KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
ELECTRICAL ENGINEERING DEPARTMENT
EE 463 – Term 121
HW # 3: Symmetrical Faults

Key Solutions

9.2. The system shown in Figure 67 shows an existing plant consisting of a generator of 100 MVA, 30 kV, with 20 percent subtransient reactance and a generator of 50 MVA, 30 kV with 15 percent subtransient reactance, connected in parallel to a 30-kV bus bar. The 30-kV bus bar feeds a transmission line via the circuit breaker

C which is rated at 1250 MVA. A grid supply is connected to the station bus bar through a 500-MVA, 400/30-kV transformer with 20 percent reactance. Determine the reactance of a current limiting reactor in ohm to be connected between the grid system and the existing bus bar such that the short-circuit MVA of the breaker C does not exceed.

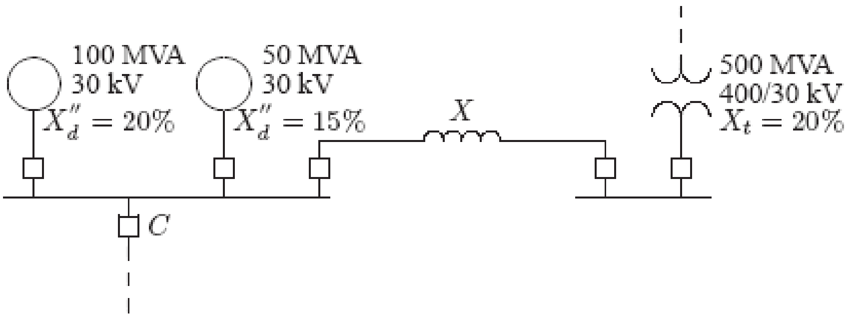

FIGURE 67
One-line diagram for Problem 9.2.

The base impedance for line is

$$Z_B = \frac{(30)^2}{100} = 9 \quad \Omega$$

The reactances on a common 100 MVA base are

$$X''_{d_{g1}} = \frac{100}{100}(0.2) = 0.2 \quad \text{pu}$$
$$X''_{d_{g2}} = \frac{100}{50}(0.15) = 0.3 \quad \text{pu}$$
$$X_t = \frac{100}{500}(0.2) = 0.04 \quad \text{pu}$$

The impedance diagram is as shown in Figure 68.
Reactance to the point of fault is

$$X_f = \frac{S_B}{\text{SCMVA}} = \frac{100}{1250} = 0.08 \;\; \text{pu}$$

Parallel reactance of the generators is

$$X_{||} = \frac{(0.2)(0.3)}{0.2 + 0.3} = 0.12 \;\; \text{pu}$$

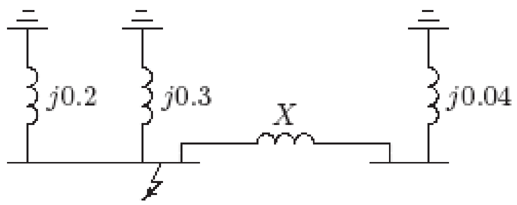

FIGURE 68
The impedance diagram for Problem 9.2.

From Figure 68, reactance to the point of fault is

$$\frac{(0.12)(X + 0.04)}{0.12 + (X + 0.04)} = 0.08$$

Solving for X, we get $X = 0.2$ pu., or

$$X_\Omega = (X)(Z_B) = (0.2)(9) = 1.8 \;\; \Omega$$

9.4. The one-line diagram of a simple three-bus power system is shown in Figure 71 Each generator is represented by an emf behind the subtransient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 3 through a fault impedance of $Z_f = j0.19$ per unit.
(a) Using Thévenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
(b) Determine the bus voltages and line currents during fault.

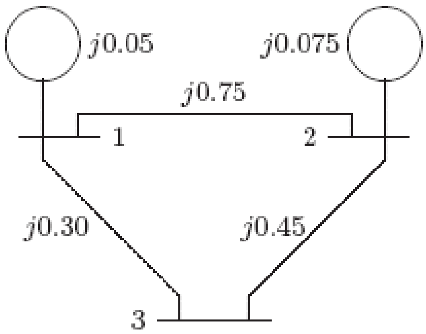

FIGURE 71
One-line diagram for Problem 9.4.

Converting the Δ formed by buses 123 to an equivalent Y as shown in Figure 72(a), we have

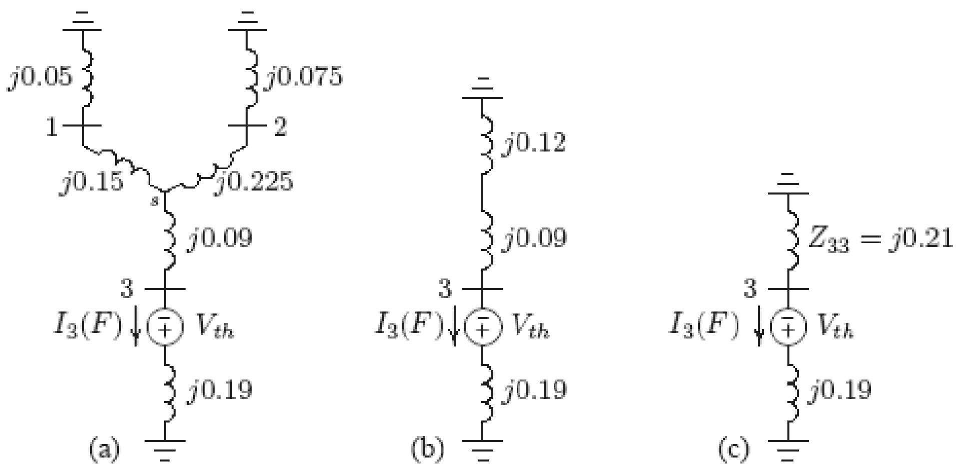

FIGURE 72
Reduction of Thévenin's equivalent network.

