King Fahd University of Petroleum & Minerals

Electrical Engineering Department

EE 360: Home Work # 5

Due Dates (May 5th , 2014)

From your textbook

Q1) Problem 5-1 (a-d)

SOLUTION

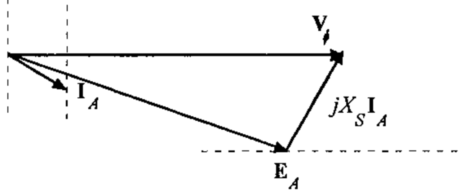
(a) The speed of this motor is given by

$$n_m = \frac{120 f_{se}}{P} = \frac{120(60 \text{ Hz})}{8} = 900 \text{ r/min}$$

(b) If losses are being ignored, the output power is equal to the input power, so the input power will be

$$P_{\text{IN}} = (400 \text{ hp})(746 \text{ W/hp}) = 298.4 \text{ kW}$$

This situation is shown in the phasor diagram below:



The line current flow under these circumstances is

$$I_L = \frac{P}{\sqrt{3}\, V_T \text{ PF}} = \frac{298.4 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.8)} = 449 \text{ A}$$

Because the motor is Δ-connected, the corresponding phase current is $I_A = 449/\sqrt{3} = 259 \text{ A}$. The angle of the current is $-\cos^{-1}(0.80) = -36.87°$, so $\mathbf{I}_A = 259\angle -36.87° \text{ A}$. The internal generated voltage $\mathbf{E}_A$ is

$$\mathbf{E}_A = \mathbf{V}_\phi - jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = (480\angle 0° \text{ V}) - j(0.6\ \Omega)(259\angle -36.87° \text{ A}) = 406\angle -17.8° \text{ V}$$

(c) This motor has 6 poles and an electrical frequency of 60 Hz, so its rotation speed is n_m = 1200 r/min. The induced torque is

$$\tau_{\text{ind}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{298.4 \text{ kW}}{(900 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)} = 3166 \text{ N}\cdot\text{m}$$

The maximum possible induced torque for the motor at this field setting is the maximum possible power divided by ω_m

$$\tau_{\text{ind,max}} = \frac{3V_\phi E_A}{\omega_m X_S} = \frac{3(480 \text{ V})(406 \text{ V})}{(900 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)(0.6\ \Omega)} = 10{,}340 \text{ N}\cdot\text{m}$$

The current operating torque is about 1/3 of the maximum possible torque.

(d) If the magnitude of the intemal generated voltage E_A is increased by 30%, the new torque angle can be found from the fact that $E_A \sin\delta \propto P = \text{constant}$.

$$E_{A2} = 1.30\ E_{A1} = 1.30(406\ \text{V}) = 487.2\ \text{V}$$

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_1\right) = \sin^{-1}\left(\frac{406\ \text{V}}{487.2\ \text{V}}\sin(-17.8°)\right) = -14.8°$$

The new armature current is

$$\mathbf{I}_{A2} = \frac{\mathbf{V}_\phi - \mathbf{E}_{A2}}{jX_S} = \frac{480\angle 0°\ \text{V} - 487.2\angle -14.8°\ \text{V}}{j0.6\ \Omega} = 208\angle -4.1°\ \text{A}$$

The magnitude of the armature current is 208 A, and the power factor is cos (-24.1°) = 0.913 lagging.

Q2) Problem 5-3

SOLUTION

(a) If this motor is assumed lossless, then the input power is equal to the output power. The input power to this motor is

$$P_{\text{IN}} = \sqrt{3}V_T I_L \cos\theta = \sqrt{3}(230\ \text{V})(40\ \text{A})(1.0) = 15.93\ \text{kW}$$

The rotational speed of the motor is

$$n_m = \frac{120 f_{se}}{P} = \frac{120(50\ \text{Hz})}{4} = 1500\ \text{r/min}$$

