King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE 360: Home Work #1 Solution

Q1)

SOLUTION This core can be divided up into five regions. Let $\mathcal{R}_1$ be the reluctance of the left-hand portion of the core, $\mathcal{R}_2$ be the reluctance of the left-hand air gap, $\mathcal{R}_3$ be the reluctance of the right-hand portion of the core, $\mathcal{R}_4$ be the reluctance of the right-hand air gap, and $\mathcal{R}_5$ be the reluctance of the center leg of the core. Then the total reluctance of the core is

$$\mathcal{R}_{TOT} = \mathcal{R}_5 + \frac{(\mathcal{R}_1 + \mathcal{R}_2)(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}$$

$$\mathcal{R}_1 = \frac{l_1}{\mu_r \mu_0 A_1} = \frac{1.11 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})} = 168 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_2 = \frac{l_2}{\mu_0 A_2} = \frac{0.0007 \text{ m}}{(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})(1.05)} = 152 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_3 = \frac{l_3}{\mu_r \mu_0 A_3} = \frac{1.11 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})} = 168 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_4 = \frac{l_4}{\mu_0 A_4} = \frac{0.0005 \text{ m}}{(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})(1.05)} = 108 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_5 = \frac{l_5}{\mu_r \mu_0 A_5} = \frac{0.37 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.05 \text{ m})} = 56.1 \text{ kA} \cdot \text{t/Wb}$$

The total reluctance is

$$\mathcal{R}_{TOT} = \mathcal{R}_5 + \frac{(\mathcal{R}_1 + \mathcal{R}_2)(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} = 56.1 + \frac{(168+152)(168+108)}{168+152+168+108} = 204 \text{ kA} \cdot \text{t/Wb}$$

The total flux in the core is equal to the flux in the center leg:

$$\phi_{center} = \phi_{TOT} = \frac{\mathcal{F}}{\mathcal{R}_{TOT}} = \frac{(300 \text{ t})(1.0 \text{ A})}{204 \text{ kA} \cdot \text{t/Wb}} = 0.00147 \text{ Wb}$$

The fluxes in the left and right legs can be found by the "flux divider rule", which is analogous to the current divider rule.

$$\phi_{left} = \frac{(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} \phi_{TOT} = \frac{(168+108)}{168+152+168+108}(0.00147 \text{ Wb}) = 0.00068 \text{ Wb}$$

$$\phi_{right} = \frac{(\mathcal{R}_1 + \mathcal{R}_2)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} \phi_{TOT} = \frac{(168+152)}{168+152+168+108}(0.00147 \text{ Wb}) = 0.00079 \text{ Wb}$$

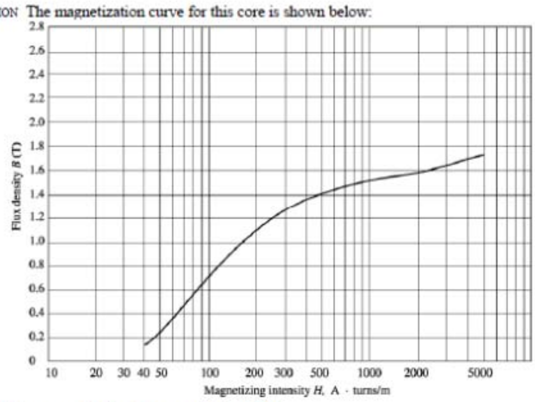
The flux density in the air gaps can be determined from the equation $\phi = BA$:

$$B_{left} = \frac{\phi_{left}}{A_{eff}} = \frac{0.00068 \text{ Wb}}{(0.07 \text{ cm})(0.05 \text{ cm})(1.05)} = 0.185 \text{ T}$$

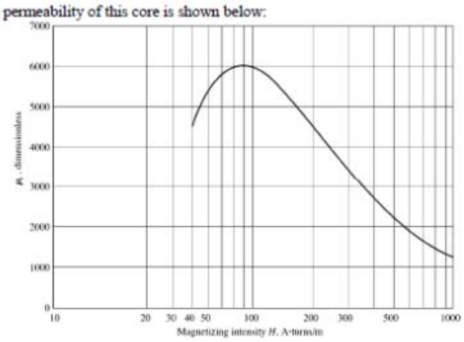
$$B_{right} = \frac{\phi_{right}}{A_{eff}} = \frac{0.00079 \text{ Wb}}{(0.07 \text{ cm})(0.05 \text{ cm})(1.05)} = 0.215 \text{ T}$$

Q2)

SOLUTION The magnetization curve for this core is shown below:



The relative permeability of this core is shown below:



> **Note:** This is a design problem, and the answer presented here is not unique. Other values could be selected for the flux density in part *(a)*, and other numbers of turns could be selected in part *(c)*. These other answers are also correct if the proper steps were followed, and if the choices were reasonable.

(a) From Figure 1-10*c*, a reasonable maximum flux density would be about 1.2 T. Notice that the saturation effects become significant for higher flux densities.

