Spurious Modes in the DuFort–Frankel Finite-Difference Beam Propagation Method

Husain M. Masoudi and John M. Arnold

***Abstract*—In this letter, we analyze the DuFort–Frankel beam propagation method (BPM) which is a modification to the known explicit finite-difference beam propagation method (EFD-BPM) and found that there are some precautions that must be taken before using the method. The accuracy and the efficiency of this method has been shown and compared with the most popular FD-BPM's.**

***Index Terms*— Beam propagation method, finite-difference analysis, modeling, numerical analysis, optical waveguide theory, partial differential equations.**

I. Introduction

THE BEAM propagation method (BPM) is one of the most attractive techniques to analyze optical devices due to its simplicity and applicability to a variety of optical devices. For modeling three-dimensional devices, parallel processing can be used to analyze long and complicated devices in a reasonable amount of time [1]–[3]. In previous work, we showed that the explicit finite-difference (EFD) BPM and the real space (RS) BPM are well suited to the parallel environment due to their explicit nature [2]–[3]; also the parallel EFD has been successfully extended to analyze nonlinear three-dimensional (3-D) optical waveguides containing second order nonlinearities [4]–[5]. Although the parallel EFD is simple and efficient, the disadvantage with this method is that it is conditionally stable. Decreasing the transverse mesh sizes requires that the longitudinal step size be decreased as well for the algorithm to remain stable, and this has the consequence of longer computational time [6]. One way to improve the stability of the EFD is to use the DuFort–Frankel technique [7] which is also explicit. The DuFort–Frankel technique uses the same equations as the EFD with small modification [1], [7]–[8], and for simplicity we will refer to this method as the modified EFD or MEFD. The MEFD has two advantages over the EFD; the first is that the stability condition is improved, resulting in a relaxed longitudinal step size. The second is that the total mesh points can be divided into two sets, odd and even (leapfrog arrangement), where only one set need be used in computations, giving a 50% reduction in execution time while retaining the same accuracy [1], [8]. Another important advantage to the MEFD is that it is still highly parallel. However, during our analysis of this method we found that there are spurious fields that are coupled with the true field [1] and this problem has not been addressed by the previously published work on this method [8]. In this work we show the analysis of the MEFD together with some numerical solutions that remove/reduce the error fields. As a first step toward 3-D parallel implementations of the MEFD we restrict the analysis in this work to 2-D because it is easier to analyze and the results can be compared with analytical solutions. Then later we compare with other FD-BPM's in terms of accuracy and efficiency.

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H. M. Masoudi is with the Department of Electrical Engineering, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia.

J. M. Arnold is with the Department of Electrical Engineering, University of Glasgow, Glasgow, Scotland.

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II. Numerical Method

Starting with the parabolic equation for a two-dimensional TE field ϕ

$$2jk_on_o\frac{\partial\phi}{\partial z}=\frac{\partial^2\phi}{\partial x^2}+k_o^2(n^2-n_o^2)\phi \tag{1}$$

where n_o is a reference refractive index, k_o is the free-space wave number and $n(x,z)$ is the refractive index and using the central finite-difference approximation for the partial derivatives leads to the discrete EFD equation [6]

$$\phi_i(z+\Delta z)=\phi_i(z-\Delta z)+a[\phi_{i-1}(z)+\phi_{i+1}(z)]+b_i\phi_i(z) \tag{2}$$

where $a=\Delta z/(jk_on_o\Delta x^2)$ and $b_i=(\Delta z/jk_on_o)[k_o^2(n_i^2-n_o^2)-2/\Delta x^2]$. i represents the discretization of the transverse coordinate x, and Δx and Δz are the transverse mesh size and the longitudinal step size, respectively. It is known that the EFD is stable under the condition $\Delta z<2k_on_o/[(4/\Delta x^2)+k_o^2|n_i^2-n_0^2|_{\max}]$ [6]. On the other hand, if the field $\phi_i(z)$ in (2) is replaced by its average value as $\phi_i(z)=[\phi_i(z+\Delta z)+\phi_i(z-\Delta z)]/2$ [7] this leads to the DuFort–Frankel method (or the MEFD) [1], [8]

$$\phi_i(z+\Delta z)=c_i\phi_i(z-\Delta z)+d_i[\phi_{i-1}(z)+\phi_{i+1}(z)] \tag{3}$$

where $c_i=(2+b_i)/(2-b_i)$ and $d_i=2a/(2-b_i)$. We may notice that both the EFD and the MEFD need two initial fields to start the algorithm. It has been observed from analyzing the MEFD (as will be seen later) that there are some spurious fields that propagate with the true field. For later reference, we will refer to this field as a *ghost* field because it takes on the shape of the true fields propagating in the waveguide. This problem is caused by the initial excitation of the method. In our analysis we use the power spectral method in [9] to calculate

