HW#6: Symmetrical Components

10.4. The line-to-line voltages in an unbalanced three-phase supply are $V_{ab} = 1000\angle 0°$, $V_{bc} = 866.0254\angle -150°$, and $V_{ca} = 500\angle 120°$. Determine the symmetrical components for line and phase voltages, then find the phase voltages V_{an}, V_{bn}, and V_{cn}.

First find the symmetrical components of line voltages, then find the symmetrical components of phase voltages. Use the inverse symmetrical components transformation to obtain the phase voltages. We use the following commands

```
a = -0.5+j*sqrt(3)/2;
Vabbcca=[1000      0        % Unbalanced line-to-line voltage
         866.0254  -150
         500        120];
VL012=abc2sc(Vabbcca); % Sym. comp. line voltages, rectangular
VL012p=rec2pol(VL012)        % Sym. comp. line voltages, polar
Va012=[ 0              % Sym. comp. phase voltages, rectangular
        VL012(2)/(sqrt(3)*(0.866+j0.5))
        VL012(3)/(sqrt(3)*(0.866-j0.5))];
Va012p=rec2pol(Va012)        % Sym. comp. phase voltage, polar
Vabc=sc2abc(Va012);    % Unbalanced phase voltages, rectangular
Vabcp=rec2pol(Vabc)          % Unbalanced phase voltages, polar
```

The result is

```
   VL012p =
              0.0000   30.0000
            763.7626  -10.8934
            288.6751   30.0000

   Va012p =
              0          0
            440.9586  -40.8934
            166.6667   60.0000
```

```
Vabcp =
        440.9586  -19.1066
        600.9252 -166.1021
        333.3333   60.0000
```

Note: The necessary relationships were derived in the class as a part of a problem. MATLAB has been used here in place of a calculator. Look inside and you will find all the relationships.

10.7. A three-phase unbalanced source with the following phase-to-neutral voltages

$$\mathbf{V}^{abc} = \begin{bmatrix} 300 & \angle -120^\circ \\ 200 & \angle 90^\circ \\ 100 & \angle -30^\circ \end{bmatrix}$$

is applied to the circuit in Figure 82. The load series impedance per phase is

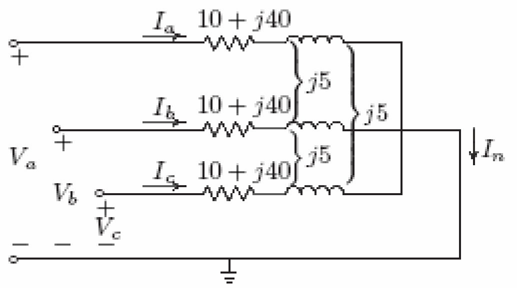

FIGURE 82
Circuit for Problem 10.7.

$Z_s = 10 + j40$ and the mutual impedance between phases is $Z_m = j5$. The load and source neutrals are solidly grounded. Determine
(a) The load sequence impedance matrix, $\mathbf{Z}^{012} = \mathbf{A}^{-1}\mathbf{Z}^{abc}\mathbf{A}$.
(b) The symmetrical components of voltage.
(c) The symmetrical components of current.
(d) The load phase currents.
(e) The complex power delivered to the load in terms of symmetrical components, $S_{3\phi} = 3(V_a^0 I_a^{0*} + V_a^1 I_a^{1*} + V_a^2 I_a^{2*})$.

(f) The complex power delivered to the load by summing up the power in each phase, $S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^*$.

