4-2. A 13.8-kV, 50-MVA, 0.9-power-factor-lagging, 60-Hz, four-pole Y-connected synchronous generator has a synchronous reactance of 2.5 Ω and an armature resistance of 0.2 Ω. At 60 Hz, its friction and windage losses are 1 MW, and its core losses are 1.5 MW. The field circuit has a dc voltage of 120 V, and the maximum I_F is 10 A. The current of the field circuit is adjustable over the range from 0 to 10 A. The OCC of this generator is shown in Figure P4-1.

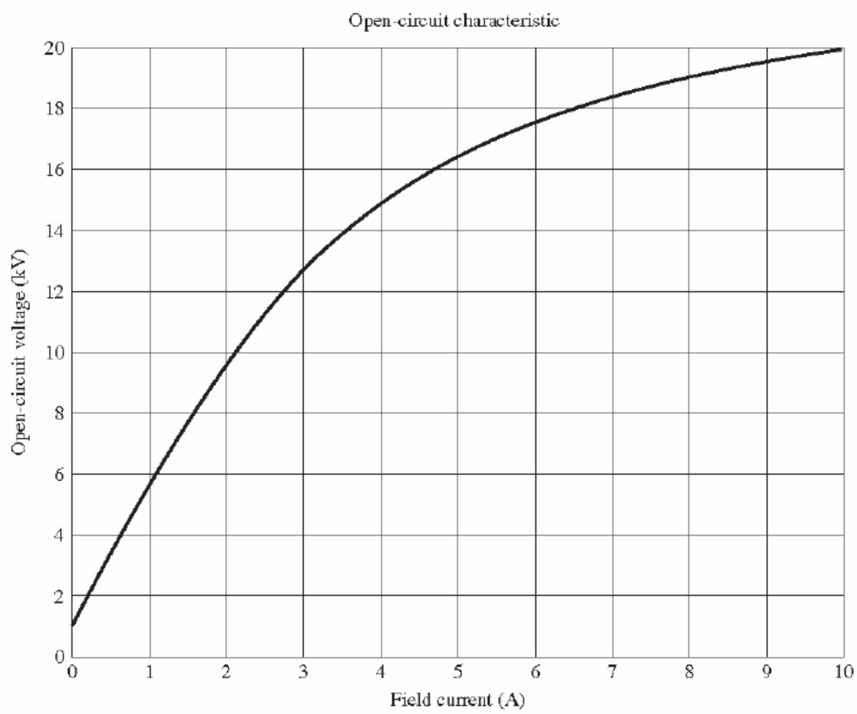


FIGURE P4–1
Open-circuit characteristic curve for the generator in Problem 4–2.

(a) How much field current is required to make the terminal voltage V_T (or line voltage V_L) equal to 13.8 kV when the generator is running at no load?

(b) What is the internal generated voltage E_A of this machine at rated conditions?

(c) What is the phase voltage V_ϕ of this generator at rated conditions?

(d) How much field current is required to make the terminal voltage V_T equal to 13.8 kV when the generator is running at rated conditions?

(e) Suppose that this generator is running at rated conditions, and then the load is removed without changing the field current. What would the terminal voltage of the generator be?

(f) How much steady-state power and torque must the generator's prime mover be capable of supplying to handle the rated conditions?

(a) If the no-load terminal voltage is 13.8 kV, the required field current can be read directly from the open-circuit characteristic. It is 3.50 A.

(b) This generator is Y-connected, so $I_L = I_A$. At rated conditions, the line and phase current in this generator is

$$I_A = I_L = \frac{P}{\sqrt{3}\ V_L} = \frac{50 \text{ MVA}}{\sqrt{3}(13800 \text{ V})} = 2092 \text{ A at an angle of } -25.8°$$

The phase voltage of this machine is $V_\phi = V_T / \sqrt{3} = 7967 \text{ V}$. The internal generated voltage of the machine is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 7967\angle 0° + (0.20\ \Omega)(2092\angle -25.8° \text{ A}) + j(2.5\ \Omega)(2092\angle -25.8° \text{ A})$$
$$\mathbf{E}_A = 11544\angle 23.1° \text{ V}$$

(c) The phase voltage of the machine at rated conditions is $V_\phi = 7967 \text{ V}$

From the OCC, the required field current is 10 A.