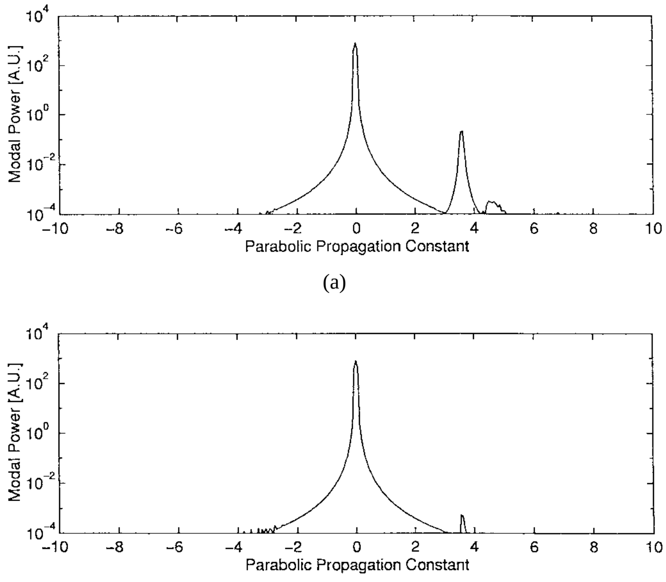

Fig. 1. The modal power spectrum for the slab waveguide excited with the TE_0 guided mode using the MEFD-BPM with $\Delta x = 0.05\ \mu$m, $\Delta z = 0.1\ \mu$m and a total distance of 102.4 μm. (a) Before correction. (b) After correction.

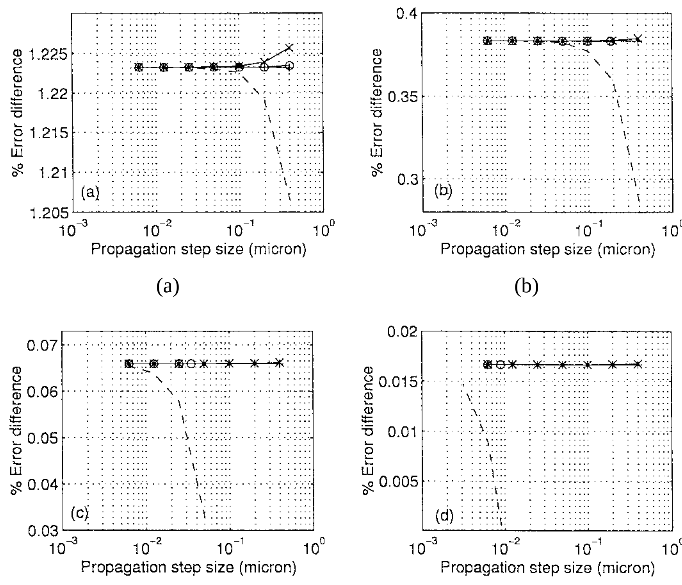

Fig. 2. The percentage error difference of the first guided-mode FD-BPM's effective index using the CN $(+)$, the RS $(-)$, the EFD (o) and the MEFD $(\times)$ as a function of the longitudinal step size Δz for (a) $\Delta x = 0.5\ \mu$m, (b) $\Delta x = 0.25\ \mu$m, (c) $\Delta x = 0.1\ \mu$m and (d) $\Delta x = 0.05\ \mu$m.

the mode indices from the BPM fields. First, the correlation function between the input and the marched field is evaluated numerically during the course of propagation, then the result is multiplied with the Hanning window function and Fourier transformed. The propagation constant is computed by locating the peak in the spectral domain. If the waveguide is excited with the first guided mode, the spectral analysis should display only one peak that belongs to this mode. However, numerical tests with the BPM in general showed that, for large Δx, some very little radiation fields are excited due to the discretization where these radiation can be minimized by reducing Δx [1].

III. Numerical Analysis

In the following, we use a symmetric slab waveguide with core and cladding refractive indices of 1.2 and air respectively. The waveguide width is 1 μm, the total window size is equal to 11 μm and the wavelength is 1 μm. In all that follows, the first guided mode is excited at the input. Fig. 1 shows the power spectrum for the propagation of the first guided mode inside the slab waveguide using the MEFD. The top figure is caused by setting the initial two fields to be equal. From the figure we may notice the existence of two peaks and a summation of other little peaks. The largest peak belongs to the true guided mode, the second largest peak belongs to the guided *ghost* mode and the little peaks belong to radiation ghost fields. The ghost fields are the error associated to the false excitation of the input fields. Analysis of these ghost fields showed that the shape of the field is similar to the true fields propagating in the waveguides and for this reason we call it a *ghost*. The peak of the power spectrum of the ghost fields reduces by reducing Δz. A full mathematical analysis, which we will elaborate elsewhere, shows that the dispersion relation of (3) for modal fields with variation $\exp(-j\beta z)$ along the z-direction allows two values of β for each transverse eigenvector. After several

