	0.5	0.75	0.75	0.3	0.3	0.3
0.3	0.5	0.75	0.9	0.3	0.3	0.3
0.3	0.3	0.3	0.3	0.46	0.46	0.46
0.3	0.3	0.3	0.3	0.46	0.81	0.81
0.3	0.3	0.3	0.3	0.46	0.81	0.87

Consider the addition of the loop line between Bus 4 and Bus 0 (reference)

$Z_{BUS\text{-}LOOP} =$ j

0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
0.3	0.5	0.5	0.5	0.3	0.3	0.3	0.5
0.3	0.5	0.75	0.75	0.3	0.3	0.3	0.75
0.3	0.5	0.75	0.9	0.3	0.3	0.3	0.9
0.3	0.3	0.3	0.3	0.46	0.46	0.46	0.3
0.3	0.3	0.3	0.3	0.46	0.81	0.81	0.3
0.3	0.3	0.3	0.3	0.46	0.81	0.87	0.3
0.3	0.5	0.75	0.9	0.3	0.3	0.3	1.17

After applying bus elimination method

$Z_{BUS} = j$

0.2231	0.1718	0.1077	0.0692	0.2231	0.2231	0.2231
0.1718	0.2863	0.1795	0.1154	0.1718	0.1718	0.1718
0.1077	0.1795	0.2692	0.1731	0.1077	0.1077	0.1077
0.0692	0.1154	0.1731	0.2077	0.0692	0.0692	0.0692
0.2231	0.1718	0.1077	0.0692	0.3831	0.3831	0.3831
0.2231	0.1718	0.1077	0.0692	0.3831	0.7331	0.7331
0.2231	0.1718	0.1077	0.0692	0.3831	0.7331	0.7931

Consider finally the addition of the loop line between Bus 7 and Bus 0 (reference)

$Z_{BUS-LOOP}$ =

0.2231	0.1718	0.1077	0.0692	0.2231	0.2231	0.2231	0.2231
0.1718	0.2863	0.1795	0.1154	0.1718	0.1718	0.1718	0.1718
0.1077	0.1795	0.2692	0.1731	0.1077	0.1077	0.1077	0.1077
0.0692	0.1154	0.1731	0.2077	0.0692	0.0692	0.0692	0.0692
0.2231	0.1718	0.1077	0.0692	0.3831	0.3831	0.3831	0.3831
0.2231	0.1718	0.1077	0.0692	0.3831	0.7331	0.7331	0.7331
0.2231	0.1718	0.1077	0.0692	0.3831	0.7331	0.7931	0.7931
0.2231	0.1718	0.1077	0.0692	0.3831	0.7331	0.7931	-9.0869

j

After applying bus elimination method

Z_{BUS} = j

0.2286	0.1760	0.1103	0.0709	0.2325	0.2411	0.2426
0.1760	0.2895	0.1815	0.1167	0.1790	0.1857	0.1868
0.1103	0.1815	0.2705	0.1739	0.1122	0.1164	0.1171
0.0709	0.1167	0.1739	0.2082	0.0721	0.0748	0.0752
0.2325	0.1790	0.1122	0.0721	0.3993	0.4140	0.4165
0.2411	0.1857	0.1164	0.0748	0.4140	0.7922	0.7971
0.2426	0.1868	0.1171	0.0752	0.4165	0.7971	0.8623

Consider the addition of the loop line between Bus 4 and Bus 7

$Z_{BUS-LOOP}$ =

0.2286	0.1760	0.1103	0.0709	0.2325	0.2411	0.2426	-0.1717
0.1760	0.2895	0.1815	0.1167	0.1790	0.1857	0.1868	-0.0701
0.1103	0.1815	0.2705	0.1739	0.1122	0.1164	0.1171	0.0568
0.0709	0.1167	0.1739	0.2082	0.0721	0.0748	0.0752	0.1330
0.2325	0.1790	0.1122	0.0721	0.3993	0.4140	0.4165	-0.3444
0.2411	0.1857	0.1164	0.0748	0.4140	0.7922	0.7971	-0.7223
0.2426	0.1868	0.1171	0.0752	0.4165	0.7971	0.8623	-0.7871
-0.1717	-0.0701	0.0568	0.1330	-0.3444	-0.7223	-0.7871	1.1001

After applying bus elimination method

Z_{BUS} = j

0.2018	0.1651	0.1192	0.0917	0.1787	0.1284	0.1198
0.1651	0.2850	0.1851	0.1252	0.1571	0.1397	0.1366
0.1192	0.1851	0.2676	0.1670	0.1300	0.1537	0.1577
0.0917	0.1252	0.1670	0.1921	0.1137	0.1621	0.1704
0.1787	0.1571	0.1300	0.1137	0.2915	0.1879	0.1701
0.1284	0.1397	0.1537	0.1621	0.1879	0.3180	0.2803
0.1198	0.1366	0.1577	0.1704	0.1701	0.2803	0.2991

Use the Chained Data Structure Method for storing Symmetric Z_{BUS} (15 Points)

	STO	IC	NX	NFIRST
1	0.2018	1	1	1
2	0.1651	2	1	8
3	0.1192	3	1	14
4	0.0917	4	1	19
5	0.1787	5	1	23
6	0.1284	6	1	26
7	0.1198	7	0	28
8	0.2850	2	1	
9	0.1851	3	1	
10	0.1252	4	1	
11	0.1571	5	1	
12	0.1397	6	1	
13	0.1366	7	0	
14	0.2676	3	1	
15	0.1670	4	1	
16	0.1300	5	1	
17	0.1537	6	1	
18	0.1577	7	0	
19	0.1921	4	1	
20	0.1137	5	1	
21	0.1621	6	1	
22	0.1704	7	0	
23	0.2915	5	1	
24	0.1879	6	1	
25	0.1701	7	0	
26	0.3180	6	1	
27	0.2803	7	0	
28	0.2991	7	1	

Q3. Check if multiplication of the bus admittance matrix of (Q2-a) and bus impedance matrix of (Q2-b) result into the identity matrix.

Multiplication of Z_{BUS} and Y_{BUS} (10 Points)

-1.0004	0.0001	0.0002	-0.0000	0.0002	0.0004	-0.0003
-0.0003	**-0.9995**	0.0000	-0.0003	0.0004	-0.0008	0.0007
0.0002	0.0000	**-1.0001**	-0.0001	0.0001	-0.0003	0.0003
0.0001	-0.0002	-0.0002	**-0.9998**	0.0002	-0.0010	0.0010
0.0004	0.0002	-0.0000	0.0001	**-1.0006**	-0.0004	0.0007
0.0006	-0.0005	0.0001	-0.0001	-0.0003	**-0.9991**	-0.0006
-0.0001	-0.0001	-0.0002	0.0004	0.0003	-0.0009	**-0.9990**

Q4. Use L_1U factorization to factor the bus admittance matrix.

L_1 =

1.0000						
-0.3429	1.0000					
0	-0.5490	1.0000				
0	0	-0.7870	1.0000			
-0.4286	-0.2941	-0.1389	-0.0867	1.0000		
0	0	0	0	-0.5144	1.0000	
0	0	0	-0.5202	-0.0867	-0.9369	1.0000

U =

-14.5830	5.0000	0	0	6.2500	0	0
	-7.2857	4.0000	0	2.1429	0	0
		-8.4709	6.6670	1.1765	0	0
			-10.6798	0.9260	0	5.5560
				-5.5544	2.8570	0.4817
					-18.0545	16.9148
						-3.3417