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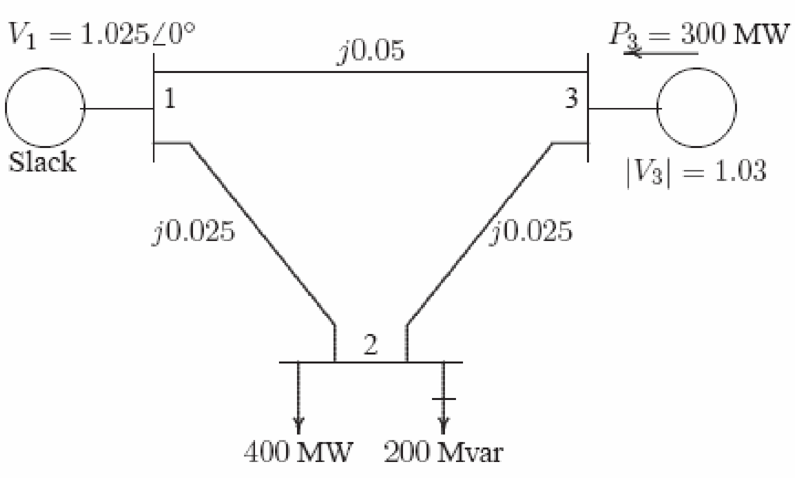
EE-463
Key Solution

Quiz # 2 Serial # Name: I.D.#

The figure below shows the one-line diagram of a simple three-bus power system with generation at buses 1 (slack bus) and 3 (PV bus). The voltage at bus 1 is $V_1 = 1.025\ \angle 0^o$ per unit. Voltage magnitude at bus 3 is fixed at 1.03 pu with a real power generation of 300 MW. A load consisting of 400 MW and 200 Mvar is taken from bus 2 (PQ bus). Line impedances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.



(a) Using Gauss-Seidel method and initial estimates of $V_2^{(0)} = 1.0 + j0$ and $V_3^{(0)} = 1.03 + j0$ and keeping $|V_3| = 1.03$ pu, determine the phasor values of V_2 and V_3. Perform one iteration.

(b) If after several iterations the bus voltages converge to

$V_2 = 1.001243\ \angle -2.1^o = 1.000571 - j0.0366898$ pu

$V_3 = 1.03\ \angle 1.37^o = 1.029706 + j0.0246$ pu

Determine the line flows and line losses and the slack bus real and reactive power. Construct a power flow diagram and show the direction of the line flows.

(a) Line impedances converted to admittances are $y_{12} = -j40$, $y_{13} = -j20$ and $y_{23} = -j40$. The load and generation expressed in per units are

$$S_2^{sch} = -\frac{(400 + j200)}{100} = -4.0 - j2.0 \text{ pu}$$

$$P_3^{sch} = \frac{300}{100} = 3.0 \text{ pu}$$

Bus 1 is taken as the reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.03 + j0.0$, V_2 and V_3 are computed from (6.28).

$$
\begin{aligned}
V_2^{(1)} &= \frac{\frac{S_2^{sch*}}{V_2^{(0)*}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}} \\
&= \frac{\frac{-4.0+j2.0}{1.0-j0} + (-j40)(1.025 + j0) + (-j40)(1.03 + j0)}{(-j80)} \\
&= 1.0025 - j0.05
\end{aligned}
$$

Bus 3 is a regulated bus where voltage magnitude and real power are specified. For the voltage-controlled bus, first the reactive power is computed from (6.30)

$$
\begin{aligned}
Q_3^{(1)} &= -\Im\{V_3^{(0)*}[V_3^{(0)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(1)}]\} \\
&= -\Im\{(1.03 - j0)[(1.03 + j0)(-j60) - (-j20)(1.025 + j0) \\
&\quad -(-j40)(1.0025 - j0.05)]\} \\
&= 1.236
\end{aligned}
$$

