EE 360-122

HW2-a Solution

Q 2.1

SOLUTION

(a) The turns ratio of this transformer is $a = 8000/277 = 28.88$. Therefore, the primary impedances referred to the low voltage (secondary) side are

$$R_P' = \frac{R_P}{a^2} = \frac{5\ \Omega}{(28.88)^2} = 0.006\ \Omega$$

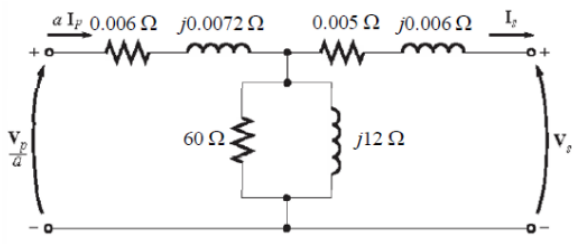
$$X_P' = \frac{R_P}{a^2} = \frac{6\ \Omega}{(28.88)^2} = 0.0072\ \Omega$$

and the excitation branch elements referred to the secondary side are

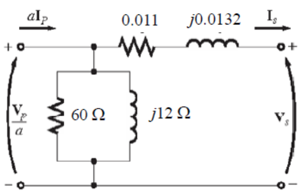
$$R_C' = \frac{R_C}{a^2} = \frac{50\ \text{k}\Omega}{(28.88)^2} = 60\ \Omega$$

$$X_M' = \frac{X_M}{a^2} = \frac{10\ \text{k}\Omega}{(28.88)^2} = 12\ \Omega$$

The resulting equivalent circuit is



(c) To simplify the calculations, use the simplified equivalent circuit referred to the secondary side of the transformer:



The secondary current in this transformer is

$$\mathbf{I}_S = \frac{100\ \text{kVA}}{277\ \text{V}} \angle -31.8° \text{ A} = 361\angle -31.8° \text{ A}$$

Therefore, the primary voltage on this transformer (referred to the secondary side) is

$$\mathbf{V}_P' = \mathbf{V}_S + (R_{EQ} + jX_{EQ})\,\mathbf{I}_S$$

$$\mathbf{V}_P' = 277\angle 0° \text{ V} + (0.011 + j0.0132)(361\angle -31.8° \text{ A}) = 283\angle 0.4° \text{ V}$$

The voltage regulation of the transformer under these conditions is

$$\text{VR} = \frac{283 - 277}{277} \times 100\% = 2.2\%$$

(d) Under the conditions of part *(c)*, the transformer's output power copper losses and core losses are:

$$P_{OUT} = S \cos\theta = (100\ \text{kVA})(0.85) = 85\ \text{kW}$$

$$P_{CU} = (I_S)^2 R_{EQ} = (361)^2 (0.11) = 1430\ \text{W}$$

$$P_{core} = \frac{V_P'^2}{R_C} = \frac{283^2}{50} = 1602\ \text{W}$$

(e) The efficiency of this transformer is

$$\eta = \frac{P_{OUT}}{P_{OUT} + P_{CU} + P_{core}} \times 100\% = \frac{85{,}000}{85{,}000 + 1430 + 1602} \times 100\% = 96.6\%$$

Q2.6

SOLUTION

(a) <u>OPEN CIRCUIT TEST</u> (referred to the low-voltage or secondary side):

$$|Y_{EX}| = |G_C - jB_M| = \frac{0.11\text{ A}}{115\text{ V}} = 0.0009565\text{ S}$$

$$\theta = \cos^{-1}\frac{P_{OC}}{V_{OC}I_{OC}} = \cos^{-1}\frac{3.9\text{ W}}{(115\text{ V})(0.11\text{ A})} = 72.0°$$

$$Y_{EX} = G_C - jB_M = 0.0009565\angle -72°\text{ S} = 0.0002956 - j0.0009096\text{ S}$$

$$R_C = \frac{1}{G_C} = 3383\ \Omega$$

$$X_M = \frac{1}{B_M} = 1099\ \Omega$$

<u>SHORT CIRCUIT TEST</u> (referred to the high-voltage or primary side):

$$|Z_{EQ}| = |R_{EQ} + jX_{EQ}| = \frac{17.1\text{ V}}{8.7\text{ A}} = 1.97\ \Omega$$