(d) The equivalent open-circuit terminal voltage corresponding to an E_A of 11544 volts is

$$V_{T,\text{oc}} = \sqrt{3}(11544 \text{ V}) = 20 \text{ kV}$$

From the OCC, the required field current is 10 A.

(e) If the load is removed without changing the field current, $V_\phi = E_A = 11544 \text{ V}$. The corresponding terminal voltage would be 20 kV.

(f) The input power to this generator is equal to the output power plus losses. The rated output power is

$$P_{\text{OUT}} = (50 \text{ MVA})(0.9) = 45 \text{ MW}$$

$$P_{\text{CU}} = 3I_A^{\,2}R_A = 3(2092 \text{ A})^2(0.2\ \Omega) = 2.6 \text{ MW}$$

$$P_{\text{F\&W}} = 1 \text{ MW}$$

$$P_{\text{core}} = 1.5 \text{ MW}$$

$$P_{\text{stray}} = (\text{assumed } 0)$$

$$P_{\text{IN}} = P_{\text{OUT}} + P_{\text{CU}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{stray}} = 50.1 \text{ MW}$$

Therefore the prime mover must be capable of supplying 50.1 MW. Since the generator is a four-pole 60 Hz machine, to must be turning at 1800 r/min. The required torque is

$$\tau_{\text{APP}} = \frac{P_{\text{IN}}}{\omega_m} = \frac{50.1 \text{ MW}}{(1800 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)} = 265{,}800 \text{ N}\cdot\text{m}$$

4-4. Assume that the field current of the generator in Problem 4-2 is adjusted to achieve rated voltage (13.8 kV) at full load conditions in each of the questions below.

(a) What is the efficiency of the generator at rated load?

(b) What is the voltage regulation of the generator if it is loaded to rated kilovoltamperes with 0.9-PF-lagging loads?

(c) What is the voltage regulation of the generator if it is loaded to rated kilovoltamperes with 0.9-PF-leading loads?

(d) What is the voltage regulation of the generator if it is loaded to rated kilovoltamperes with unity-power-factor loads?

SOLUTION

(a) This generator is Y-connected, so $I_L = I_A$. At rated conditions, the line and phase current in this generator is

$$I_A = I_L = \frac{P}{\sqrt{3}\, V_L} = \frac{50 \text{ MVA}}{\sqrt{3}\left(13800 \text{ V}\right)} = 2092 \text{ A} \text{ at an angle of } -25.8°$$

The phase voltage of this machine is $V_\phi = V_T / \sqrt{3} = 7967 \text{ V}$. The internal generated voltage of the machine is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A \mathbf{I}_A + jX_S \mathbf{I}_A$$

$$\mathbf{E}_A = 7967\angle 0° + \left(0.20\ \Omega\right)\left(251\angle -36.87° \text{ A}\right) + j\left(2.5\ \Omega\right)\left(2092\angle -25.8° \text{ A}\right)$$

$$\mathbf{E}_A = 11544\angle 23.1° \text{ V}$$

The input power to this generator is equal to the output power plus losses. The rated output power is

$$P_{\text{OUT}} = (50 \text{ MVA})(0.9) = 45 \text{ MW}$$

$$P_{\text{CU}} = 3I_A{}^2R_A = 3(2092 \text{ A})^2(0.2\ \Omega) = 2.6 \text{ MW}$$

$$P_{\text{F\&W}} = 1 \text{ MW}$$

$$P_{\text{core}} = 1.5 \text{ MW}$$

$$P_{\text{stray}} = (\text{assumed } 0)$$

$$P_{\text{IN}} = P_{\text{OUT}} + P_{\text{CU}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{stray}} = 50.1 \text{ MW}$$

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{45 \text{ MW}}{50.1 \text{ MW}} \times 100\% = 89.8\%$$

