King Fahd University of Petroleum & Minerals
Department of Electrical Engineering
EE 360: Home Work # 2
Due Date ( 3rd March )

# Key Solutions

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**Q1) Problem 2-1 (a, c, d, e)**

SOLUTION

*(a)* The turns ratio of this transformer is $a$ = 8000/277 = 28.88. Therefore, the primary impedances referred to the low voltage (secondary) side are

$$R_P' = \frac{R_P}{a^2} = \frac{5\ \Omega}{(28.88)^2} = 0.006\ \Omega$$

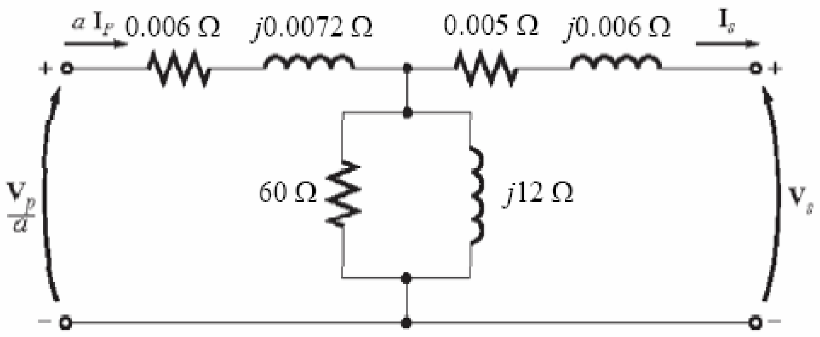
$$X_P' = \frac{R_P}{a^2} = \frac{6\ \Omega}{(28.88)^2} = 0.0072\ \Omega$$

and the excitation branch elements referred to the secondary side are

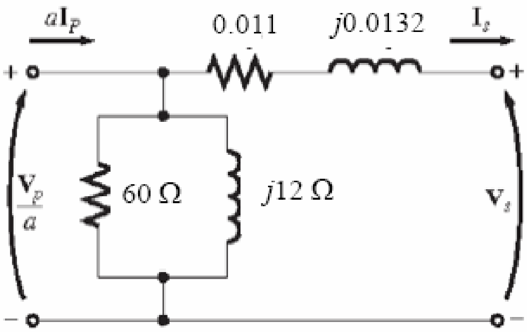
$$R_C' = \frac{R_C}{a^2} = \frac{50\ \text{k}\Omega}{(28.88)^2} = 60\ \Omega$$

$$X_M' = \frac{X_M}{a^2} = \frac{10\ \text{k}\Omega}{(28.88)^2} = 12\ \Omega$$

The resulting equivalent circuit is



*(c)* To simplify the calculations, use the simplified equivalent circuit referred to the secondary side of the transformer:




The secondary current in this transformer is

$$\mathbf{I}_S = \frac{100 \text{ kVA}}{277 \text{ V}} \angle -31.8° \text{ A} = 361\angle -31.8° \text{ A}$$

Therefore, the primary voltage on this transformer (referred to the secondary side) is

$$\mathbf{V}_P' = \mathbf{V}_S + \left(R_{\text{EQ}} + jX_{\text{EQ}}\right) \mathbf{I}_S$$

$$\mathbf{V}_P' = 277\angle 0° \text{ V} + \left(0.011 + j0.0132\right)\left(361\angle -31.8° \text{ A}\right) = 283\angle 0.4° \text{ V}$$

The voltage regulation of the transformer under these conditions is

$$\text{VR} = \frac{283 - 277}{277} \times 100\% = 2.2\%$$

*(d)* Under the conditions of part *(c)*, the transformer's output power copper losses and core losses are:

$$P_{\text{OUT}} = S \cos \theta = (100 \text{ kVA})(0.85) = 85 \text{ kW}$$

$$P_{\text{CU}} = (I_S)^2 R_{\text{EQ}} = (361)^2 (0.11) = 1430 \text{ W}$$

$$P_{\text{core}} = \frac{V_P'^2}{R_C} = \frac{283^2}{50} = 1602 \text{ W}$$

*(e)* The efficiency of this transformer is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{OUT}} + P_{\text{CU}} + P_{\text{core}}} \times 100\% = \frac{85{,}000}{85{,}000 + 1430 + 1602} \times 100\% = 96.6\%$$

