

# Problem 1

360 360 360

## EE 360 Problem Session #5 Induction Motor

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①  $V_i = 220/\sqrt{3} = 127 \text{ V}$

$$P_{in} = 3V_i I_i \cos \phi = 3 \times 127 \times 77 \times 0.88$$

$$P_{in} = 25.8 \text{ kW}$$

$$\begin{aligned} P_{cu1} &= 1033 \text{ W} \\ P_{cu2} &= 1299 \text{ W} \\ P_{core} &= 485 \text{ W} \\ P_{F\&W} &= 540 \text{ W} \end{aligned}$$

$$P_g = P_{in} - P_{cu1} - P_{core} = 25.8 \times 10^3 - 1033 - 485$$

$$P_g = 24.3 \text{ kW}$$

②  $T_d = \frac{P_g}{\omega_s}$ ,  $\omega_s = \frac{2\pi n_s}{60}$ ,  $n_s = \frac{120 \text{ pf}}{P} = \frac{120 \times 60}{4} = 1800 \text{ rpm}$

$$\omega_s = \frac{2\pi \times 1800}{60} = 188.5 \text{ rad/sec}$$

$$T_d = \frac{24.3 \times 10^3}{188.5}$$

$$T_d = 128.9 \text{ N-m}$$

③  $S = \frac{P_{cu2}}{P_g} = \frac{1299}{24.3 \times 10^3} \Rightarrow S = 0.0534$

$$S = 5.34\%$$

④  $P_d = (1-S) \times P_g = (1-0.0534) \times 24.3 \times 10^3$

$$P_d = 23.0 \text{ kW}$$

⑤  $P_o = P_d - P_{F\&W} = 23 \times 10^3 - 540 = 22.46 \text{ kW}$

$$P_o = \frac{22.46 \times 10^3}{746} \Rightarrow P_o = 30 \text{ hp}$$

⑥  $n_m = (1-S) \times n_s = (1-0.0534) \times 1800 \Rightarrow n_m = 1704 \text{ rpm}$

⑦  $T_o = \frac{P_o}{\omega_m} = \frac{22.46 \times 10^3}{1704} \Rightarrow T_o = 13.2 \text{ N-m}$       ⑧  $\eta = \frac{P_o}{P_{in}} = \frac{22.46}{25.8}$

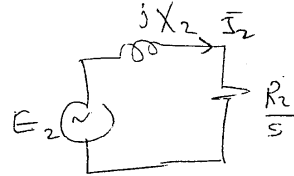
## Problem 2

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$$n = 1425 \text{ rpm}, \quad n_s = \frac{120 \times f}{P} = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

$$s = \frac{n_s - n}{n_s} = \frac{1800 - 1425}{1800} = 0.05$$

$$s = 5\%$$



(i) at full-load condition  $s = 0.05$

$$\bar{Z}_2 = \frac{R_2}{s} + jX_2 = \frac{0.4}{0.05} + j4 = 8 + j4 \Omega$$

$$\bar{Z}_2 = 8.94 \angle 26.6^\circ \Omega$$

$$\bar{I}_2 = \frac{\bar{E}_2}{\bar{Z}_2} = \frac{203 \angle 0^\circ}{8.94 \angle 26.6^\circ} \Rightarrow \bar{I}_2 = 22.67 \angle -26.6^\circ \text{ A}$$

$$P_g = 3 I_2^2 \frac{R_2}{s} = 3 (22.67)^2 \left( \frac{0.4}{0.05} \right)$$

$$P_g = 12.36 \text{ kW} \Rightarrow T_d = \frac{P_g}{\omega_s}, \quad \omega_s = \frac{2\pi n_s}{60} = 157 \text{ rad/s}$$

$$T_d = \frac{12.36 \times 10^3}{157} \Rightarrow T_d = 78.7 \text{ N-m}$$

$$T_d = \frac{P_d}{\omega_m}, \quad P_d = 3 I_2^2 R_2 \frac{(1-s)}{s}, \quad \omega_m = \frac{2\pi n}{60}$$