(b) If the generator is loaded to rated MVA with lagging loads, the phase voltage is $\mathbf{V}_\phi = 7967\angle 0° \text{ V}$ and the internal generated voltage is $\mathbf{E}_A = 11544\angle 23.1° \text{ V}$. Therefore, the phase voltage at no-load would be $\mathbf{V}_\phi = 11544\angle 0° \text{ V}$. The voltage regulation would be:

$$\text{VR} = \frac{11544 - 7967}{7967} \times 100\% = 44.9\%$$

(c) If the generator is loaded to rated kVA with leading loads, the phase voltage is $\mathbf{V}_\phi = 7967\angle 0° \text{ V}$ and the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 7967\angle 0° + (0.20\ \Omega)(2092\angle 25.8° \text{ A}) + j(2.5\ \Omega)(2092\angle 25.8° \text{ A})$$
$$\mathbf{E}_A = 7793\angle 38.8° \text{ V}$$

The voltage regulation would be:

$$\text{VR} = \frac{7793 - 7967}{7967} \times 100\% = -2.2\%$$

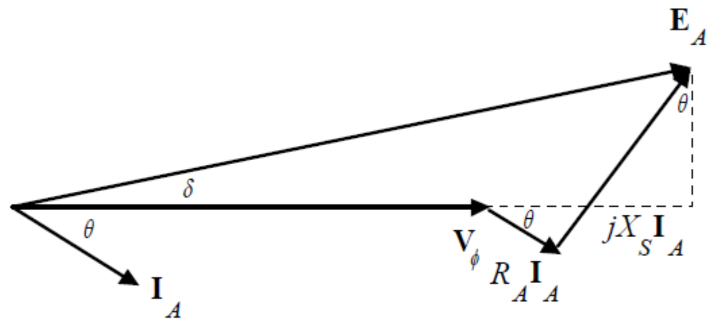
(d) If the generator is loaded to rated kVA at unity power factor, the phase voltage is $\mathbf{V}_\phi = 7967\angle 0° \text{ V}$ and the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 7967\angle 0° + (0.20\ \Omega)(2092\angle 0° \text{ A}) + j(2.5\ \Omega)(2092\angle 0° \text{ A})$$
$$\mathbf{E}_A = 9883\angle 32° \text{ V}$$

The voltage regulation would be:

$$\text{VR} = \frac{9883 - 7967}{7967} \times 100\% = 24\%$$

(e) For this problem, we will assume that the terminal voltage is adjusted to 13.8 kV at no load conditions, and see what happens to the voltage as load increases at 0.9 lagging, unity, and 0.9 leading power factors. Note that the maximum current will be 2092 A in any case. A phasor diagram representing the situation at lagging power factor is shown below:



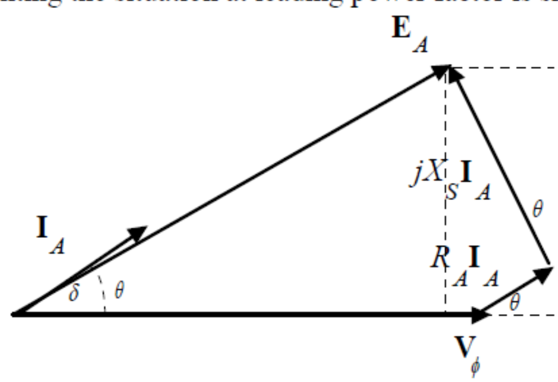
By the Pythagorean Theorem,

$$E_A^{\,2} = \left(V_\phi + R_A I_A \cos\theta + X_S I_A \sin\theta\right)^2 + \left(X_S I_A \cos\theta - R_A I_S \sin\theta\right)^2$$

$$V_\phi = \sqrt{E_A^{\,2} - \left(X_S I_A \cos\theta - R_A I_S \sin\theta\right)^2} - R_A I_A \cos\theta - X_S I_A \sin\theta$$

In this case, $\theta = 25.84°$, so $\cos\theta = 0.9$ and $\sin\theta = 0.6512$.

