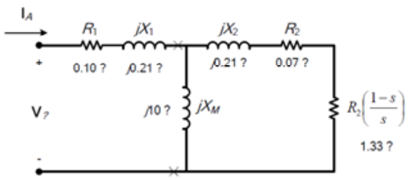
EE 360 (122)

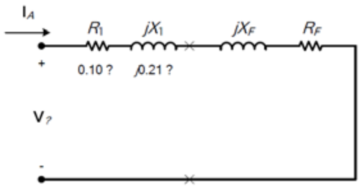
Solution: HW-5

Q1) P6-5

SOLUTION The equivalent circuit of this induction motor is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j10\ \Omega} + \frac{1}{1.40 + j0.21}} = 1.318 + j0.386 = 1.374\angle 16.3^\circ\ \Omega$$

The phase voltage is $208/\sqrt{3} = 120$ V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{120\angle 0^\circ\ \text{V}}{0.10\ \Omega + j0.21\ \Omega + 1.318\ \Omega + j0.386\ \Omega}$$

$$I_L = I_A = 78.0\angle -22.8^\circ\ \text{A}$$

(b) The stator copper losses are

$$P_{SCL} = 3I_A^2 R_1 = 3(78.0\ \text{A})^2 (0.10\ \Omega) = 1825\ \text{W}$$

(c) The air gap power is $P_{AG} = 3I_2^2 \frac{R_2}{s} = 3I_A^2 R_F$

(Note that $3I_A^2 R_F$ is equal to $3I_2^2 \frac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{AG} = 3I_2^2 \frac{R_2}{s} = 3I_A^2 R_F = 3(78.0\ \text{A})^2 (1.318\ \Omega) = 24.0\ \text{kW}$$

(d) The power converted from electrical to mechanical form is

$$P_{conv} = (1-s)P_{AG} = (1-0.05)(24.0\ \text{kW}) = 22.8\ \text{kW}$$

(e) The induced torque in the motor is

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{24.0\ \text{kW}}{(1800\ \text{r/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)} = 127.4\ \text{N}\cdot\text{m}$$

(f) The output power of this motor is

$$P_{OUT} = P_{conv} - P_{mech} - P_{core} - P_{misc} = 22.8\ \text{kW} - 500\ \text{W} - 400\ \text{W} - 0\ \text{W} = 21.9\ \text{kW}$$

The output speed is

$$n_m = (1-s)\ n_{sync} = (1-0.05)(1800\ \text{r/min}) = 1710\ \text{r/min}$$

Therefore the load torque is

$$\tau_{load} = \frac{P_{OUT}}{\omega_m} = \frac{21.9 \text{ kW}}{(1710 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 122.3 \text{ N}\cdot\text{m}$$

(g) The overall efficiency is

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{P_{OUT}}{3V_\phi I_A \cos\theta} \times 100\%$$

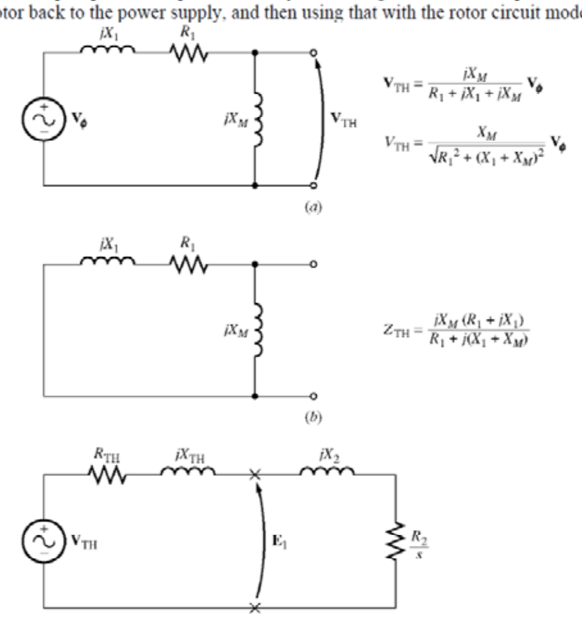
$$\eta = \frac{21.9 \text{ kW}}{3(120 \text{ V})(78.0 \text{ A})\cos 22.8°} \times 100\% = 84.6\%$$

(h) The motor speed in revolutions per minute is 1710 r/min. The motor speed in radians per second is

$$\omega_m = (1710 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 179 \text{ rad/s}$$

Q2) P-6-6

Solution The slip at pullout torque is found by calculating the Thevenin equivalent of the input circuit from the rotor back to the power supply, and then using that with the rotor circuit model.



$$Z_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)} = \frac{(j10\ \Omega)(0.10\ \Omega + j0.21\ \Omega)}{0.10\ \Omega + j(0.21\ \Omega + 10\ \Omega)} = 0.0959 + j0.2066\ \Omega = 0.2278\angle 65.1°\ \Omega$$

$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + j(X_1 + X_M)}\mathbf{V}_\phi = \frac{(j10\ \Omega)}{0.1\ \Omega + j(0.23\ \Omega + 10\ \Omega)}(120\angle 0°\ \text{V}) = 117.5\angle 0.6°\ \text{V}$$

The slip at pullout torque is

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$s_{max} = \frac{0.070\ \Omega}{\sqrt{(0.0959\ \Omega)^2 + (0.2066\ \Omega + 0.210\ \Omega)^2}} = 0.164$$