$$= (1-s) P_g$$

(ii)  $I_2 = 22.67 \text{ A}$

$$P_{F_2} = \cos(26.6) \Rightarrow P_{F_2} = 0.894 \text{ lagging}$$

(iii)  $P_o = P_d - P_{F_3W}$

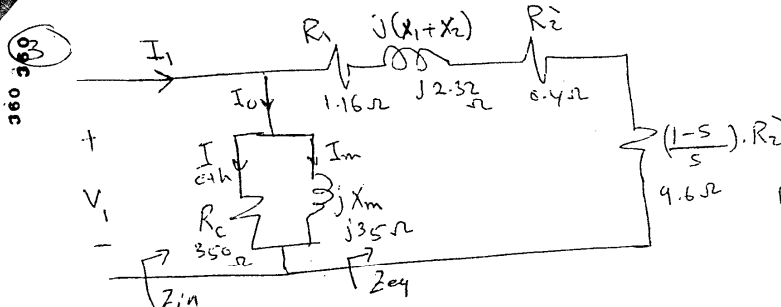
$$P_d = (1-s) P_g = (1-0.05) \times 12.36 \text{ K}$$

$$P_d = 11.74 \text{ kW}$$

$$P_o = 11.74 + 10^3 - 500 \Rightarrow P_o = 11.24 \text{ kW}$$

$$P_o = 11.24 \times 10^3 \Rightarrow P_o = 15 \text{ hp}$$

**Problem 3**



$P_{Fsw} = 250 \text{ W}$   
 $n = 1440 \text{ rpm}$   
 $V_1 = \frac{400}{\sqrt{3}} = 230 \text{ V}$

$n_s = \frac{120 \times f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$   
 $s = \frac{n_s - n}{n_s} = \frac{1500 - 1440}{1500} = 0.04$   
 $\frac{1-s}{s} \cdot R_2 = \frac{1-0.04}{0.04} \times 0.4 = 9.6 \Omega$

$(a) \Rightarrow \vec{I}_2 = \frac{V_1}{Z_{eq}}, \quad Z_{eq} = (R_1 + \frac{R_2}{s}) + j(X_1 + X_2)$   
 $= 11.16 + j2.32 = 11.4 \angle 11.74^\circ \Omega$

$\vec{I}_2 = \frac{230 \angle 0^\circ}{11.4 \angle 11.74^\circ} = 20.26 \angle -11.74^\circ \text{ A}$

$\vec{I}_0 = \vec{I}_{e+h} + \vec{I}_m, \quad \vec{I}_{e+h} = \frac{\vec{V}_1}{R_c} = \frac{230}{350} = 0.65 \angle 0^\circ \text{ A}$   
 $\vec{I}_m = \frac{\vec{V}_1}{X_m} = \frac{230}{35 \angle 40^\circ} = 6.6 \angle -90^\circ \text{ A}$

$\vec{I}_0 = 0.65 - j6.6 \text{ A}$

$\vec{I}_1 = \vec{I}_0 + \vec{I}_2 = 0.65 - j6.6 + 20.26 \angle -11.74^\circ$

$\vec{I}_1 = 23.1 \angle -27.6^\circ \text{ A}$

$|I_1| = 23.1 \text{ A}$   
 $PF = \cos(27.6)$   
 $PF_1 = 0.886 \text{ lagging}$

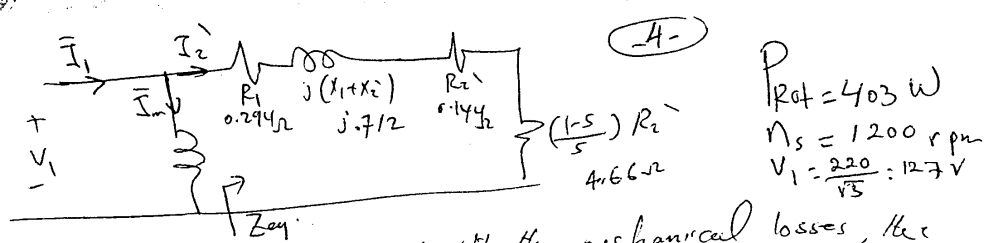
$P_{in} = 3 \times 23.1 \times 230 \times 0.886$   
 $P_{in} = 14.19 \text{ kW}$