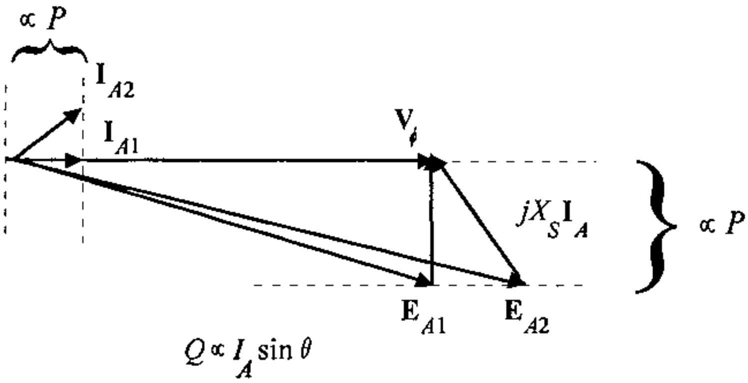
The output torque would be

$$\tau_{\text{LOAD}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{15.93\ \text{kW}}{(1500\ \text{r/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{r}}\right)} = 101.4\ \text{N}\cdot\text{m}$$

In English units,

$$\tau_{\text{LOAD}} = \frac{7.04\ P_{\text{OUT}}}{n_m} = \frac{(7.04)(15.93\ \text{kW})}{(1500\ \text{r/min})} = 74.8\ \text{lb}\cdot\text{ft}$$

(b) To change the motor's power factor to 0.8 leading, its field current must be increased. Since the power supplied to the load is independent of the field current level, an increase in field current increases $|\mathbf{E}_A|$ while keeping the distance $E_A \sin\delta$ constant. This increase in E_A changes the angle of the current $\mathbf{I}_A$, eventually causing it to reach a power factor of 0.8 leading.



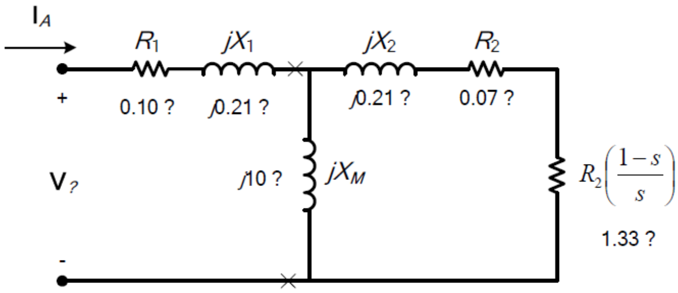
$$Q \propto I_A \sin\theta$$

(c) The magnitude of the line current will be

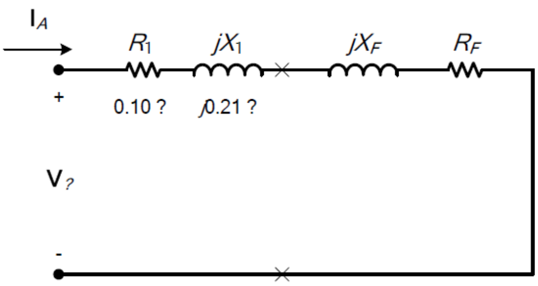
$$I_L = \frac{P}{\sqrt{3}\, V_T \text{ PF}} = \frac{15.93 \text{ kW}}{\sqrt{3}(230 \text{ V})(0.8)} = 50.0 \text{ A}$$

Q3) Problem 6.5

SOLUTION The equivalent circuit of this induction motor is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j10\ \Omega} + \frac{1}{1.40 + j0.21}} = 1.318 + j0.386 = 1.374\angle 16.3^\circ\ \Omega$$

The phase voltage is $208/\sqrt{3}$ = 120 V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{120\angle 0^\circ\ \text{V}}{0.10\ \Omega\ + j0.21\ \Omega +\ 1.318\ \Omega + j0.386\ \Omega}$$

$$I_L = I_A = 78.0\angle -22.8^\circ\ \text{A}$$

(b) The stator copper losses are

$$P_{\text{SCL}} = 3I_A^{\,2}R_1 = 3(78.0\ \text{A})^2(0.10\ \Omega) = 1825\ \text{W}$$

(c) The air gap power is $P_{\text{AG}} = 3I_2^{\,2}\frac{R_2}{s} = 3I_A^{\,2}R_F$

(Note that $3I_A^{\,2}R_F$ is equal to $3I_2^{\,2}\frac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{\text{AG}} = 3I_2^{\,2}\frac{R_2}{s} = 3I_A^{\,2}R_F = 3(78.0\ \text{A})^2(1.318\ \Omega) = 24.0\ \text{kW}$$

(d) The power converted from electrical to mechanical form is

$$P_{\text{conv}} = (1-s)P_{\text{AG}} = (1-0.05)(24.0\ \text{kW}) = 22.8\ \text{kW}$$

