EE 360: Homework # 4

Problem 1a:

Ea is proportional to the multiplication of both flux and frequency. So, higher frequency implies the need for less flux in order to produce the same amount of induced voltage. The need for less flux means the need for a smaller field winding, a smaller exciter, and hence a smaller rotor. This, in turn, implies a smaller machine.

Problem 1b:

1- Both machines must have the same phase sequence
2- Both must have the same voltage magnitudes
3- Both must have the same voltage phase shift.
4- Both must have *almost* the same frequency.

Problem 2:

(a) If the no-load terminal voltage is 13.8 kV, the required field current can be read directly from the open-circuit characteristic. It is 3.50 A.

(b) This generator is Y-connected, so $I_L = I_A$. At rated conditions, the line and phase current in this generator is

$$I_A = I_L = \frac{P}{\sqrt{3}\ V_L} = \frac{50 \text{ MVA}}{\sqrt{3}(13800 \text{ V})} = 2092 \text{ A at an angle of } -25.8°$$

The phase voltage of this machine is $V_\phi = V_T / \sqrt{3} = 7967 \text{ V}$. The internal generated voltage of the machine is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A \mathbf{I}_A + jX_S \mathbf{I}_A$$
$$\mathbf{E}_A = 7967\angle 0° + (0.20\ \Omega)(2092\angle -25.8° \text{ A}) + j(2.5\ \Omega)(2092\angle -25.8° \text{ A})$$
$$\mathbf{E}_A = 11544\angle 23.1° \text{ V}$$

(c) The phase voltage of the machine at rated conditions is $V_\phi = 7967 \text{ V}$

From the OCC, the required field current is 10 A.

(d) The equivalent open-circuit terminal voltage corresponding to an E_A of 11544 volts is

$$V_{T,oc} = \sqrt{3}(11544 \text{ V}) = 20 \text{ kV}$$

From the OCC, the required field current is 10 A.

(e) If the load is removed without changing the field current, $V_\phi = E_A = 11544 \text{ V}$. The corresponding terminal voltage would be 20 kV.

(f) The input power to this generator is equal to the output power plus losses. The rated output power is

$$P_{OUT} = (50 \text{ MVA})(0.9) = 45 \text{ MW}$$

$$P_{CU} = 3I_A^2 R_A = 3(2092 \text{ A})^2 (0.2\ \Omega) = 2.6 \text{ MW}$$

$P_{\text{F\&W}} = 1 \text{ MW}$

$P_{\text{core}} = 1.5 \text{ MW}$

$P_{\text{stray}} = \text{(assumed 0)}$

$P_{\text{IN}} = P_{\text{OUT}} + P_{\text{CU}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{stray}} = 50.1 \text{ MW}$

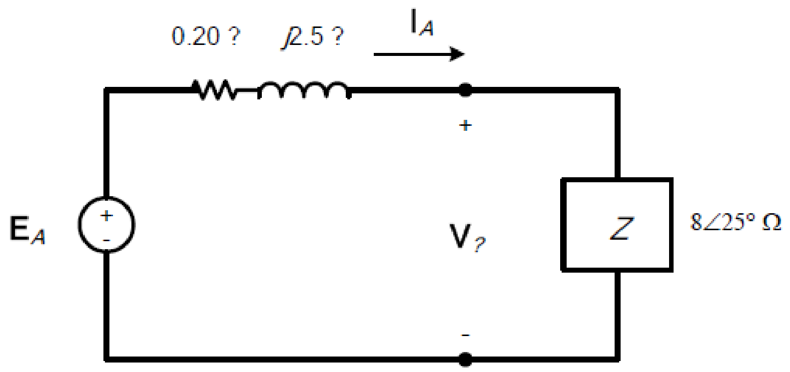
Therefore the prime mover must be capable of supplying 50.1 MW. Since the generator is a four-pole 60 Hz machine, to must be turning at 1800 r/min. The required torque is

$$\tau_{\text{APP}} = \frac{P_{\text{IN}}}{\omega_m} = \frac{50.1 \text{ MW}}{(1800 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)} = 265,800 \text{ N}\cdot\text{m}$$

Problem 3:

(a) If the field current is 5.0 A, the open-circuit terminal voltage will be about 16,500 V, and the open-circuit phase voltage in the generator (and hence E_A) will be $16,500/\sqrt{3} = 9526 \text{ V}$.