(a)  $\rightarrow$

$(b) P_0 = P_d - P_{Fsw}$   
 $P_d = 3 \times I_2^2 \cdot R_2 \cdot \frac{1-s}{s} = 3 \times (20.26)^2 \times \frac{0.4}{0.04} = 11.82 \text{ kW}$

$(c) \gamma = \frac{P_0}{P_{in}} \times 100 = \frac{11.82}{14.19} \times 100$

Problem 4



Since the core loss is lumped with the mechanical losses, the core resistance will be removed from the equivalent circuit.

$$s = 0.03 \Rightarrow \frac{1-s}{s} \times R_2' = \frac{1-0.03}{0.03} \times 0.144 = 4.66 \Omega$$

$$Z_{eq} = (R_1 + \frac{R_2'}{s}) + j(X_1 + X_2) = (0.294 + \frac{0.144}{0.03}) + j(0.503 + 2.09)$$

$$Z_{eq} = 5.14 \angle 7.95^\circ \Omega$$

(a)  $V_1 = \frac{220}{\sqrt{3}} = 127 \text{ V}$

$$\bar{I}_2 = \frac{V_1}{Z_{eq}} = \frac{127 \angle 0^\circ}{5.14 \angle 7.95^\circ} \Rightarrow \bar{I}_2 = 24.7 \angle -7.95^\circ \text{ A}$$

(b)  $P_d = 3 I_2'^2 \frac{(1-s) R_2'}{s} = 3 (24.7)^2 (4.66) \Rightarrow P_d = 8.52 \text{ kW}$

$$n = (1-s) n_s = (1-0.03) \times 1200 \Rightarrow n = 1164 \text{ rpm}$$

$$T_d = \frac{P_d}{\omega_m} = \frac{P_d}{\frac{2\pi n}{60}} = \frac{8.52 \times 10^3}{\frac{2\pi (1164)}{60}} \Rightarrow T_d = 69.9 \text{ N}\cdot\text{m}$$

(b)  $T_{max} = \frac{3 V_1^2}{2 \omega_s} \cdot \frac{1}{[R_1 + \sqrt{R_1^2 + (X_1 + X_2)^2}]} = \frac{3 \times (127)^2}{2 \times \frac{2\pi \times 1200}{60}} \times \frac{1}{[0.294 + \sqrt{0.294^2 + (0.712)^2}]}$

$$T_{max} = 181 \text{ N}\cdot\text{m}$$

$$s_{max} = \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2)^2}} = \frac{0.144}{\sqrt{(0.294)^2 + (0.712)^2}} = 0.187$$

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$$|\bar{I}_2^{\text{start}}| = \frac{V_1}{\sqrt{(R_1 + R_2')^2 + (X_1 + X_2')^2}} = \frac{127}{\sqrt{(0.438)^2 + (0.712)^2}} \quad -5-$$

$$\bar{I}_2^{\text{start}} = 151.9 \angle -58.5^\circ \text{ A}$$

$$\alpha = \tan^{-1} \frac{(X_1 + X_2')}{(R_1 + R_2')} = 58.5^\circ$$

$$\bar{I}_m = \frac{\bar{V}_1}{jX_m} = \frac{127 \angle 0^\circ}{j13.25 \angle 90^\circ} = -j9.6 \text{ A}$$

$$\bar{I}_{\text{start}} = \bar{I}_m + \bar{I}_2^{\text{start}} = 9.6 \angle -90^\circ + 151.9 \angle -58.5^\circ$$

$$\bar{I}_1 = 159.6 \angle -60.3^\circ \text{ A}$$

$$T_{\text{start}} = \frac{3 I_2^{\text{start}2} R_2'}{\omega_s} = \frac{3 V_1^2}{\omega_s} \frac{R_2'}{(R_1 + R_2')^2 + (X_1 + X_2')^2}$$

$$T_{\text{start}} = \frac{3(127)^2 (0.144)}{(0.438)^2 + (0.712)^2} \times \frac{1}{2\pi \times 1200 / 60}$$

$$T_{\text{start}} = 79 \text{ N-m}$$

$$(d) \frac{n}{T_{\text{max}}} = 900 \text{ rpm} \Rightarrow s_{\text{max}} = \frac{n_s - n}{n_s} = \frac{1200 - 900}{1200} = 0.25$$