(b) At a flux density of 1.2 T, the total flux in the core would be

$$\phi = BA = (1.2\ \text{T})(0.05\ \text{m})(0.05\ \text{m}) = 0.0030\ \text{Wb}$$

(c) The total reluctance of the core is:

$$\mathcal{R}_{TOT} = \mathcal{R}_{stator} + \mathcal{R}_{air\ gap\ 1} + \mathcal{R}_{rotor} + \mathcal{R}_{air\ gap\ 2}$$

At a flux density of 1.2 T, the relative permeability μ_r of the stator is about 3800, so the stator reluctance is

$$\mathcal{R}_{stator} = \frac{l_{stator}}{\mu_{stator} A_{stator}} = \frac{0.60\ \text{m}}{(3800)(4\pi \times 10^{-7}\ \text{H/m})(0.05\ \text{m})(0.05\ \text{m})} = 50.3\ \text{kA} \cdot \text{t/Wb}$$

At a flux density of 1.2 T, the relative permeability μ_r of the rotor is 3800, so the rotor reluctance is

$$\mathcal{R}_{rotor} = \frac{l_{rotor}}{\mu_{stator} A_{rotor}} = \frac{0.05\ \text{m}}{(3800)(4\pi \times 10^{-7}\ \text{H/m})(0.05\ \text{m})(0.05\ \text{m})} = 4.2\ \text{kA} \cdot \text{t/Wb}$$

The reluctance of both air gap 1 and air gap 2 is

$$\mathcal{R}_{air\ gap\ 1} = \mathcal{R}_{air\ gap\ 2} = \frac{l_{air\ gap}}{\mu_{air\ gap} A_{air\ gap}} = \frac{0.0005\ \text{m}}{(4\pi \times 10^{-7}\ \text{H/m})(0.0018\ \text{m}^2)} = 221\ \text{kA} \cdot \text{t/Wb}$$

Therefore, the total reluctance of the core is

$$\mathcal{R}_{TOT} = \mathcal{R}_{stator} + \mathcal{R}_{air\ gap\ 1} + \mathcal{R}_{rotor} + \mathcal{R}_{air\ gap\ 2}$$

$$\mathcal{R}_{TOT} = 50.3 + 221 + 4.2 + 221 = 496\ \text{kA} \cdot \text{t/Wb}$$

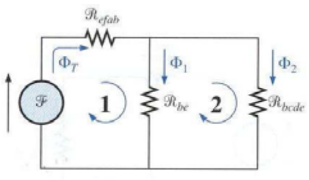
The required MMF is

$$\mathcal{F}_{TOT} = \phi \mathcal{R}_{TOT} = (0.003\ \text{Wb})(496\ \text{kA} \cdot \text{t/Wb}) = 1488\ \text{A} \cdot \text{t}$$

Since $\mathcal{F} = Ni$, and the current is limited to 1 A, one possible choice for the number of turns is $N = 2000$. This would allow the desired flux density to be achieved with a current of about 0.74 A.

Q3)

The above device can be analysed by its magnetic circuit equivalent



For loop 2, the magnetic flux density is

$$B_2 = \frac{\Phi_2}{A} = \frac{1.5\times10^{-4}\ \text{Wb}}{6\times10^{-4}\ \text{m}^2} = 0.25\,\text{T}.$$

and the corresponding magnetic field intensity is

$$H_{bcde} = \frac{B_2}{\mu_{bcde}} = \frac{B_2}{\mu_2\mu_0} = \frac{0.25\ \text{T}}{4972\times4\pi\times10^{-7}\ \text{Hm}^{-1}} = 40\ \text{At/m}.$$

Applying Ampère circuital law around loop 2 of figure 4.19,

$$H_{be}l_{be} - H_{bcde}l_{bcde} = 0$$
$$H_{be}(0.05\ \text{m}) - (40\ \text{At/m})(0.2\ \text{m}) = 0$$
$$H_{be} = \frac{8\ \text{At}}{0.05\ \text{m}}$$
$$= 160\ \text{At/m}.$$

Then, the magnetic flux density in region 'be' is

$$B_1 = B_{be} = \mu_{be}H_{be} = \mu_1\mu_0H_{be} = 4821\times4\pi\times10^{-7}\ \text{H/m}\times160\ \text{At/m} = 0.97\,\text{T}.$$

The flux in this region is

$$\Phi_1 = B_1A = (0.97\ \text{T})(6\times10^{-4}\ \text{m}^2) = 5.82\times10^{-4}\ \text{Wb}.$$

For loop 1, apply Gauss law (analogy to KCL in electric circuit) gives

$$\Phi_T = \Phi_1 + \Phi_2 = (5.82 + 1.5)\times10^{-4}\ \text{Wb}$$
$$= 7.32\times10^{-4}\ \text{Wb}$$
$$\text{B} = \frac{\Phi_T}{A} = \frac{7.32\times10^{-4}\ \text{Wb}}{6\times10^{-4}\ \text{m}^2} = 1.22\ \text{T}.$$