We write the following commands

```
Vabc=[300    -120                    % Phase-to-neutral voltages
      200      90
      100     -30];
Zabc=[10+j*40    j*5    j*5 %Self and mutual impedances matrix
         j*5  10+j*40   j*5
         j*5     j*5 10+j*40];
Z012 = zabc2sc(Zabc)      % Symmetrical components of impedance
V012 = abc2sc(Vabc);        % Symmetrical components of voltage
V012p= rec2pol(V012)    % Converts rectangular phasors to polar
I012 = inv(Z012)*V012 ;     % Symmetrical components of current
I012p= rec2pol(I012)    % Converts rectangular phasors to polar
Iabc = sc2abc(I012);                            % Phase currents
Iabcp= rec2pol(Iabc)    % Converts rectangular phasors to polar
S3ph=3*(V012.')*conj(I012) %Power using symmetrical components
Vabcr = Vabc(:,1).*(cos(pi/180*Vabc(:,2))+...
j*sin(pi/180*Vabc(:,2)));
S3ph=(Vabcr.')*conj(Iabc)        % Power using phase quantities
```

The result is

```
Z012 =
      10.0+50.0i     0              0
       0            10.0 +35.0i     0
       0             0             10.0+35.0i
V012p =
       42.2650   -120.0000
      193.1852   -135.0000
       86.9473    -84.8961

I012p =
        0.8289    161.3099
        5.3072    150.9454
        2.3886   -158.9507
Iabcp =
        7.9070    165.4600
        5.8190     14.8676
        2.7011    -96.9315
S3ph =
        1036.8+3659.6i
S3ph =
        1036.8+3659.6i
```

10.8. The line-to-line voltages in an unbalanced three-phase supply are $V_{ab} = 600\angle 36.87^\circ$, $V_{bc} = 800\angle 126.87^\circ$, and $V_{ca} = 1000\angle -90^\circ$. A Y-connected load with a resistance of 37 Ω per phase is connected to the supply. Determine
(a) The symmetrical components of voltage.
(b) The phase voltages.
(c) The line currents.

We use the following statements

```
Vabbcca=[600  36.87                   % Unbalanced line voltages
         800  126.87
         1000  -90];
VL012=abc2sc(Vabbcca); % Sym. comp. line voltages, rectangular
VL012p=rec2pol(VL012)        % Sym. comp. line voltages, polar
Va012=[0
       VL012(2)/(sqrt(3)*(0.866+j*.5))
       VL012(3)/(sqrt(3)*(0.866-j*.5))];  % Sym. components of
                                 % phase voltages, rectangular
Va012p=rec2pol(Va012)     % Sym. comp. of phase voltages, polar
Vabc=sc2abc(Va012);              % Phase voltages, rectangular
Vabcp=rec2pol(Vabc)                    % Phase voltages, polar
Iabc=Vabc/37;                     % Line currents, rectangular
Iabcp=rec2pol(Iabc)                     % Line currents, polar
```

which result in

```
VL012p =
          0.0006   -179.9999
        237.0762    169.9342
        781.3204     24.0621

Va012p =
          0           0
        136.8790    139.9335
        451.1055     54.0628

Vabcp =
        480.7542     70.5606
        333.3386    163.7411
        569.6111    -73.6857

Iabcp =
         12.9934     70.5606
          9.0092    163.7411
         15.3949    -73.6857
```

10.9. A generator having a solidly grounded neutral and rated 50-MVA, 30-kV has positive-, negative-, and zero-sequence reactances of 25, 15, and 5 percent, respectively. What reactance must be placed in the generator neutral to limit the fault current for a bolted line-to-ground fault to that for a bolted three-phase fault?

The generator base impedance is

$$Z_B = \frac{(30)^2}{50} = 18 \ \ \Omega$$

The three-phase fault current is

$$I_{f\,3\phi} = \frac{1}{0.25} = 4.0 \ \ \text{pu}$$

The line-to-ground fault current is

$$I_{f\,LG} = \frac{3}{0.25 + 0.15 + 0.05 + 3X_n} = 4.0 \ \ \text{pu}$$

Solving for X_n, results in

$$\begin{aligned} X_n &= 0.1 \quad \text{pu} \\ &= (0.1)(18) = 1.8 \ \ \Omega \end{aligned}$$