A phasor diagram representing the situation at leading power factor is shown below:



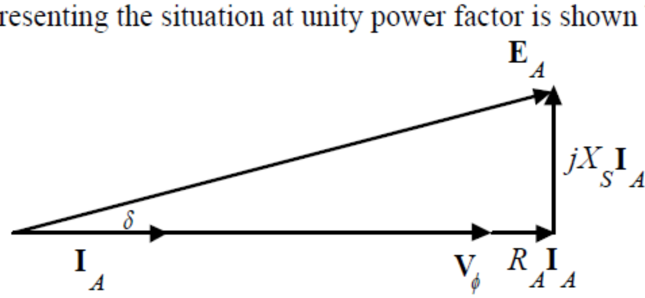
By the Pythagorean Theorem,

$$E_A^{\,2} = \left(V_\phi + R_A I_A \cos\theta - X_S I_A \sin\theta\right)^2 + \left(X_S I_A \cos\theta + R_A I_S \sin\theta\right)^2$$

$$V_\phi = \sqrt{E_A^{\,2} - \left(X_S I_A \cos\theta + R_A I_S \sin\theta\right)^2} - R_A I_A \cos\theta + X_S I_A \sin\theta$$

In this case, $\theta = -25.84°$, so $\cos\theta = 0.9$ and $\sin\theta = -0.6512$.

A phasor diagram representing the situation at unity power factor is shown below:



By the Pythagorean Theorem,

$$E_A^{\,2} = V_\phi^{\,2} + \left(X_S I_A\right)^2$$

$$V_\phi = \sqrt{E_A^{\,2} - \left(X_S I_A\right)^2}$$

In this case, $\theta = 0°$, so $\cos\theta = 1.0$ and $\sin\theta = 0$.

4-6. The internal generated voltage E_A of a **2-pole, Δ-connected, 60 Hz,** three phase synchronous generator is 14.4 kV, and the terminal voltage V_T is 12.8 kV. The synchronous reactance of this machine is 4 Ω, and the armature resistance can be ignored.

(a) If the torque angle of the generator $\delta = 18°$, how much power is being supplied by this generator at the current time?

(b) What is the power factor of the generator at this time?

(c) Sketch the phasor diagram under these circumstances.

(d) Ignoring losses in this generator, what torque must be applied to its shaft by the prime mover at these conditions?

SOLUTION

(a) If resistance is ignored, the output power from this generator is given by

$$P = \frac{3V_\phi E_A}{X_S} \sin\delta = \frac{3(12.8 \text{ kV})(14.4 \text{ kV})}{4\ \Omega} \sin 18° = 42.7 \text{ MW}$$

(b) The phase current flowing in this generator can be calculated from

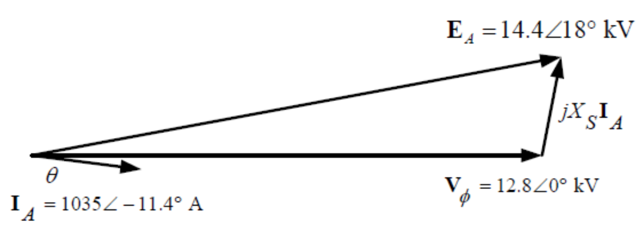
$$\mathbf{E}_A = \mathbf{V}_\phi + jX_S\mathbf{I}_A$$

$$\mathbf{I}_A = \frac{\mathbf{E}_A - \mathbf{V}_\phi}{jX_S}$$

$$\mathbf{I}_A = \frac{14.4\angle 18° \text{ kV} - 12.8\angle 0° \text{ kV}}{j4\ \Omega} = 1135\angle -11.4° \text{ A}$$

Therefore the impedance angle $\theta = 11.4°$, and the power factor is $\cos(11.4°) = 0.98$ lagging.

(c) The phasor diagram is



(d) The induced torque is given by the equation

$$P_{\text{conv}} = \tau_{\text{ind}}\omega_m$$

With no losses,

$$\tau_{\text{app}} = \tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{42.7 \text{ MW}}{2\pi(60 \text{ hz})} = 113{,}300 \text{ N•m}$$