$$Z_{1s} = \frac{(j0.3)(j0.75)}{j1.5} = j0.15 \qquad Z_{2s} = \frac{(j0.75)(j0.45)}{j1.5} = j0.225$$

$$Z_{3s} = \frac{(j0.3)(j0.45)}{j1.5} = j0.09$$

Combining the parallel branches, Thévenin's impedance is

$$\begin{aligned} Z_{33} &= \frac{(j0.2)(j0.3)}{j0.2 + j0.3} + j0.09 \\ &= j0.12 + j0.09 = j0.21 \end{aligned}$$

From Figure 72(c), the fault current is

$$I_3(F) = \frac{V_3(F)}{Z_{33} + Z_f} = \frac{1.0}{j0.21 + j0.19} = -j2.5 \quad \text{pu}$$

With reference to Figure 72(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.3}{j0.2 + j0.3} I_3(F) = -j1.5 \quad \text{pu}$$

$$I_{G2} = \frac{j0.2}{j0.2 + j0.3} I_3(F) = -j1.0 \quad \text{pu}$$

For the bus voltage changes from Figure 72(a), we get

$$\begin{aligned} \Delta V_1 &= 0 - (j0.05)(-j1.5) \ = -0.075 \ \text{pu} \\ \Delta V_2 &= 0 - (j0.075)(-j1) \ = -0.075 \ \text{pu} \\ \Delta V_3 &= (j0.19)(-j2.5) - 1.0 = -0.525 \ \text{pu} \end{aligned}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, i.e.,

$$\begin{aligned} V_1(F) &= V_1(0) + \Delta V_1 = 1.0 - 0.075 = 0.925 \ \text{pu} \\ V_2(F) &= V_2(0) + \Delta V_2 = 1.0 - 0.075 = 0.925 \ \text{pu} \\ V_3(F) &= V_3(0) + \Delta V_3 = 1.0 - 0.525 = 0.475 \ \text{pu} \end{aligned}$$

The short circuit-currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.925 - 0.925}{j0.75} = 0 \quad \text{pu}$$

$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.925 - 0.475}{j0.3} = -j1.5 \quad \text{pu}$$

$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.925 - 0.475}{j0.45} = -j1.0 \quad \text{pu}$$

9.6. Using the method of building algorithm find the bus impedance matrix for the network shown in Figure 76.

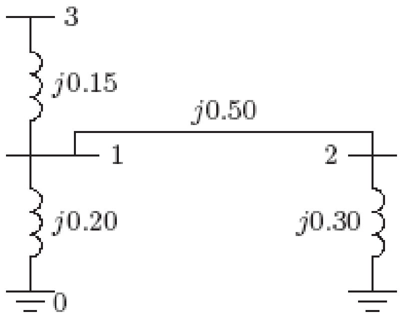

FIGURE 76
One-line diagram for Problem 9.6.

Add branch 1, $z_{10} = j0.2$ between node $q = 1$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{11} = z_{10} = j0.20$$

Next, add branch 2, $z_{20} = j0.3$ between node $q = 2$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 \\ 0 & j0.3 \end{bmatrix}$$

Add branch 3, $z_{13} = j0.15$ between the new node $q = 3$ and the existing node $p = 1$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{11} \\ Z_{21} & Z_{22} & Z_{21} \\ Z_{11} & Z_{12} & Z_{11} + z_{13} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 & j0.2 \\ 0 & j0.3 & 0 \\ j0.2 & 0 & j0.35 \end{bmatrix}$$

Add link 4, $z_{12} = j0.5$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\begin{aligned} \mathbf{Z}_{bus}^{(4)} &= \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix} \\ &= \begin{bmatrix} j0.2 & 0 & j0.2 & -j0.2 \\ 0 & j0.3 & 0 & j0.3 \\ j0.2 & 0 & j0.35 & -j0.2 \\ -j0.2 & j0.3 & -j0.2 & Z_{44} \end{bmatrix} \end{aligned}$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = j0.5 + j0.2 + j0.3 - 2(j0) = j1.0$$

and

$$\frac{\Delta \mathbf{Z}\, \Delta \mathbf{Z}^T}{Z_{44}} = \frac{1}{j1.0}\begin{bmatrix} -j0.2 \\ j0.3 \\ -j0.2 \end{bmatrix}\begin{bmatrix} -j0.2 & j0.3 & -j0.2 \end{bmatrix}$$

$$= \begin{bmatrix} j0.04 & -j0.06 & j0.04 \\ -j0.06 & j0.09 & -j0.06 \\ j0.04 & -j0.06 & j0.04 \end{bmatrix}$$

From (9.59), the new bus impedance matrix is

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.2 & 0 & j0.2 \\ 0 & j0.3 & 0 \\ j0.2 & 0 & j0.35 \end{bmatrix} - \begin{bmatrix} j0.04 & -j0.06 & j0.04 \\ -j0.06 & j0.09 & -j0.06 \\ j0.04 & -j0.06 & j0.04 \end{bmatrix}$$

$$= \begin{bmatrix} j0.16 & j0.06 & j0.16 \\ j0.06 & j0.21 & j0.06 \\ j0.16 & j0.06 & j0.31 \end{bmatrix}$$