The value of $Q_3^{(1)}$ is used as Q_3^{sch} for the computation of voltage at bus 3. The complex voltage at bus 3, denoted by $V_{c3}^{(1)}$, is calculated

$$
\begin{aligned}
V_{c3}^{(1)} &= \frac{\frac{S_3^{sch}{}_*}{V_3^{(0)}{}_*} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}} \\
&= \frac{\frac{3.0-j1.236}{1.03-j0} + (-j20)(1.025 + j0) + (-j40)(1.0025 - j0.05)}{(-j60)} \\
&= 1.0300 + j0.0152
\end{aligned}
$$

Since $|V_3|$ is held constant at 1.03 pu, only the imaginary part of $V_{c3}^{(1)}$ is retained, i.e, $f_3^{(1)} = 0.0152$, and its real part is obtained from

$$
e_3^{(1)} = \sqrt{(1.03)^2 - (0.0152)^2} = 1.0299
$$

Thus

$$
V_3^{(1)} = 1.0299 + j0.0152
$$

For the second iteration, we have

$$
\begin{aligned}
V_2^{(2)} &= \frac{\frac{S_2^{sch*}}{V_2^{(1)*}} + y_{12}V_1 + y_{23}V_3^{(1)}}{y_{12}+y_{23}} \\
&= \frac{\frac{-4.0+j2.0}{1.0025+j.05} + (-j40)(1.025) + (-j40)(1.0299+j0.0152)}{(-j80)} \\
&= 1.0001 - j0.0409
\end{aligned}
$$

$$
\begin{aligned}
Q_3^{(2)} &= -\Im\{V_3^{(1)^*}[V_3^{(1)}(y_{13}+y_{23}) - y_{13}V_1 - y_{23}V_2^{(2)}]\} \\
&= -\Im\{(1.0299 - j0.0152)[(1.0299+j0.0152)(-j60) \\
&\quad -(-j20)(1.025+j0) - (-j40)(1.0001 - j0.0409)]\} \\
&= 1.3671
\end{aligned}
$$

$$
\begin{aligned}
V_{c3}^{(2)} &= \frac{\frac{S_3^{sch*}}{V_3^{(1)*}} + y_{13}V_1 + y_{23}V_2^{(2)}}{y_{13}+y_{23}} \\
&= \frac{\frac{3.0-j1.3671}{1.0299-j0.0152} + (-j20)(1.025) + (-j40)(1.0001 - j.0409)}{(-j60)} \\
&= 1.0298 + j0.0216
\end{aligned}
$$

Since $|V_3|$ is held constant at 1.03 pu, only the imaginary part of $V_{c3}^{(2)}$ is retained, i.e, $f_3^{(2)} = 0.0216$, and its real part is obtained from

$$
e_3^{(2)} = \sqrt{(1.03)^2 - (0.0216)^2} = 1.0298
$$

or

$$
V_3^{(2)} = 1.0298 + j0.0216
$$

The process is continued and a solution is converged with an accuracy of 5×10^{-5} pu to

$$
\begin{aligned}
V_2 &= 1.001243\angle{-2.1^\circ} = 1.000571 - j.0366898 \ \text{ pu} \\
S_3 &= 3.0 + j1.3694 \ \text{ pu} = 300 \ \text{ MW} + j136.94 \ \text{ Mvar} \\
V_3 &= 1.03\angle 1.36851^\circ \ \text{ pu} = 1.029706 + j0.0246
\end{aligned}
$$

(b) Line flows and line losses are computed as in Problem 6.7, and the results expressed in MW and Mvar are

$$
\begin{array}{lll}
S_{12} = 150.43 + j100.16 & S_{21} = -150.43 - j92.39 & S_{L\,12} = 0.0 + j7.77 \\
S_{13} = -50.43 - j9.65 & S_{31} = \quad 50.43 + j10.90 & S_{L\,13} = 0.0 + j1.25 \\
S_{23} = -249.58 - j107.61 & S_{32} = 249.58 + j126.03 & S_{L\,23} = 0.0 + j18.42
\end{array}
$$

The slack bus real and reactive powers are

$$
\begin{aligned}
S_1 &= S_{12} + S_{13} = (150.43 + j100.16) + (-50.43 - j9.65) \\
&= 100 \text{ MW} + j90.51 \text{ Mvar}
\end{aligned}
$$