The load is Δ-connected with three impedances of $24\angle 25° \ \Omega$. From the Y-Δ transform, this load is equivalent to a Y-connected load with three impedances of $8\angle 25° \ \Omega$. The resulting per-phase equivalent circuit is shown below:



The magnitude of the phase current flowing in this generator is

$$I_A = \frac{E_A}{|R_A + jX_S + Z|} = \frac{9526 \text{ V}}{|0.2 + j2.5 + 8\angle 25°|} = \frac{9526 \text{ V}}{9.49\ \Omega} = 1004 \text{ A}$$

Therefore, the magnitude of the phase voltage is

$$V_\phi = I_A Z = (1004 \text{ A})(8\ \Omega) = 8032 \text{ V}$$

and the terminal voltage is

$$V_T = \sqrt{3}\ V_\phi = \sqrt{3}\ (8032 \text{ V}) = 13{,}910 \text{ V}$$

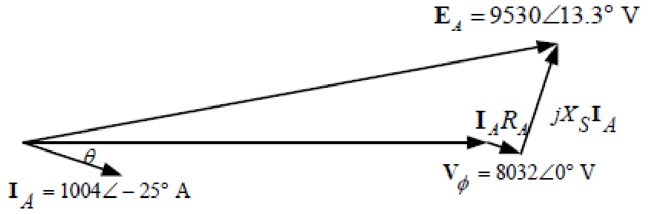
(b) Armature current is $\mathbf{I}_A = 1004\angle -25° \text{ A}$, and the phase voltage is $\mathbf{V}_\phi = 8032\angle 0° \text{ V}$. Therefore, the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = 8032\angle 0° + (0.20\ \Omega)(1004\angle -25° \text{ A}) + j(2.5\ \Omega)(1004\angle -25° \text{ A})$$

$$\mathbf{E}_A = 9530\angle 13.3° \text{ V}$$

The resulting phasor diagram is shown below (not to scale):



(c) The efficiency of the generator under these conditions can be found as follows:

$$P_{\text{OUT}} = 3\ V_\phi\ I_A\ \cos\theta = 3(8032 \text{ V})(1004 \text{ A})\cos(-25°) = 21.9 \text{ MW}$$

$$P_{\text{CU}} = 3 I_A^2 R_A = 3(1004 \text{ A})^2 (0.2\ \Omega) = 605 \text{ kW}$$

$$P_{\text{F\&W}} = 1 \text{ MW}$$

$$P_{\text{core}} = 1.5 \text{ MW}$$

$$P_{\text{stray}} = (\text{assumed } 0)$$

$$P_{\text{IN}} = P_{\text{OUT}} + P_{\text{CU}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{stray}} = 25 \text{ MW}$$

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{21.9 \text{ MW}}{25 \text{ MW}} \times 100\% = 87.6\%$$

(d) To get the basic idea of what happens, we will ignore the armature resistance for the moment. If the field current and the rotational speed of the generator are constant, then the magnitude of $\mathbf{E}_A\ (= K\phi\omega)$ is constant. The quantity $jX_S\mathbf{I}_A$ increases in length at the same angle, while the magnitude of $\mathbf{E}_A$ must remain constant. Therefore, $\mathbf{E}_A$ "swings" out along the arc of constant magnitude until the new $jX_S\mathbf{I}_S$ fits exactly between $\mathbf{V}_\phi$ and $\mathbf{E}_A$.

