6-1. A 220-V three-phase six-pole 50-Hz induction motor is running at a slip of 3.5 percent. Find:

(a) The speed of the magnetic fields in revolutions per minute

(b) The speed of the rotor in revolutions per minute

(c) The slip speed of the rotor

(d) The rotor frequency in hertz

Solution

(a) The speed of the magnetic fields is

$$n_{sync} = \frac{120 f_{se}}{P} = \frac{120(50\text{ Hz})}{6} = 1000\text{ r/min}$$

(b) The speed of the rotor is

$$n_m = (1-s)\, n_{sync} = (1-0.035)(1000\text{ r/min}) = 965\text{ r/min}$$

(c) The slip speed of the rotor is

$$n_{slip} = s n_{sync} = (0.035)(1000\text{ r/min}) = 35\text{ r/min}$$

(d) The rotor frequency is

$$f_{re} = \frac{n_{slip} P}{120} = \frac{(35\text{ r/min})(6)}{120} = 1.75\text{ Hz}$$

6-3. A three-phase 60-Hz induction motor runs at 715 r/min at no load and at 670 r/min at full load.

(a) How many poles does this motor have?

(b) What is the slip at rated load?

(c) What is the speed at one-quarter of the rated load?

(d) What is the rotor's electrical frequency at one-quarter of the rated load?

Solution

(a) This machine has 10 poles, which produces a synchronous speed of

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(60\text{ Hz})}{10} = 720\text{ r/min}$$

(b) The slip at rated load is

$$s = \frac{n_{sync} - n_m}{n_{sync}} \times 100\% = \frac{720-670}{720} \times 100\% = 6.94\%$$

(c) The motor is operating in the linear region of its torque-speed curve, so the slip at ¼ load will be

$$s = 0.25(0.0694) = 0.0171$$

The resulting speed is

$$n_m = (1-s)\, n_{sync} = (1-0.0171)(720\text{ r/min}) = 708\text{ r/min}$$

(d) The electrical frequency at ¼ load is

$$f_r = s f_e = (0.0171)(60\text{ Hz}) = 1.03\text{ Hz}$$

6-5. A 208-V four-pole 60-Hz Y-connected wound-rotor induction motor is rated at 30 hp. Its equivalent circuit components are

$R_1 = 0.100\ \Omega$ $\quad R_2 = 0.070\ \Omega$ $\quad X_M = 10.0\ \Omega$

$X_1 = 0.210\ \Omega$ $\quad X_2 = 0.210\ \Omega$

$P_{mech} = 500\ \text{W}$ $\quad P_{misc} \approx 0$ $\quad P_{core} = 400\ \text{W}$

For a slip of 0.05, find

(a) The line current I_L

(b) The stator copper losses P_{SCL}

(c) The air-gap power P_{AG}

(d) The power converted from electrical to mechanical form P_{conv}

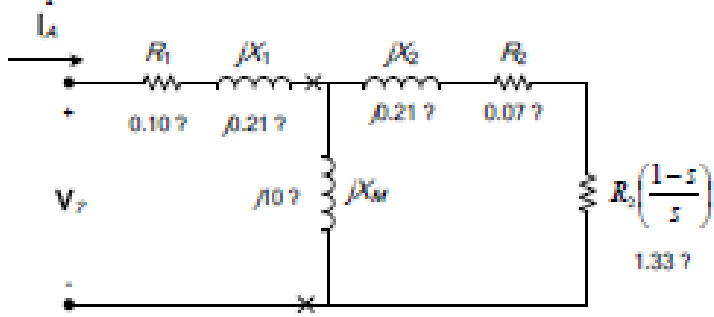
(e) The induced torque τ_{ind}

(f) The load torque τ_{load}

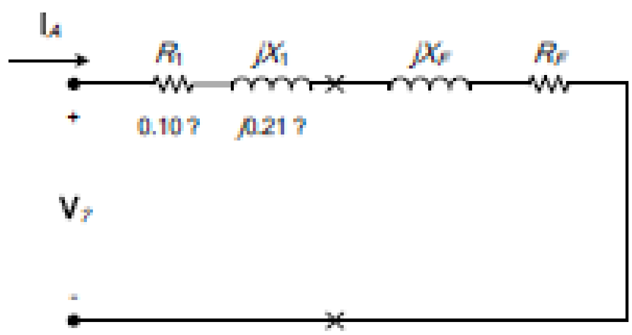
(g) The overall machine efficiency

(h) The motor speed in revolutions per minute and radians per second

SOLUTION The equivalent circuit of this induction motor is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j10\ \Omega} + \frac{1}{1.40 + j0.21}} = 1.318 + j0.386 = 1.374\angle 16.3^\circ\ \Omega$$