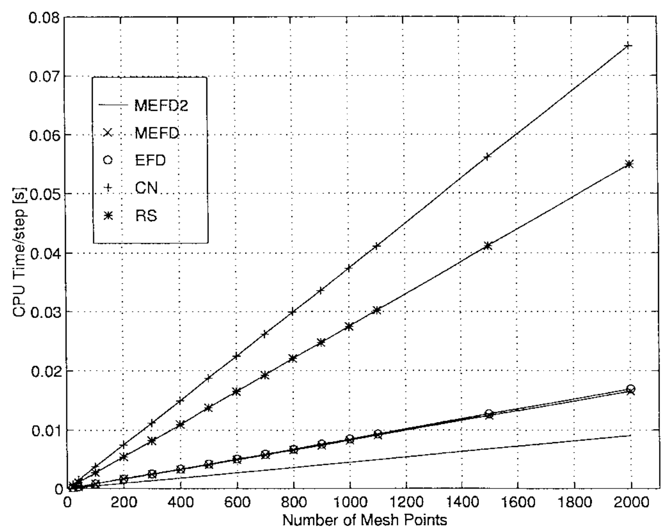

Fig. 3. Comparison between the speed of the CN, RS, EFD, MEFD, and MEFD2 as a function of the transverse mesh points. MEFD2 is the MEFD using half of the discretised mesh points.

numerical tests, we found two ways to reduce or eliminate the error caused by the initial excitation. Our numerical investigations include excitation using the unstable forward Finite-Difference instead of the central difference, for the partial derivative with respect to z, for the first step of the propagation. Also other BPM's (Crank–Nicolson (CN), real space (RS), fast Fourier transform (FFT) and EFD) have been used to provide two initial fields for the MEFD. All these techniques produce very little difference compared to the results in Fig. 1(a). If the initial field is a guided mode, one can find two fields spaced Δz from each other by multiplying one of the field with the modal phase factor assuming that the medium does not change over that distance. Tests on this technique showed the complete removal of the spurious field. The second technique is to use two equal initial fields with a very small initial step size increasing gradually during the course of propagation to the desired Δz. In other words, if the desired $\Delta z = 0.1$, then an initial $\Delta z = 0.001$ could be used increasing uniformly to 0.1, which involves 100 pre-propagation steps. Of course, the smaller initial Δz is better. Fig. 1(b) is the power spectrum of the same as the top figure with the second technique of removing the error applied, using an initial $\Delta z = 0.0001$ μm. The computation of the correlation function for the spectral technique starts when Δz becomes 0.1 μm. The comparison between the two figures shows that the error has reduced drastically. To show the advantage of the MEFD, we compare the accuracy and the efficiency of the MEFD with the most popular FD-BPM's: the CN, RS and EFD. Fig. 2 shows the percentage error of the effective refractive index as a function of Δz for different transverse mesh spacing using the four methods. From the figure, we can observe that the error generally decreases as Δx is decreased for the four algorithms. The figure also shows that the accuracy of the CN, the MEFD and the EFD are not very sensitive to the change in Δz, but Δz for the EFD must be less than the limit of the stability condition. We note from this stability condition of the EFD that as Δx is decreased, Δz must be also decreased. On the other hand, Fig. 2 shows the convergence of the RS as a function of Δz and shows, as for the EFD, that Δz must decrease as Δx is reduced. Fig. 3 shows the CPU time per propagational step as a function of the total transverse mesh points. From the figure, the speed of the MEFD is almost the same the EFD, however when half of the number of mesh points is considered (using the leapfrog ordering [1], [8]) the speed as expected increases two fold. Comparison between the MEFD2 and the CN shows that the MEFD2 is always more than eight times faster per propagational step than the CN. All the simulations in this work were executed using a Sparc Classic Sun workstation.

In conclusion, we have analyzed the DuFort–Frankel BPM (or the MEFD), which is a modification to the existing EFD, and found that in using this method care must be exercised due to the existence of spurious fields that must be cleared before using the method. The analysis of the MEFD has been shown and the accuracy is compared with other FD-BPM's. From the comparisons, we conclude that the MEFD is very efficient; its accuracy is comparable to the best FD-BPM's and its speed is higher. Since the MEFD is still explicit, the 3-D version will be highly parallel and useful for analyzing long and complicated devices. The work of parallelizing the 3-D MEFD is under investigation.

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