Problem 4:

(a)

$$I_A = I_L = \frac{S}{\sqrt{3}\ V_T} = \frac{120 \text{ MVA}}{\sqrt{3}(13.8 \text{ kV})} = 5020 \text{ A}$$

The power factor is 0.8 lagging, so $\mathbf{I}_A = 5020\angle -36.87° \text{ A}$. The phase voltage is 13.8 kV / $\sqrt{3}$ = 7967 V. Therefore, the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 7967\angle 0° + (0.1\ \Omega)(5020\angle -36.87° \text{ A}) + j(1.2\ \Omega)(5020\angle -36.87° \text{ A})$$
$$\mathbf{E}_A = 12{,}800\angle 20.7° \text{ V}$$

The resulting voltage regulation is

$$\text{VR} = \frac{12{,}800 - 7967}{7967} \times 100\% = 60.7\%$$

(b) If the generator is to be operated at 50 Hz with the same armature and field losses as at 60 Hz (so that the windings do not overheat), then its armature and field currents must not change. Since the voltage of the generator is directly proportional to the speed of the generator, the voltage rating (and hence the apparent power rating) of the generator will be reduced by a factor of 5/6.

$$V_{T,\text{rated}} = \frac{5}{6}(13.8 \text{ kV}) = 11.5 \text{ kV}$$

$$S_{\text{rated}} = \frac{5}{6}(120 \text{ MVA}) = 100 \text{ MVA}$$

Also, the synchronous reactance will be reduced by a factor of 5/6.

$$X_S = \frac{5}{6}(1.2\ \Omega) = 1.00\ \Omega$$

(c) At 50 Hz rated conditions, the armature current would be

$$I_A = I_L = \frac{S}{\sqrt{3}\ V_T} = \frac{100 \text{ MVA}}{\sqrt{3}(11.5 \text{ kV})} = 5020 \text{ A}$$

The power factor is 0.8 lagging, so $\mathbf{I}_A = 5020\angle -36.87° \text{ A}$. The phase voltage is 11.5 kV / $\sqrt{3}$ = 6640 V. Therefore, the internal generated voltage is

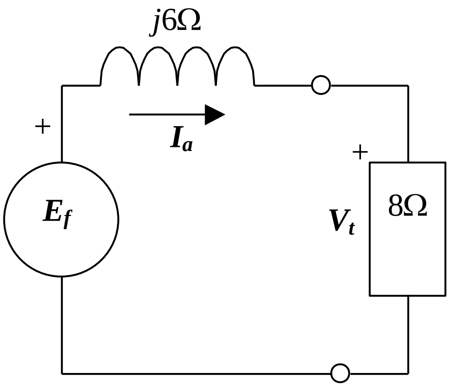
$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 6640\angle 0° + (0.1\ \Omega)(5020\angle -36.87° \text{ A}) + j(1.0\ \Omega)(5020\angle -36.87° \text{ A})$$
$$\mathbf{E}_A = 10{,}300\angle 18.8° \text{ V}$$

The resulting voltage regulation is

$$\text{VR} = \frac{10{,}300 - 6640}{6640} \times 100\% = 55.1\%$$

Problem 5:

a. The circuit is as shown.



At the resistor

$$\bar{I}_a = \frac{\bar{V}_t}{R_L} = \frac{3000/\sqrt{3}}{8} = 216.5\angle 0A$$

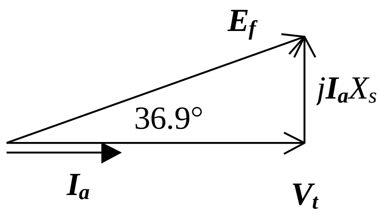
Then

$$\bar{E}_f = \bar{V}_t + j\bar{I}_a X_s = \frac{3000}{\sqrt{3}} + j216.5\cdot 6$$

$$\bar{E}_f = \bar{V}_t + j\bar{I}_a X_s = \frac{3000}{\sqrt{3}} + j216.5\cdot 6$$

$$\bar{E}_f = 1732 + j1300 = 2165\angle 36.9°$$

b.



c. For the 60 Hz machine,

$$f_{60} = \frac{np_{60}}{120} = 60$$

For the 50 Hz machine

$$f_{50} = \frac{np_{50}}{120} = 50$$

Note that *n*, the shaft speed, is the same for each machine. Dividing the two equations