The phase voltage is $208/\sqrt{3} = 120$ V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{120\angle 0^\circ \text{ V}}{0.10\ \Omega + j0.21\ \Omega + 1.318\ \Omega + j0.386\ \Omega}$$

$$I_L = I_A = 78.0\angle -22.8^\circ \text{ A}$$

(b) The stator copper losses are

$$P_{SCL} = 3I_A^2 R_1 = 3(78.0\text{ A})^2(0.10\ \Omega) = 1825\text{ W}$$

(c) The air gap power is $P_{AG} = 3I_2^2 \frac{R_2}{s} = 3I_A^2 R_F$

(Note that $3I_A^2 R_F$ is equal to $3I_2^2 \frac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{AG} = 3I_2^2 \frac{R_2}{s} = 3I_A^2 R_F = 3(78.0\text{ A})^2(1.318\ \Omega) = 24.0\text{ kW}$$

(d) The power converted from electrical to mechanical form is

$$P_{conv} = (1-s)P_{AG} = (1-0.05)(24.0\text{ kW}) = 22.8\text{ kW}$$

(e) The induced torque in the motor is

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{24.0\text{ kW}}{(1800\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = 127.4\text{ N}\cdot\text{m}$$

(f) The output power of this motor is

$$P_{OUT} = P_{conv} - P_{mech} - P_{core} - P_{misc} = 22.8\text{ kW} - 500\text{ W} - 400\text{ W} - 0\text{ W} = 21.9\text{ kW}$$

The output speed is

$$n_m = (1-s)\ n_{sync} = (1-0.05)(1800\text{ r/min}) = 1710\text{ r/min}$$

Therefore the load torque is

$$\tau_{load} = \frac{P_{OUT}}{\omega_m} = \frac{21.9\text{ kW}}{(1710\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = 122.3\text{ N}\cdot\text{m}$$

(g) The overall efficiency is

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{P_{OUT}}{3V_\phi I_A \cos\theta} \times 100\%$$

$$\eta = \frac{21.9\text{ kW}}{3(120\text{ V})(78.0\text{ A})\cos 22.8^\circ} \times 100\% = 84.6\%$$

(h) The motor speed in revolutions per minute is 1710 r/min. The motor speed in radians per second is

$$\omega_m = (1710\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 179\text{ rad/s}$$

6-8. For the motor of Problem 6-5, how much additional resistance (referred to the stator circuit) would it be necessary to add to the rotor circuit to make the maximum torque occur at starting conditions (when the shaft is not moving)? Plot the torque-speed characteristic of this motor with the additional resistance inserted.

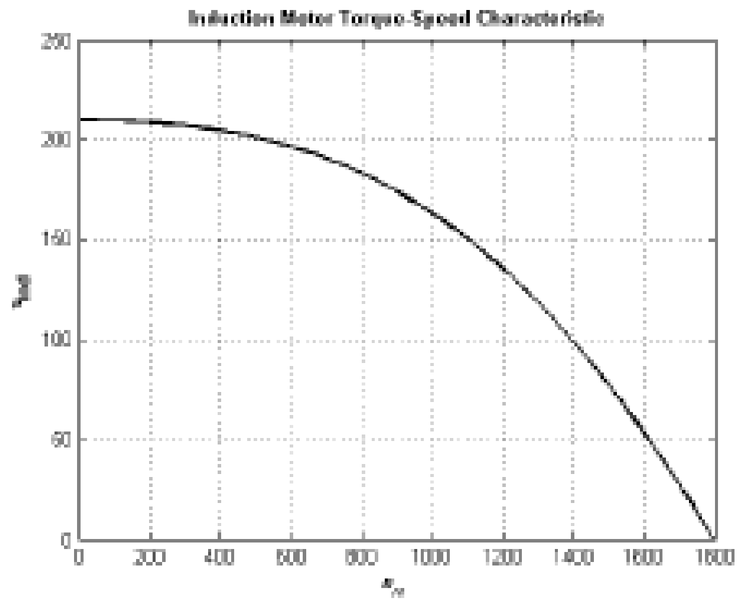
SOLUTION To get the maximum torque at starting, the s_{max} must be 1.00. Therefore,