**Q2) Problem 2-2**

SOLUTION

To solve this problem, we will refer the circuit to the secondary (low-voltage) side. The feeder's impedance referred to the secondary side is

$$Z_{\text{line}}' = \left(\frac{2.4 \text{ kV}}{14 \text{ kV}}\right)^2 (38.2\ \Omega + j140\ \Omega) = 1.12 + j4.11\ \Omega$$

The secondary current $\mathbf{I}_S$ is given by

$$I_S = \frac{90 \text{ kW}}{(2400 \text{ V})(0.8)} = 46.88 \text{ A}$$

The power factor is 0.80 lagging, so the impedance angle $\theta = \cos^{-1}(0.8) = 36.87°$, and the phasor current is

$$\mathbf{I}_S = 46.88\angle -36.87° \text{ A}$$

*(a)* The voltage at the power source of this system (referred to the secondary side) is

$$\mathbf{V}_{\text{source}}' = \mathbf{V}_S + \mathbf{I}_S Z_{\text{line}}' + \mathbf{I}_S Z_{\text{EQ}}$$

$$\mathbf{V}_{\text{source}}' = 2400\angle 0° \text{ V} + (46.88\angle -36.87° \text{ A})(1.12 + j4.11\ \Omega) + (46.88\angle -36.87° \text{ A})(0.10 + j0.40\ \Omega)$$

$$\mathbf{V}_{\text{source}}' = 2576\angle 3.0° \text{ V}$$

Therefore, the voltage at the power source is

$$\mathbf{V}_{\text{source}} = (2576\angle 3.0° \text{ V})\frac{14 \text{ kV}}{2.4 \text{ kV}} = 15.5\angle 3.0° \text{ kV}$$

*(b)* To find the voltage regulation of the transformer, we must find the voltage at the primary side of the transformer (referred to the secondary side) under these conditions:

$$\mathbf{V}_P' = \mathbf{V}_S + \mathbf{I}_S Z_{\text{EQ}}$$

$$\mathbf{V}_P' = 2400\angle 0° \text{ V} + (46.88\angle -36.87° \text{ A})(0.10 + j0.40\ \Omega) = 2415\angle 0.3° \text{ V}$$

There is a voltage drop of 15 V under these load conditions. Therefore the voltage regulation of the transformer is

$$\text{VR} = \frac{2415 - 2400}{2400} \times 100\% = 0.63\%$$

*(c)* The overall efficiency of the power system will be the ratio of the output power to the input power. The output power supplied to the load is $P_{\text{OUT}} = 90$ kW. The input power supplied by the source is

$$P_{\text{IN}} = P_{\text{OUT}} + P_{\text{LOSS}} = P_{\text{OUT}} + I^2 R = (90 \text{ kW}) + (46.88 \text{ A})^2 (1.22\ \Omega) = 92.68 \text{ kW}$$

$$P_{\text{IN}} = V_{\text{source}}' I_S \cos\theta = (2415 \text{ V})(46.88 \text{ A}) \cos 36.57° = 90.93 \text{ kW}$$

Therefore, the efficiency of the power system is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{90 \text{ kW}}{92.68 \text{ kW}} \times 100\% = 97.1\%$$

**<u>Extra Problems:</u>**

**Q3)** A 2.2-kVA, 440/220-V, step down transformer has the following equivalent parameters as referred to the primary side:

| | |
|---|---|
| Reqp = 3-Ω, | Rcp = 2.5-kΩ |
| Xeqp = 4- Ω, | Xmp = 2-kΩ |

The transformer is operating at full load with a power factor of 0.707 lagging. Determine the efficiency and the voltage regulation of the transformer

**Solution:**

For lagging power of 0.707

$$\theta = -45^{\circ}$$

Secondary current referred to primary

$$I_s^{'} = \frac{I_s}{a} = \frac{2200\ kVA\ /\ 220\ V}{440\ V\ \ /\ 220\ V} = 5\angle -45^{\circ}\ A$$