Then,

$$H_{efab} = \frac{B}{\mu_{efab}} = \frac{B}{\mu_T\mu_0} = \frac{1.22\ \text{T}}{2426\times4\pi\times10^{-7}\ \text{Hm}^{-1}} = 400\ \text{At/m}.$$

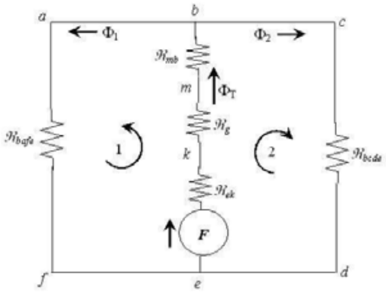
Applying Ampère circuital law (analogy to KVL in electric circuit),

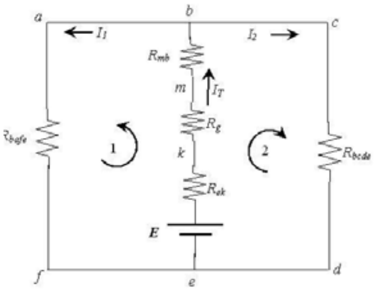
$$NI = H_{efab}l_{efab} + H_{be}l_{be}$$
$$NI = (400\ \text{At/m})(0.2\ \text{m}) + (160\ \text{At/m})(0.05\ \text{m})$$
$$(50\ \text{t})I = 80\ \text{At} + 8\ \text{At}$$
$$I = 1.76\ \text{A}.$$

Q4)

Solution:

The magnetic circuit equivalent and its electric circuit analogy are shown in below.





Since the magnetic device is symmetry and made by the same material at both left and right arms, we can concentrate the analysis of the circuit on either loop 1 or loop 2.

From Gauss law, one gets

$$\Phi_1 = \Phi_2.$$
$$\Phi_T = \Phi_1 + \Phi_2.$$
$$= 2\Phi_2.$$

Apply Ampère circuital law at left hand arm,

$$\sum_{loop1} NI = \sum_{loop1} Hl$$
$$NI = H_{ek}l_{ek} + H_g l_g + H_{mb}l_{mb} + H_{bafe}l_{bafe}$$
$$NI = H_{ek}l_{ek} + H_g l_g + H_{mb}l_{mb} + H_{bafe}\left(l_{ba} + l_{af} + l_{fe}\right) \qquad (1)$$
$$B_{ek} = \frac{\Phi_T}{A} = \frac{2\Phi_2}{A} = \frac{2\times30\times10^{-6}\text{ Wb}}{(0.01\times0.01)\text{m}^2} = 0.6\text{ T}$$
$$H_{ek} = H_{mb} = 2600\text{ At/m} \qquad \text{(from figure 4.26)} \qquad (2)$$
$$H_g = \frac{B_g}{\mu_0} = \frac{\Phi_T}{\mu_0 A} = \frac{2\Phi_2}{\mu_0 A} = \frac{2\times30\times10^{-6}\text{ Wb}}{(4\pi\times10^{-7}\text{ H/m})(0.01\times0.01)\text{m}^2} = 4.77\times10^5\text{ At/m}. \qquad (3)$$
$$B_{bafe} = \frac{\Phi_1}{A} = \frac{30\times10^{-6}\text{ Wb}}{(0.01\times0.01)\text{m}^2} = 0.3\text{ T}$$

$$H_{bafe} = 750\text{ At/m}$$

Substitute (2), (3), and (4) in (1) yields

$$I = \frac{1}{N}\begin{bmatrix}(2600\text{ At/m}\times0.03\text{ m}) + (4.77\times10^5\text{ At/m}\times0.5\times10^{-2}\text{ m}) \\ + (2600\text{ At/m}\times0.5\times10^{-2}\text{ m}) + (750\text{ At/m}\times3\times0.04\text{ m})\end{bmatrix}$$
$$= \frac{2566}{400}$$
$$= 6.415\text{ A.}$$

Q5)

$P_{h_{50}} = 300W$, $P_{e_{50}} = 250$ } at 500V and 50 Hz

$P_{h_{100}} = ?$, $P_{e_{100}} = ?$ } at 1000V and 100 Hz

$P_h = k_h B^2 f$

$P_e = k_e B^2 f^2$

It is assumed that $B \propto V \Rightarrow$

$P_{h_{50}} = 300 = k_h (500)^2 50$

$\Rightarrow k_h = 2.4 \times 10^{-6}$

$P_{e_{50}} = 250 = k_e (500)^2 (50)^2$

$= 0.4 \times 10^{-6}$

at 1000V & 100 Hz

$P_h = 2.4 \times 10^{-6} (1000)^2 (100) = 240 W$

$P_e = 0.4 \times 10^{-6} (1000)^2 (100)^2 = 4000 W$