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}}$$

$$1.00 = \frac{R_2}{\sqrt{\left(0.0959\ \Omega\right)^2 + \left(0.2066\ \Omega\ + 0.210\ \Omega\right)^2}}$$

$$R_2 = 0.428\ \Omega$$

Since the existing resistance is 0.070 Ω, an additional 0.358 Ω must be added to the rotor circuit. The resulting torque-speed characteristic is:



6-10. A three-phase 60-Hz two-pole induction motor runs at a no-load speed of 3580 r/min and a full-load speed of 3440 r/min. Calculate the slip and the electrical frequency of the rotor at no-load and full-load conditions. What is the speed regulation of this motor [Equation (3-68)]?

SOLUTION The synchronous speed of this machine is 3600 r/min. The slip and electrical frequency at no-load conditions is

$$s_{nl} = \frac{n_{sync} - n_{nl}}{n_{sync}} \times 100\% = \frac{3600 - 3580}{3600} \times 100\% = 0.56\%$$

$$f_{r,nl} = sf_e = \left(0.0056\right)\left(60\ \text{Hz}\right) = 0.33\ \text{Hz}$$

The slip and electrical frequency at full load conditions is

$$s_{fl} = \frac{n_{sync} - n_{nl}}{n_{sync}} \times 100\% = \frac{3600 - 3440}{3600} \times 100\% = 4.44\%$$

$$f_{r,fl} = sf_e = (0.0444)(60 \text{ Hz}) = 2.67 \text{ Hz}$$

The speed regulation is

$$\text{SR} = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\% = \frac{3580 - 3440}{3440} \times 100\% = 4.07\%$$

6-12. The power crossing the air gap of a 60 Hz, four-pole induction motor is 25 kW, and the power converted from electrical to mechanical form in the motor is 23.2 kW.

(a) What is the slip of the motor at this time?

(b) What is the induced torque in this motor?

(c) Assuming that the mechanical losses are 300 W at this slip, what is the load torque of this motor?

SOLUTION

(a) The synchronous speed of this motor is

$$n_{sync} = \frac{120 f_{se}}{P} = \frac{120(60 \text{ Hz})}{4} = 1800 \text{ r/min}$$

The power converted from electrical to mechanical form is

$$P_{conv} = (1-s) P_{AG}$$

so

$$s = 1 - \frac{P_{conv}}{P_{AG}} = 1 - \frac{23.4 \text{ kW}}{25 \text{ kW}} = 0.064$$

or 6.4%.

(b) The speed of the motor is

$$n_m = (1-s) n_{sync} = (1 - 0.064)(1800 \text{ r/min}) = 1685 \text{ r/min}$$

The induced torque of the motor is

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{23.4 \text{ kW}}{(1685 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 132.6 \text{ N}\cdot\text{m}$$

Alternately, the induced torque can be found as

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{25.0 \text{ kW}}{(1800 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 132.6 \text{ N}\cdot\text{m}$$

(c) The output power of this motor is

$$P_{out} = P_{conv} - P_{mech} = 23,400 \text{ W} - 300 \text{ W} = 23,100 \text{ W}$$

$$\tau_{load} = \frac{P_{out}}{\omega_m} = \frac{23.1 \text{ kW}}{(1685 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 130.9 \text{ N}\cdot\text{m}$$

6-15. A 460-V 60-Hz four-pole Y-connected induction motor is rated at 25 hp. The equivalent circuit parameters are