$$\text{max. Torque} \Rightarrow s_{\text{max}} = \frac{R_2'_{\text{new}}}{\sqrt{R_1^2 + (X_1 + X_2')^2}} = 0.25$$

$$R_2'_{\text{new}} = 0.25 \times \sqrt{(0.294)^2 + (0.712)^2} = 0.193 \Omega$$

$$R_2 \text{ added} = R_2'_{\text{new}} - R_2' = 0.193 - 0.144$$

$$R_2 \text{ added} = 0.0486 \Omega$$

112  $R_2'_{\text{new}}$

$$\Rightarrow T_{\text{start}} = 100 \text{ N-m}$$

Problem 5

360  
0.9E  
0.9E

I- No-Load test

$V_{L-L} = 400V, I_0 = 7.5A, P_0 = 700W$

$P_{core} = \frac{1}{3}(P_0 - P_{fw}) = \frac{700 - 200}{3} \Rightarrow P_{core} = 166.7W$

$V_{0/ph} = \frac{400}{\sqrt{3}} = 230V$

$R_c = \frac{V_{0/ph}^2}{P_{core}} = \frac{(230)^2}{166.7} \Rightarrow R_c = 320 \Omega / \text{phase}$

$Q_0 = \sqrt{S_0^2 - P_0^2} = \sqrt{V_0^2 I_0^2 - P_0^2} = \sqrt{(230)(7.5)^2 - (700)^2}$

$Q_0 = 1709 \text{ VAR/phase} = \frac{V_{0/ph}^2}{X_m} \Rightarrow X_m = \frac{(230)^2}{1709}$

$X_m = 30.9 \Omega$

II- Blocked-Rotor Test

$V_{L-L} = 150V, I_B = 35A, P_B = 4000W$

$V_{B/ph} = \frac{150}{\sqrt{3}} = 86.6V, P_{B/ph} = \frac{4000}{3} = 1333.3W$

$R_{eq} = R_1 + R_2 = \frac{P_{B/ph}}{I_B^2} = \frac{(1333.3)}{(35)^2} \Rightarrow R_{eq} = 1.088 \Omega / \text{phase}$

Since  $P_{cu1} = P_{cu2} \Rightarrow R_1 = R_2 = \frac{R_{eq}}{2} = \frac{1.088}{2} = 0.544 \Omega / \text{phase}$

$Z_{ph} = \frac{V_B}{I_B} = \frac{86.6}{35} = 2.47 \Omega = \sqrt{R_{eq}^2 + X_{eq}^2}$

$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = \sqrt{(2.47)^2 - (1.088)^2} = 2.22 \Omega = X_1 + X_2$

But  $X_1 = X_2 \rightarrow 2X_1 = X_1 + X_2 = 2.22$

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$$S = 0.04$$

$$Z_{eq} = (R_1 + \frac{R_2'}{S}) + j(X_1 + X_2') = (0.544 + \frac{0.544}{0.04}) + j2.22$$

$$Z_{eq} = 14.32 \angle 8.9^\circ \Omega$$

$$I_2' = \frac{|V_1|}{|Z_{eq}|} = \frac{230}{14.32} \Rightarrow I_2' = 16.13 \text{ A}$$

$$P_d = 3 I_2'^2 \frac{(1-S)}{S} R_2' = 3 (16.13)^2 \frac{(1-0.04)}{0.04} (0.544)$$

$$P_d = 10.19 \text{ Kw}$$

$$P_o = P_d - P_{mech} = 10.19 \times 10^3 - 200$$

$$P_o = 9.99 \text{ Kw}$$

$$T_o = \frac{P_o}{\omega_m} = \frac{P_o}{2\pi n / 60}, \quad n = (1-S)n_s, \quad n_s = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$n = (1-0.04) \times 1000 \Rightarrow n = 960 \text{ rpm} \Rightarrow \omega_m = 100.5 \text{ rad/sec}$$

$$T_o = \frac{9.99 \times 10^3}{100.5} \Rightarrow T_o = 99.37 \text{ N-m}$$