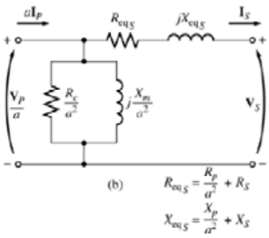
$$\theta = \cos^{-1}\frac{P_{SC}}{V_{SC}I_{SC}} = \cos^{-1}\frac{38.1\text{ W}}{(17.1\text{ V})(8.7\text{ A})} = 75.2°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 1.97\angle 75.2°\ \Omega = 0.503 + j1.905\ \Omega$$

$$R_{EQ} = 0.503\ \Omega$$

$$X_{EQ} = j1.905\ \Omega$$

To convert the equivalent circuit to the secondary side, divide each series impedance by the square of the turns ratio ($a = 230/115 = 2$). Note that the excitation branch elements are already on the secondary side. The resulting equivalent circuit is shown below:



$R_{EQ,S} = 0.126\ \Omega$ $\qquad$ $X_{EQ,S} = j0.476\ \Omega$

$R_{C,S} = 3383\ \Omega$ $\qquad$ $X_{M,S} = 1099\ \Omega$

(b) To find the required voltage regulation, we will use the equivalent circuit of the transformer referred to the secondary side. The rated secondary current is

$$I_S = \frac{1000\text{ VA}}{115\text{ V}} = 8.70\text{ A}$$

We will now calculate the primary voltage referred to the secondary side and use the voltage regulation equation for each power factor.

(1) **<u>0.8 PF Lagging:</u>**

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{EQ}\mathbf{I}_S = 115\angle 0°\text{ V} + (0.126 + j0.476\ \Omega)(8.7\angle -36.87°\text{ A})$$

$$\mathbf{V}_P' = 118.4\angle 1.3°\text{ V}$$

$$\text{VR} = \frac{118.4\text{-}115}{115} \times 100\% = 2.96\%$$

(2) **<u>1.0 PF:</u>**

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{EQ}\mathbf{I}_S = 115\angle 0°\text{ V} + (0.126 + j0.476\ \Omega)(8.7\angle 0.0°\text{ A})$$

$$\mathbf{V}_P' = 116.2\angle 2.04°\text{ V}$$

$$VR = \frac{116.2\text{-}115}{115} \times 100\% = 1.04\%$$

(3) <u>0.8 PF Leading:</u>

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{EQ}\mathbf{I}_S = 115\angle 0° \text{ V} + (0.126 + j0.476\ \Omega)(8.7\angle 36.87° \text{ A})$$

$$\mathbf{V}_P' = 113.5\angle 2.0° \text{ V}$$

$$VR = \frac{113.5\text{-}115}{115} \times 100\% = -1.3\%$$

(c) At rated conditions and 0.8 PF lagging, the output power of this transformer is

$$P_{OUT} = V_S I_S \cos\theta = (115 \text{ V})(8.7 \text{ A})(0.8) = 800 \text{ W}$$

The copper and core losses of this transformer are

$$P_{CU} = I_S^2 R_{EQ,S} = (8.7 \text{ A})^2 (0.126\ \Omega) = 9.5 \text{ W}$$

$$P_{core} = \frac{(V_P')^2}{R_C} = \frac{(118.4 \text{ V})^2}{3383\ \Omega} = 4.1 \text{ W}$$

Therefore the efficiency of this transformer at these conditions is

$$\eta = \frac{P_{OUT}}{P_{OUT} + P_{CU} + P_{core}} \times 100\% = \frac{800 \text{ W}}{800 \text{ W} + 9.5 \text{ W} + 4.1 \text{ W}} = 98.3\%$$

Q2.7

SOLUTION

(a) The easiest way to solve this problem is to refer all components to the *primary* side of the transformer. The turns ratio is $a = 8000/230 = 34.78$. Thus the load impedance referred to the primary side is

$$Z_L' = (34.78)^2 (2.0 + j0.7\ \Omega) = 2419 + j847\ \Omega$$

The referred secondary current is

$$\mathbf{I}_S' = \frac{7967\angle 0° \text{ V}}{(20 + j100\ \Omega) + (2419 + j847\ \Omega)} = \frac{7967\angle 0° \text{ V}}{2616\angle 21.2°\ \Omega} = 3.045\angle -21.2° \text{ A}$$

and the referred secondary voltage is

$$\mathbf{V}_S' = \mathbf{I}_S' Z_L' = (3.045\angle -21.2° \text{ A})(2419 + j847\ \Omega) = 7804\angle -1.9° \text{ V}$$

The actual secondary voltage is thus

$$\mathbf{V}_S = \frac{\mathbf{V}_S'}{a} = \frac{7804\angle -1.9° \text{ V}}{34.78} = 224.4\angle -1.9° \text{ V}$$

