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PARALLEL THREE-DIMENSIONAL FINITE DIFFERENCE BEAM PROPAGATION METHODS

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SUMMARY

In this work, we show the implementation of two explicit three-dimensional finite-difference beam propagation methods (BPM) on two different parallel computers, namely a transputer array and a Connection Machine (CM). To assess the performance of using parallel computers, serial computer codes of the two methods have been implemented and a comparison between the speed of the serial and parallel codes has been made. Large gains in the speed of the parallel FD-BPMs have been obtained compared to the serial implementations. In addition, a comparison between the performance of the transputer array and the CM in executing the two FD-BPMs has been discussed. Finally, to assess and compare the two methods, three different rib waveguides and three different directional couplers have been analysed and the results compared with published results.

1. INTRODUCTION

Most large numerical algorithms for the analysis of optoelectronic devices consume a lot of time to produce one set of data in solving a single problem; however, most of the computations involved may contain parts that could be computed independently. By breaking the problem into small pieces and arranging for each piece to be solved simultaneously, using parallel computers, the computation of the problem could be solved in a smaller amount of time. The beam propagation method (BPM), which is a standard method used to model optical devices, is one of the algorithms that can run efficiently on parallel computers.

The classical BPM has been widely used to analyse optical waveguide structures.[1–3] The BPM is attractive to the designer of optical devices because it overcomes the difficulties of mode theory when applied to complicated structures, and because of its flexibility as a propagational technique. It is essentially an approximate numerical method which solves the scalar wave equation in its approximate parabolic form. The original method consisted of marching the input optical field over small distances in dielectric media with the use of the fast Fourier transform (thus called FFT-BPM).[1] The role of the FFT is to provide a transformation between the spectral domain and the spatial domain. For each propagational step, the optical field is simulated by a spectrum of plane waves in the spectral domain and a phase correction due to the medium inhomogeneity is introduced in the spatial domain. The FFT-BPM has limitations that restrict its application. For instance, in addition to the poor efficiency of the FFT-BPM, a large variation in the transverse refractive index profile of the semiconductor waveguides will force the method to use extremely small propagational steps. Above all, the restriction imposed by the FFT makes the method incapable of using non-uniform grid spacing and radiation boundary conditions.

An alternative numerical technique to solve the parabolic equation in the spatial domain is to use a finite-difference approximation to replace the partial derivatives in the equation (thus called FD-BPM). Recently, this approach has received wide attention from many workers.[2–7] Lately, the vectorial finite-difference beam propagation method has been reported.[6] All FD-BPM techniques have shown that this approach is much more efficient than the FFT-BPM in terms of accuracy, speed and storage required. In addition, some of these techniques have succeeded in

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overcoming the main limitation of low-contrast media in the FFT-BPM, and allow propagation in strongly guiding structures.

The FD-BPM is very efficient when applied to the analysis of 2-D structures, because the computation involves only one-dimensional numerical arrays. On the other hand the FD-BPM is time-consuming when used to analyse three-dimensional devices accurately, for several reasons. First, the 3-D nature of the rectangular waveguides leads to a large computational problem; second, some practical devices contain multiple coupled linear and non-linear waveguides; and third, the existence of large-contrast media will force the BPM to use small transverse mesh sizes and small longitudinal step sizes for convergence or stability reasons. Obviously, this will multiply the computational effort many times over. The computational resources used (conventional serial computers) are not adequate for this kind of problem. The best way to solve large problems quickly is to use parallel computers.[7–10] Already, substantial improvements in the speed of parallel over serial computation have been achieved for a number of major mathematical applications. In this work we show that the implementation of two finite-difference BPMs on a transputer array and a Connection Machine (CM) speed up the execution of these algorithms tremendously. Owing to the fact that parallel computers can execute large problems rapidly, the parallel FD-BPM allows optical devices to be modelled accurately, by increasing the number of mesh points, and gives the freedom to study complicated devices that contain multiple waveguides. The next section shows the formulation of three finite-difference BPMs in three dimensions, in which two of them are highly parallel. Then Section 3 shows the implementation of these two methods on parallel computers where full comparisons between the performance of these parallel methods have been made. Section 4 contains the accuracy assessment of the two parallel BPMs where three different rib waveguides and three different directional couplers have been analysed and the results are compared with other techniques.