(e) The induced torque in the motor is

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}} = \frac{24.0\ \text{kW}}{(1800\ \text{r/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)} = 127.4\ \text{N}\cdot\text{m}$$

(f) The output power of this motor is

$$P_{\text{OUT}} = P_{\text{conv}} - P_{\text{mech}} - P_{\text{core}} - P_{\text{misc}} = 22.8\ \text{kW} - 500\ \text{W}\ -\ 400\ \text{W}\ -\ 0\ \text{W}\ =\ 21.9\ \text{kW}$$

The output speed is

$$n_m = (1-s)\ n_{\text{sync}} = (1-0.05)(1800\ \text{r/min}) = 1710\ \text{r/min}$$

Therefore the load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{21.9\ \text{kW}}{(1710\ \text{r/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)} = 122.3\ \text{N}\cdot\text{m}$$

(g) The overall efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{P_{\text{OUT}}}{3V_\phi I_A \cos\theta} \times 100\%$$

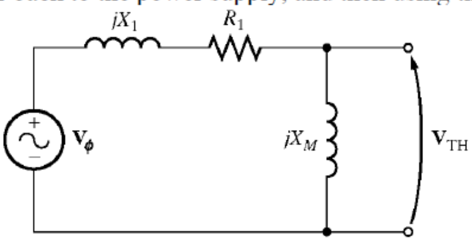
$$\eta = \frac{21.9\ \text{kW}}{3(120\ \text{V})(78.0\ \text{A})\cos 22.8^\circ} \times 100\% = 84.6\%$$

(h) The motor speed in revolutions per minute is 1710 r/min. The motor speed in radians per second is

$$\omega_m = (1710\ \text{r/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 179\ \text{rad/s}$$

Q4) Problem 6.6

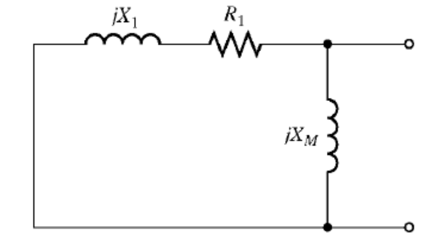
SOLUTION The slip at pullout torque is found by calculating the Thevenin equivalent of the input circuit from the rotor back to the power supply, and then using that with the rotor circuit model.



$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + jX_1 + jX_M}\mathbf{V}_\phi$$

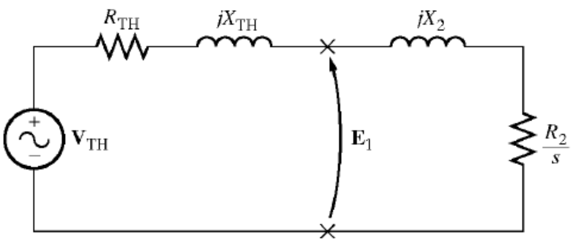
$$V_{TH} = \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}\mathbf{V}_\phi$$

(a)



$$Z_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$

(b)



(c)

$$Z_{TH} = \frac{jX_M\left(R_1 + jX_1\right)}{R_1 + j\left(X_1 + X_M\right)} = \frac{\left(j10\ \Omega\right)\left(0.10\ \Omega\ + j0.21\ \Omega\right)}{0.10\ \Omega\ + j\left(0.21\ \Omega + 10\ \Omega\right)} = 0.0959 + j0.2066\ \Omega = 0.2278\angle 65.1°\ \Omega$$

$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + j\left(X_1 + X_M\right)}\mathbf{V}_\phi = \frac{\left(j10\ \Omega\right)}{0.1\ \Omega\ + j\left(0.23\ \Omega + 10\ \Omega\right)}\left(120\angle 0°\ \text{V}\right) = 117.5\angle 0.6°\ \text{V}$$

The slip at pullout torque is

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^{\ 2} + \left(X_{TH} + X_2\right)^2}}$$

$$s_{max} = \frac{0.070\ \Omega}{\sqrt{\left(0.0959\ \Omega\right)^2 + \left(0.2066\ \Omega\ + 0.210\ \Omega\right)^2}} = 0.164$$