The voltage regulation is

$$VR = \frac{7967\text{-}7804}{7804} \times 100\% = 2.09\%$$

(b) The easiest way to solve this problem is to refer all components to the *primary* side of the transformer. The turns ratio is again $a = 34.78$. Thus the load impedance referred to the primary side is

$$Z_L' = (34.78)^2 (-j3.0\ \Omega) = -j3629\ \Omega$$

The referred secondary current is

$$\mathbf{I}_S' = \frac{7967\angle 0° \text{ V}}{(20 + j100\ \Omega) + (-j3629\ \Omega)} = \frac{7967\angle 0° \text{ V}}{3529\angle -89.7°\ \Omega} = 2.258\angle 89.7° \text{ A}$$

and the referred secondary voltage is

$$\mathbf{V}_S' = \mathbf{I}_S' Z_L' = (2.258\angle 89.7° \text{ A})(-j3629\ \Omega) = 8194\angle -0.3° \text{ V}$$

The actual secondary voltage is thus

$$\mathbf{V}_S = \frac{\mathbf{V}_S'}{a} = \frac{8194\angle -0.3° \text{ V}}{34.78} = 235.6\angle -0.3° \text{ V}$$

The voltage regulation is

$$VR = \frac{7967 - 8914}{8194} \times 100\% = -10.6\%$$

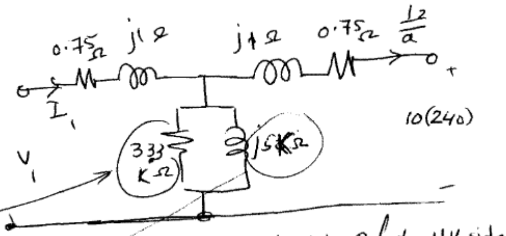
P1)

$a = \frac{2400}{240} = 10$

$R_2' = 0.0075(10)^2 = 0.75\,\Omega$

$X_2' = 0.01(10)^2 = 1\,\Omega.$

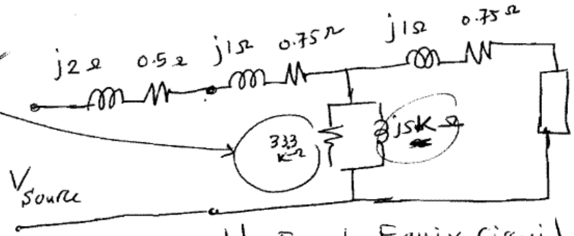
a)



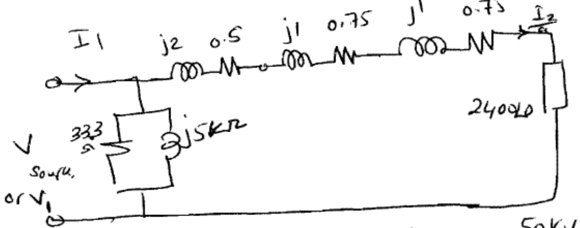
Exact Equiv. Circuit Ref. to HV side

Note: $R_c \approx 33.3\,K\Omega$, $|X_M = 5\,K\Omega|$

b)



⇓ Exact Equiv. Circuit



Approximate Equiv. Circuit

50KVA @ 0.8 pf Lagg.

$\frac{\vec{I}_2}{a} = \frac{50\times10^3}{10(240\angle 0)} \angle -\cos^{-1} 0.8$

$\frac{\vec{I}_2}{a} = 20.8\angle -36.9^\circ\ A$

$\vec{V}_1 = 2400\angle 0 + (20.8\angle -36.9) Z_{eq}$

$Z_{eq} = (0.75+0.75+0.5) + j(1+1+2)$

$\vec{V}_1 = 2400\angle 0 + (20.8\angle -36.9)(2+j4)$

$|\vec{V}_1| = 2483.5 \angle 0.96\ V$

$\%VR = \frac{|V_1| - |aV_2|}{|aV_2|} \times 100\%$

$= \frac{2483.5 - 2400}{2400} \times 100\%$

$\%VR = 3.48\%$

$\%\ \text{efficiency} = \frac{P_{out}}{P_{out} + P_{loss}} = \%\eta$

$P_{out} = 50\times10^3(0.8) = 40\,kW.$

$P_{loss} = (0.75+0.75+0.5)\left|\frac{I_2}{a}\right|^2 + \frac{|V_1|^2}{R_c}$

$= (20.8)^2(2) + \frac{(2483.5)^2}{333\times10^3}$

$P_{loss} = 1050.5\,W$

$\Rightarrow \%\eta = \frac{40\times10^3}{40\times10^3 + 1050.5} \times 100\% = 97.4\%$