2. THEORY

We begin with the parabolic wave equation in three dimensions for a scalar monochromatic electric field, E, which can be written as

$$2jkn_0 \frac{\partial E}{\partial z} = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + k^2(n^2 - n_0^2)E \tag{1}$$

where k is the free space wave number, n_0 is a reference refractive index and $n(x,y,z)$ is the refractive index profile. x and y are the transverse directions and z is the propagational direction. Generally the FD-BPM has two methods of expressing the operator formulation based on equation (1), the implicit approach and the explicit approach.[1–7] Implicit methods use the following equation to march the field from z to $z + \Delta z^2$

$$E(x,y,z+\Delta z) = \exp\left(-\mathrm{j}\frac{\Delta z}{2a}\nabla_x^2\right)\exp\left(-\mathrm{j}\frac{\Delta z}{2a}\nabla_y^2\right)\exp\left(-\mathrm{j}\frac{\Delta z}{a}d\right)\exp\left(-\mathrm{j}\frac{\Delta z}{2a}\nabla_x^2\right)$$
$$\exp\left(-\mathrm{j}\frac{\Delta z}{2a}\nabla_y^2\right)E(x,y,z) + O((\Delta z)^3) \tag{2}$$

where

$$a = 2\,kn_0$$
$$d(x,y,z) = k^2[n^2(x,y,z) - n_0^2]$$
$$\nabla_\rho^2 = \frac{\partial^2}{\partial \rho^2}, \qquad (\rho = x,y)$$

Δz is a small step in the propagation direction. Replacing the operators which contain the Laplacian with the relation

$$\exp\left(-\mathrm{j}\frac{\Delta z}{2a}\nabla_{\rho}^{2}\right)=\frac{\left[1-\mathrm{j}\frac{\Delta z}{4a}\frac{\partial^{2}}{\partial\rho^{2}}\right]}{\left[1+\mathrm{j}\frac{\Delta z}{4a}\frac{\partial^{2}}{\partial\rho^{2}}\right]}+O((\Delta z)^{3}) \qquad (3)$$

gives rise to the most popular BPM, which is based on the alternating direction implicit approximation (ADI-BPM). The ADI-BPM is unconditionally stable, but requires the solution of a large block tridiagonal system of equations for each propagational step.[8] On the other hand, the explicit approach has mainly two ways of formulating the problem. The first is the real space method (RS-BPM), which uses the finite-difference matrix splitting operators to approximate equation (1), and can be written as[2,7,8]

$$\begin{aligned}E_{i,m}(z+\Delta z)=&\exp(\alpha_y S_y^{o})\exp(\beta_y S_y^{e})\exp(\alpha_y S_y^{o})\exp(\alpha_x S_x^{o})\exp(\beta_x S_x^{e})\exp(\alpha_x S_x^{o})\exp\left(-\mathrm{j}\frac{\Delta z}{a}U\right)\\&\exp(\alpha_y S_y^{o})\exp(\beta_y S_y^{e})\exp(\alpha_y S_y^{o})\exp(\alpha_x S_x^{o})\exp(\beta_x S_x^{e})\exp(\alpha_x S_x^{o})\\&E_{i,m}(z)+O((\Delta z)^{3})\end{aligned} \qquad (4)$$