$R_1 = 0.15\ \Omega$ $\quad R_2 = 0.154\ \Omega$ $\quad X_M = 20\ \Omega$

$X_1 = 0.852\ \Omega$ $\quad X_2 = 1.066\ \Omega$

$P_{F\&W} = 400\text{ W}$ $\quad P_{misc} = 150\text{ W}$ $\quad P_{core} = 400\text{ W}$

For a slip of 0.02, find

(a) The line current I_L

(b) The stator power factor

(c) The rotor power factor

(d) The rotor frequency

(e) The stator copper losses P_{SCL}

(f) The air-gap power P_{AG}

(g) The power converted from electrical to mechanical form P_{conv}

(h) The induced torque τ_{ind}

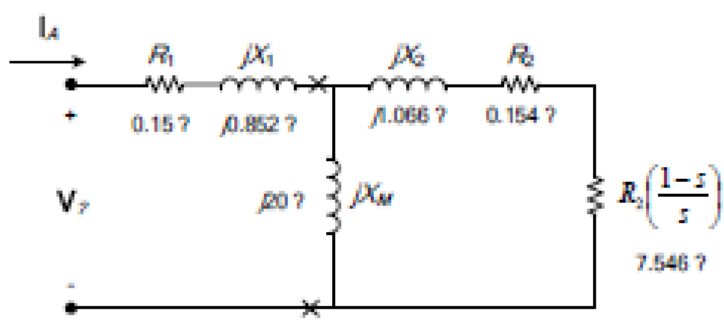
(i) The load torque τ_{load}

(j) The overall machine efficiency η

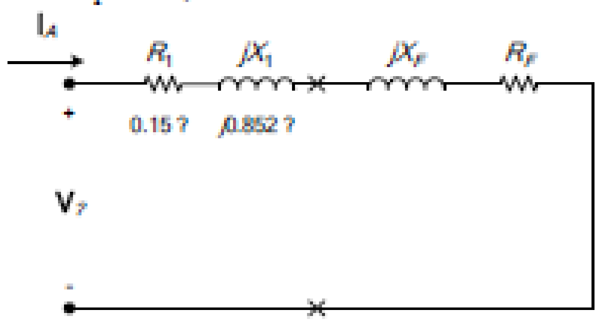
(k) The motor speed in revolutions per minute and radians per second

(l) What is the starting code letter for this motor?

SOLUTION The equivalent circuit of this induction motor is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M}+\frac{1}{Z_2}} = \frac{1}{\frac{1}{j20\ \Omega}+\frac{1}{7.70+j1.066}} = 6.123+j3.25 = 6.932\angle 28.0^\circ\ \Omega$$

The phase voltage is $460/\sqrt{3}$ = 266 V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1+jX_1+R_F+jX_F} = \frac{266\angle 0^\circ\ \text{V}}{0.15\ \Omega\ +j0.852\ \Omega+\ 6.123\ \Omega+j3.25\ \Omega}$$

$$I_L = I_A = 35.5\angle -33.2^\circ\ \text{A}$$

(b) The stator power factor is

$$\text{PF} = \cos(33.2^\circ) = 0.837\ \text{lagging}$$

(c) To find the rotor power factor, we must find the impedance angle of the rotor

$$\theta_R = \tan^{-1}\frac{X_2}{R_2/s} = \tan^{-1}\frac{1.066}{7.70} = 7.88^\circ$$

(d) The rotor frequency is

$$f_r = sf_s = (0.02)(60\ \text{Hz}) = 1.2\ \text{Hz}$$

Therefore the rotor power factor is

$$\text{PF}_R = \cos 7.88^\circ = 0.991\ \text{lagging}$$

(e) The stator copper losses are

$$P_{SCL} = 3I_A^2R_1 = 3(35.5\ \text{A})^2(0.15\ \Omega) = 567\ \text{W}$$

(f) The air gap power is $P_{AG} = 3I_2^2\frac{R_2}{s} = 3I_A^2R_F$

(Note that $3I_A^2R_F$ is equal to $3I_2^2\frac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{AG} = 3I_2^2\frac{R_2}{s} = 3I_A^2R_F = 3(35.5\ \text{A})^2(6.123\ \Omega) = 23.15\ \text{kW}$$