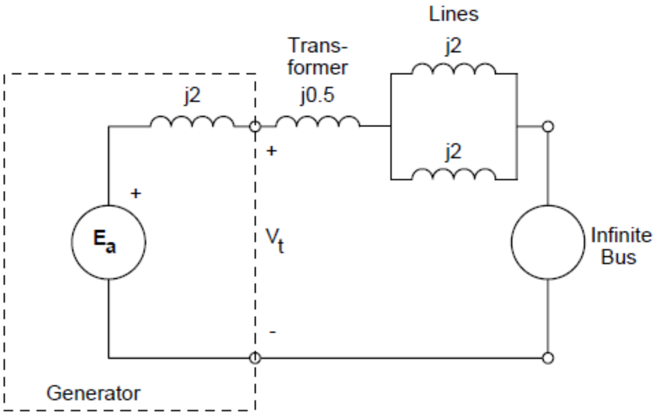
$$\frac{\frac{np_{60}}{120}}{\frac{np_{50}}{120}} = \frac{60}{50}$$

$$\frac{p_{60}}{p_{50}} = \frac{6}{5} = \frac{12}{10}$$

So 12 poles on the 60 Hz machine, 10 poles on the 50 Hz machine.
A common error is using 6 and 5 for the number of poles. Can't have an odd number of poles.

Problem 6:

The key to this problem is to think of the combination of X_s and the transmission lines as an equivalent X_s'. Then the problem is to work to find $\boldsymbol{E_a}$ (note, E_f could be used instead of E_a in calculations - different subscript for same quantity, open circuit voltage) before the outage, then the new value of δ after the outage.



Before the outage, the current supplied to the infinite bus is

$$\left|\bar{I}_a\right| = \frac{S_\phi}{V_\phi} = \frac{100\,MVA/3}{25kV/\sqrt{3}} = 2.31kA$$

The angle is

$$\theta = -\cos^{-1} pf = -\cos^{-1} 0.8 = -36.9° \text{ (which seems to be a popular angle)}$$

The original value of $\boldsymbol{E_a}$ is

$$\bar{E}_a = \bar{V}_t + j\bar{I}_a X_s' = \frac{25}{\sqrt{3}} + j2.31\angle -36.9 \cdot 3.5$$

Note that $X_s' = 3.5\Omega$

$$\bar{E}_a = 14.4 + 8.08\angle 53.1° = 14.4 + 4.85 + j6.46 = 19.25 + j6.46 = 20.3\angle 18.6° kV$$

Now consider what happens when the line trips. X_s' goes up, and becomes $X_s'' = 4.5\Omega$. The torque does not change, so the real power stays constant. The field current does not change, so the magnitude of E_a does not change. The power angle δ must therefore change. (This problem illustrates the danger of learning rules like the locus of constant power. That locus assumes that X_s stays constant. In this problem, X_s changed.)

So

$$P = \frac{E_a V_t}{X_s'} \sin\delta = \frac{E_a V_t}{X_s''} \sin\delta'$$

$$\sin\delta' = \frac{X_s''}{X_s'} \sin\delta = \frac{4.5}{3.5} \sin 18.6° = 0.41$$

$$\delta' = \sin^{-1} 0.41 = 24.2°$$

Let's just note that

$P = S \cdot pf = 1MVA \cdot 0.8 = 800MW$ and does not change. Reactive power Q, however, changes. The new Q, Q', is

$$Q' = \frac{E_a V_t}{X_s''} \cos\delta' - \frac{V_t^2}{X_s''} = \frac{20.3 \cdot 14.4}{4.5} \cos 24.2 - \frac{14.4^2}{4.5} = 65 \cdot 0.91 - 46 = 13.2MVAR$$

Now, line to neutral voltage values were used in this calculation, so this is actually the per-phase reactive power. The total reactive power is

$$Q_{3\phi}' = 3 \cdot 13.2 = 39.6MVAR$$

Complex power

$$S = 800 + j39.6 MVA = 801 MVA \angle 2.83°$$

or 801 MVA at a power factor of 0.999 current lagging - the outage has improved the power factor the generator and line deliver to the infinite bus.