Secondary voltage referred to primary

$$V_s^{'} = aV_s = 2\times 220\ V\ = 440\angle 0^{\circ}\ V$$

Thus,

$$V_p = V_s^{'} + I_s^{'}(R_{eqp} + jX_{eqp}) = 440 + (5\angle -45^{\circ})(3 + \text{j}4)$$

$$= 464.76\angle 0.44^{\circ}\ V$$

The core-loss and magnetizing currents are,

$$I_{cp} = \frac{464.76\angle 0.44^{\circ}\ V}{2500} = 0.186\angle 0.44^{\circ}\ A$$

$$I_{mp} = \frac{464.76\angle 0.44^{\circ}\ V}{j\,2000} = 0.232\angle -89.56^{\circ}\ A$$

Total primary current,

$$I_p = I_s^{'} + I_{cp} + I_{mp} = 5\angle -45^{\circ} + 0.186\angle 0.44^{\circ} + 0.232\angle -89.56^{\circ}$$
$$= 5.296\angle -45.33^{\circ}\ A$$

The output power, input power and the efficiency are,

$$P_o = \mathrm{Re}\left[(440)(5\angle -45^{\circ})\right] = 1555.63\ W$$
$$P_{in} = \mathrm{Re}\left[(464.72\angle 044^{\circ})(5.296\angle -45.33^{\circ})\right] = 1716.91\ W$$

$$\eta = \frac{1555.63}{1716.91} \times 100 = 90.6\ \%$$

The voltage regulation is,

$$VR = \frac{464.76 - 440}{440} \times 100 = 5.63\ \%$$

**Q4)** A 120-kVA, 2400/240-V, step down transformer has the following parameters:

R1 = 0.75-Ω,   R2 = 0.01-Ω
X1 = 0.8- Ω,   X2 = 0.02 Ω

The transformer is designed to operate at maximum efficiency at 70% of its rated load with 0.8 power factor lagging. Determine:

a) the kVA rating of the transformer, copper loss and core loss at maximum efficiency
b) the maximum efficiency
c) the efficiency at full load and 0.8 power factor is lagging and the equivalent core-loss resistance

**Solution:**

The rated load current

$$I_{s\_rated} = \frac{kVA_{rated}}{V_{s\_rated}} = \frac{120\ kVA}{240\ V} = 500\ A$$

The load current at maximum efficiency

$$I_{s\eta} = 0.7 \times I_{s\_rated} = 0.7 \times 500 = 350\ A$$

a) The kVA rating of the transformer at maximum efficiency:

$$kVA\big|_{\max-eff} = I_{s\eta} \times V_{s\_rated} = 350\ A \times 240\ V = 84\ kVA$$

the copper loss at maximum efficiency:

$$P_{Cu\eta} = I_{s\eta}^2 \left( \frac{R_1}{a^2} + R_2 \right) = 350^2 \left( \frac{0.75}{10^2} + 0.01 \right) = 2.14\ kW$$

the core loss at maximum efficiency:

$$P_{core\eta} = P_{Cu\eta} = 2.14\ kW$$

b) The output power, input power and efficiency when the transformer delivers load at maximum efficiency:

$$P_o = V_{s\_rated} \times I_{s\eta} \times PF = 240\ V \times 350\ A \times 0.8 = 67.2\ kW$$

$$P_{in} = P_o + P_{core\eta} + P_{Cu\eta} = 67.2 + 2.14 + 2.14 = 71.48\ kW$$

$$\eta = \frac{P_o}{P_{in}} \times 100 = \frac{67.2}{71.48} \times 100 = 94\ \%$$

c) The output power, input power and efficiency at full load:

$$P_o = V_{s\_rated} \times I_{s\_rated} \times PF = 240\ V \times 500\ A \times 0.8 = 96\ kW$$

$$P_{Cu} = I_{s\_rated}^2 \left( \frac{R_1}{a^2} + R_2 \right) = 500^2 \left( \frac{0.75}{10^2} + 0.01 \right) = 4.375\ kW$$

$$\eta = \frac{P_o}{P_o + P_{Cu} + P_{core}} \times 100 = \frac{96}{96 + 4.375 + 2.14} \times 100 = 93.64\ \%$$

d) The equivalent core loss resistance at no load is:

$$R_C = \frac{V_p^2}{P_{core}} = \frac{2400^2\ V}{2.14\ kW} = 2691\ \Omega$$