(g) The power converted from electrical to mechanical form is

$$P_{conv} = (1-s)P_{AG} = (1-0.02)\ (23.15\ \text{kW}) = 22.69\ \text{kW}$$

(h) The synchronous speed of this motor is

$$n_{sync} = \frac{120f_{se}}{P} = \frac{120(60\ \text{Hz})}{4} = 1800\ \text{r/min}$$

$$\omega_{sync} = (1800\ \text{r/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 188.5\ \text{rad/s}$$

Therefore the induced torque in the motor is

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{23.15\ \text{kW}}{(1800\ \text{r/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)} = 122.8\ \text{N}\cdot\text{m}$$

(i) The output power of this motor is

$$P_{OUT} = P_{conv} - P_{mech} - P_{core} - P_{misc} = 22.69 \text{ kW} - 400 \text{ W} - 400 \text{ W} - 150 \text{ W} = 21.74 \text{ kW}$$

The output speed is

$$n_m = (1 - s)\, n_{sync} = (1 - 0.02)\, (1800 \text{ r/min}) = 1764 \text{ r/min}$$

Therefore the load torque is

$$\tau_{load} = \frac{P_{OUT}}{\omega_m} = \frac{21.74 \text{ kW}}{(1764 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 117.7 \text{ N}\cdot\text{m}$$

(j) The overall efficiency is

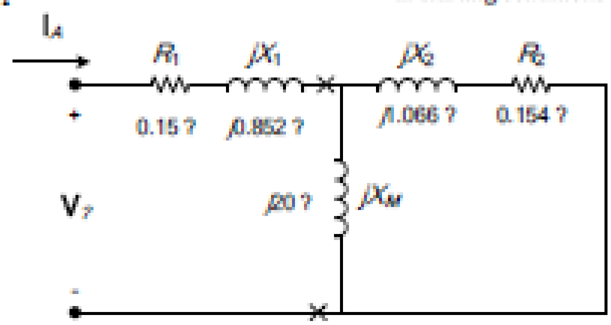
$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{P_{OUT}}{3V_\phi I_A \cos\theta} \times 100\%$$

$$\eta = \frac{21.74 \text{ kW}}{3(266 \text{ V})(35.5 \text{ A})\cos(33.2°)} \times 100\% = 91.7\%$$

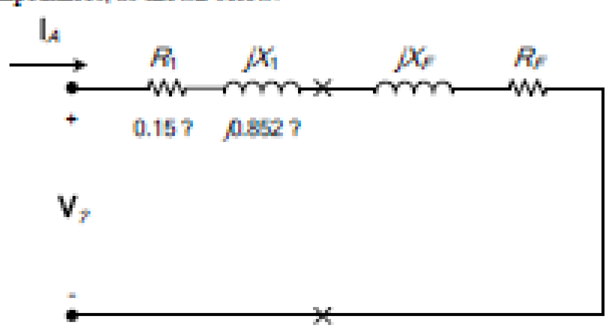
(k) The motor speed in revolutions per minute is 1764 r/min. The motor speed in radians per second is

$$\omega_m = (1764 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 184.7 \text{ rad/s}$$

(l) The equivalent circuit of this induction motor *at starting conditions* is shown below:



The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M}+\frac{1}{Z_2}} = \frac{1}{\frac{1}{j20\ \Omega}+\frac{1}{0.154+j1.066}} = 0.139+j1.013 = 1.023\angle 82.2^\circ\ \Omega$$

The phase voltage is $460/\sqrt{3} = 266$ V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{266\angle 0^\circ\ \text{V}}{0.15\ \Omega\ + j0.852\ \Omega +\ 0.139\ \Omega + j1.023\ \Omega}$$

$$I_L = I_A = 140.2\angle -81.2^\circ\ \text{A}$$

The starting kVA of the motor is

$$S_{start} = 3V_\phi I_A = 3(266\ \text{V})(140\ \text{A}) = 111.7\ \text{kVA}$$

The locked rotor kVA/hp is

$$\text{kVA/hp} = \frac{111.7\ \text{kVA}}{25\ \text{hp}} = 4.47$$

Therefore this motor is Starting Code Letter D.