The pullout torque of the motor is

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}\right]}$$

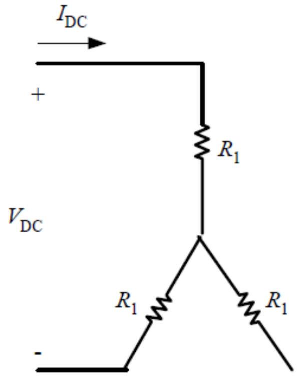
$$\tau_{max} = \frac{3\left(117.5\ \text{V}\right)^2}{2\left(188.5\ \text{rad/s}\right)\left[0.0959\ \Omega + \sqrt{\left(0.0959\ \Omega\right)^2 + \left(0.2066\ \Omega\ + 0.210\ \Omega\right)^2}\right]}$$

$$\tau_{max} = 210\ \text{N}\cdot\text{m}$$

Q5) Problem 6-20

SOLUTION From the DC test,

$$2R_1 = \frac{13.5\text{ V}}{64\text{ A}} \qquad \Rightarrow \qquad R_1 = 0.105\ \Omega$$



In the no-load test, the line voltage is 208 V, so the phase voltage is 120 V. Therefore,

$$X_1 + X_M = \frac{V_\phi}{I_{A,\text{nl}}} = \frac{120\text{ V}}{24.0\text{ A}} = 5.00\ \Omega \quad @ \quad 60\text{ Hz}$$

In the locked-rotor test, the line voltage is 24.6 V, so the phase voltage is 14.2 V. From the locked-rotor test *at 15 Hz*,

$$\left|Z'_{\text{LR}}\right| = \left|R_{\text{LR}} + jX'_{\text{LR}}\right| = \frac{V_\phi}{I_{A,\text{LR}}} = \frac{14.2\text{ V}}{64.5\text{ A}} = 0.220\ \Omega$$

$$\theta'_{\text{LR}} = \cos^{-1}\frac{P_{\text{LR}}}{S_{\text{LR}}} = \cos^{-1}\left[\frac{2200\text{ W}}{\sqrt{3}(24.6\text{ V})(64.5\text{ A})}\right] = 36.82°$$

Therefore,

$$R_{\text{LR}} = \left|Z'_{\text{LR}}\right| \cos\theta_{\text{LR}} = (0.220\ \Omega)\cos(36.82°) = 0.176\ \Omega$$

$$\Rightarrow \qquad R_1 + R_2 = 0.176\ \Omega$$

$$\Rightarrow \qquad R_2 = 0.071\ \Omega$$

$$X'_{\text{LR}} = \left|Z'_{\text{LR}}\right| \sin\theta_{\text{LR}} = (0.2202\ \Omega)\sin(36.82°) = 0.132\ \Omega$$

At a frequency of 60 Hz,

$$X_{\text{LR}} = \left(\frac{60\text{ Hz}}{15\text{ Hz}}\right) X'_{\text{LR}} = 0.528\ \Omega$$

For a Design Class B motor, the split is $X_1 = 0.211\ \Omega$ and $X_2 = 0.317\ \Omega$. Therefore,

$$X_M = 5.000\ \Omega - 0.211\ \Omega \;=\; 4.789\ \Omega$$

The resulting equivalent circuit is shown below:

Extra Problems

Q6) A 400 V, 60 Hz, 3-phase, 50 hp, Y-connected synchronous motor has a full load efficiency of 88%. The synchronous impedance of the motor is (0.2+j1.6) ohs/phase. If the excitation of the motor is adjusted to give it a leading pf of 0.9, calculate for full load,

a) The induced emf
b) The total mechanical power developed

Solution:

$$I_a = \frac{50 \times 746}{\sqrt{3} \times 400 \times 0.88 \times 0.9} \angle \cos^{-1} 0.9$$

$$= 68 \angle 25.84^\circ \text{ A}$$

a) $E_a = 231 \angle 0^\circ - (68 \angle 25.84^\circ)(0.2 + j1.6)$

$= 286.4 \angle -23.45^\circ \text{ V}$

$E_{L-L} = 495 \text{V}$

b) $P_{input} = 50 \times 746 / 0.88$

$= 42380 \text{ W}$

Total Cu Loss $= 3 I_a^2 R_a = 2774 \text{ W}$

$P_{em} = 42380 - 2774$

$= 39.3 \text{ kW}$

Also, Rotational Losses $= 39.3 \text{ kW} - 50 \times .746 \text{ kW}$

$= 2 \text{ kW}$

Q7)

A three phase , 4-pole , 60 Hz, Y- connected induction motor is rated at 10 hp, 208 V, 1755 rpm. The motor parameters are:

$R1 = 0.15\ \Omega, R2 = 0.15\ \Omega,$

$X1 = 0.4\ \Omega, X2 = 0.25\ \Omega, Xm = 30\Omega$

The combined rotational losses amounts to 500 W. The motor operates at rated speed when connected to a 208 V and 60-Hz source. Calculate:

a) Line current and PF
b) Output torque
c) Efficiency of the motor
d) Starting current and torque

Solution:

Refer to Fig. 8.8 ; $V_1 = \frac{208}{\sqrt{3}} \angle 0^\circ = 120 \angle 0^\circ$ V

(a) $n_s = \frac{120 f}{P} = \frac{(120)(60)}{4} = 1800$ rpm ; $\omega_s = \frac{2\pi n_s}{60} = \frac{2\pi(1800)}{60} = 188.5$ rad/s

$$s = \frac{n_s - n}{n_s} = \frac{1800 - 1755}{1800} = 0.025$$

$$I_2 = \frac{V_1}{R_1 + \frac{R_2}{s} + j(X_1 + X_2)} = \frac{120\angle 0^\circ}{0.15 + \frac{0.15}{0.025} + j(0.4 + 0.25)} = 19.4 \angle -6^\circ \text{ A}$$

$$I_m = \frac{V}{jX_m} = \frac{120\angle 0^\circ}{j30} = 4 \angle -90^\circ \text{ A}$$

$$I_1 = I_2 + I_m = 19.4 \angle -6^\circ + 4 \angle -90^\circ = 20.22 \angle -17.4^\circ \text{ A}$$

$$PF_1 = \cos 17.4^\circ = 0.954 \text{ lagging}$$

(b) $P_{in} = 3 V_1 I_1 \cos\theta = (3)(120)(20.22) \cos 17.4^\circ = 6946$ W

$$P_{ag} = 3 I_2^2 \frac{R_2}{s} = (3)(19.4)^2 \left(\frac{0.15}{0.025}\right) = 6774.5 \text{ W}$$

$$P_{dev} = (1-s) P_{ag} = (1 - 0.025)(6774.5) = 6605 \text{ W}$$

$$P_{out} = P_{dev} - P_{rot} = 6605 - 500 = 6105 \text{ W}$$

$$\omega_m = \frac{2\pi n}{60} = \frac{2\pi(1755)}{60} = 183.78 \text{ rad/s}$$

$$T_{out} = \frac{P_{out}}{\omega_m} = \frac{6105}{183.78} = 33.2 \text{ N-m}$$

(c) $\eta = \frac{P_{out}}{P_{in}} = \frac{6105}{6946} 100\% = 87.9\%$

(d) $I_{2,start} = \frac{V_1}{R_1 + R_2 + j(X_1 + X_2)} = \frac{120\angle 0^\circ}{0.15 + 0.15 + j(0.4 + 0.25)} = 167.6 \angle -65^\circ$ A

$$T_{start} = \frac{3 I_{2,start}^2 R_2}{\omega_s} = \frac{(3)(167.6)^2 (0.15)}{188.5} = 67 \text{ N-m}$$

Q8) A 500 V, 3 φ, 50 Hz induction motor develops an output of 15 KW at 950 r.p.m. If the input p.f. is 0.86 lagging, Mechanical (considered as Rotational in this problem) losses are 7.30 W and stator losses 1500W, Find

i) the slip
ii) the rotor cooper loss
iii) the motor input
iv) the line current

Solution:

i) VL = 500V, motor output Pr = 15KW

N = 950 r.p.m. P.f. = cos Ø = 0.86lags

Mech. Loss = 730 W

Stator loss = 1500 W

Ns = 120f/P = 120 * 50/6 = 1000r.p.m.

S = (Ns-N)/Ns * 100 = (1000 – 960)/1000 *100 = 0.05*100 = 5%

ii) Rotor output = Motor output + Mechanical output

= 15 + .730 watt = 15.73 KWatt

Therefore (Rotor Cu loss)/(Rotor output) = s/(s-1)

Or Rotor Cu loss = 15.73 * (0.05)/(1-0.05) = 827.89 watt

iii) Use power flow diagram for finding the motor input

Motor input = 15kw + 730 + 1500 + 827.89 = 18.058KW

iv) Line Current = $\sqrt{3}V_2I_2\cos\varphi$ I2 = 24.25A