The pullout torque of the motor is

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}\right]}$$

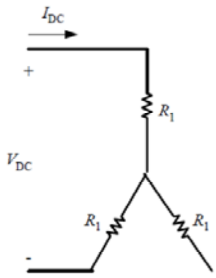
$$\tau_{max} = \frac{3(117.5\ \text{V})^2}{2(188.5\ \text{rad/s})\left[0.0959\ \Omega + \sqrt{(0.0959\ \Omega)^2 + (0.2066\ \Omega + 0.210\ \Omega)^2}\right]}$$

$$\tau_{max} = 210\ \text{N}\cdot\text{m}$$

Q3) P-6-20

SOLUTION From the DC test,

$$2R_1 = \frac{13.5\text{ V}}{64\text{ A}} \qquad \Rightarrow \qquad R_1 = 0.105\ \Omega$$



In the no-load test, the line voltage is 208 V, so the phase voltage is 120 V. Therefore,

$$X_1 + X_M = \frac{V_\phi}{I_{A,nl}} = \frac{120\text{ V}}{24.0\text{ A}} = 5.00\ \Omega \quad @ \quad 60\text{ Hz}$$

In the locked-rotor test, the line voltage is 24.6 V, so the phase voltage is 14.2 V. From the locked-rotor test *at 15 Hz*,

$$|Z'_{LR}| = |R_{LR} + jX'_{LR}| = \frac{V_\phi}{I_{A,LR}} = \frac{14.2\text{ V}}{64.5\text{ A}} = 0.220\ \Omega$$

$$\theta'_{LR} = \cos^{-1}\frac{P_{LR}}{S_{LR}} = \cos^{-1}\left[\frac{2200\text{ W}}{\sqrt{3}(24.6\text{ V})(64.5\text{ A})}\right] = 36.82°$$

Therefore,

$$R_{LR} = |Z'_{LR}|\ \cos\theta_{LR} = (0.220\ \Omega)\cos(36.82°) = 0.176\ \Omega$$

$$\Rightarrow \qquad R_1 + R_2 = 0.176\ \Omega$$

$$\Rightarrow \qquad R_2 = 0.071\ \Omega$$

$$X'_{LR} = |Z'_{LR}|\ \sin\theta_{LR} = (0.2202\ \Omega)\sin(36.82°) = 0.132\ \Omega$$

At a frequency of 60 Hz,

$$X_{LR} = \left(\frac{60\text{ Hz}}{15\text{ Hz}}\right) X'_{LR} = 0.528\ \Omega$$

For a Design Class B motor, the split is $X_1 = 0.211\ \Omega$ and $X_2 = 0.317\ \Omega$. Therefore,

$$X_M = 5.000\ \Omega - 0.211\ \Omega = 4.789\ \Omega$$

The resulting equivalent circuit is shown below:

Q4)

Refer to Fig. 8.8 ; $V_1 = \frac{208}{\sqrt{3}} \angle 0° = 120 \angle 0°$ V

(a) $n_s = \frac{120 f}{P} = \frac{(120)(60)}{4} = 1800$ rpm ; $\omega_s = \frac{2\pi n_s}{60} = \frac{2\pi(1800)}{60} = 188.5$ rad/s

$s = \frac{n_s - n}{n_s} = \frac{1800 - 1755}{1800} = 0.025$

$I_2 = \frac{V_1}{R_1 + \frac{R_2}{s} + j(X_1 + X_2)} = \frac{120 \angle 0°}{0.15 + \frac{0.15}{0.025} + j(0.4 + 0.25)} = 19.4 \angle -6°$ A

$I_m = \frac{V}{jX_m} = \frac{120\angle 0°}{j30} = 4 \angle -90°$ A

$I_1 = I_2 + I_m = 19.4 \angle -6° + 4 \angle -90° = 20.22 \angle -17.4°$ A

$PF_1 = \cos 17.4° = 0.954$ lagging

(b) $P_{in} = 3 V_1 I_1 \cos\theta = (3)(120)(20.22) \cos 17.4° = 6946$ W

$P_{ag} = 3 I_2^2 \frac{R_2}{s} = (3)(19.4)^2 \left(\frac{0.15}{0.025}\right) = 6774.5$ W

$P_{dev} = (1-s) P_{ag} = (1 - 0.025)(6774.5) = 6605$ W

$P_{out} = P_{dev} - P_{rot} = 6605 - 500 = 6105$ W ;

$\omega_m = \frac{2\pi n}{60} = \frac{2\pi(1755)}{60} = 183.78$ rad/s

$T_{out} = \frac{P_{out}}{\omega_m} = \frac{6105}{183.78} = 33.2$ N-m

(c) $\eta = \frac{P_{out}}{P_{in}} = \frac{6105}{6946} 100\% = 87.9\%$

(d) $I_{2,start} = \frac{V_1}{R_1 + R_2 + j(X_1 + X_2)} = \frac{120 \angle 0°}{0.15 + 0.15 + j(0.4 + 0.25)} = 167.6 \angle -65°$ A

$T_{start} = \frac{3 I_{2,start}^2 R_2}{\omega_s} = \frac{(3)(167.6)^2 (0.15)}{188.5} = 67$ N-m