where

$$U(x,y,z)=d(x,y,z)-\frac{2}{\Delta x^{2}}-\frac{2}{\Delta y^{2}}$$

$$\beta_{\rho}=-\mathrm{j}\frac{\Delta z}{2a\,\Delta\rho^{2}}=2\alpha_{\rho}$$

Δx and Δy are the mesh sizes in the x- and the y-directions respectively. The symbols i and m represent the sampled values of both the field and the refractive index profile in the x- and y-direction respectively. The matrix splitting in equation (4) is chosen such that each matrix S_{ρ}^{o} and S_{ρ}^{e} is block-diagonal, where each block contains a small submatrix (e.g. 2 by 2) which may be exponentiated analytically. This method is unconditionally stable but, similar to the FFT-BPM, requires small propagational steps to converge when applied to large constrast media. However, it proves to be much more efficient per propagational step than the ADI-BPM because it does not involve the solution of a system of equations, but instead multiplication of independent small matrices. Full details of the RS method can be found in Reference 8. The second approach is truly explicit and therefore called the explicit finite difference method (EFD-BPM), and is based on applying the central finite-difference approximation directly to the parabolic equation; it can be formulated, in a discretized fashion, as[5]

$$\begin{aligned}E_{i,m}(z+\Delta z)=&E_{i,m}(z-\Delta z)+4\beta_x[E_{i-1,m}(z)+E_{i+1,m}(z)]\\&+4\beta_y[E_{i,m-1}(z)+E_{i,m+1}(z)]-\mathrm{j}\frac{2\Delta z}{a}U_{i,m}E_{i,m}(z)\end{aligned} \qquad (5)$$

The propagation of the optical field in equation (5) is straightforward since it involves multiplication of the input field with a very sparse matrix, which makes the method very efficient. However, this algorithm is only stable under the condition[5]

$$\Delta z<2kn_0\left[\frac{4}{\Delta x^{2}}+\frac{4}{\Delta y^{2}}+k^{2}|n_{i,m}^{2}-n_0^{2}|_{\max}\right]^{-1}$$

Comparison between all of the FD-BPMs described above shows that both the RS-BPM and the EFD-BPM are highly parallel due to the locality of the spatial points, which reduces the movement of data between processors.[7–10] In contrast, the ADI-BPM will not gain as much in speed when implemented in parallel because of the required inversion of large matrices for each propagational step; in other words, to perform the propagation on any given spatial point, information is required from all parts of the transverse space. Although the propagational step of the ADI-BPM is considerably larger than that of the explicit methods, the explicit methods are more efficient than the implicit method in two ways: first, they are much more efficient per propagational step in normal serial form; and second, they gain a larger speed-up when run on parallel computers. These two reasons more than compensate for the constraints imposed on both methods. This is why we have not fully implemented the ADI-BPM on parallel machines.

3. PARALLEL IMPLEMENTATIONS

In this section we show how to implement both of the explicit methods, the EFD-BPM and the RS-BPM, on a transputer array and a connection machine. The transputer array is a Parsytec super-cluster consisting of 64 IMS-T800 processors (MIMD machine) each with 4 megabytes. The Connection Machine is a CM-200 consisting of 16 kbit (16,384-bit) serial processors (SIMD machine) with a total of 0·5 gigabytes. The front-end computer to both of these machines is a SUN (SPARC Station 2).

3.1. *The transputer array*

In order to implement the explicit methods on a transputer cluster, a topology for connecting the processors has to be carefully selected to ensure that maximum efficiency is gained from parallelizing these methods. We have used the 2-D grid topology shown in Figure 1 for the implementation of both the EFD-BPM and the RS-BPM on the transputer array. We believe that this topology is the best arrangement to parallelize these algorithms in terms of efficiency and transputer memory distribution. Excluding the processors at the borders of the topology shown, each processor has four links connected to its neighbours, where each link is a bidirectional communication channel for exchanging information. We have implemented both the EFD-BPM and the RS-BPM on the 2-D grid topology, by dividing the total 2-D transverse mesh into 2-D identical blocks of mesh points where the size of each block is equal to

$$\left(\frac{M_x}{\dim(x)}, \frac{M_y}{\dim(y)}\right)$$

M_x and M_y are the number of mesh points in the transverse direction x and y respectively and $\dim(x)$ and $\dim(y)$ are the number of processors in the x- and y-direction, respectively, of the 2-D grid topology. Each block is assigned to one processor for computation. The arrangement of the 2-D topology in Figure 1 will ensure that all processors carry out equal amounts of computation, without the need to load-balance the transputer system. In addition, it gives the freedom to change both the number of processors and the number of mesh points without altering

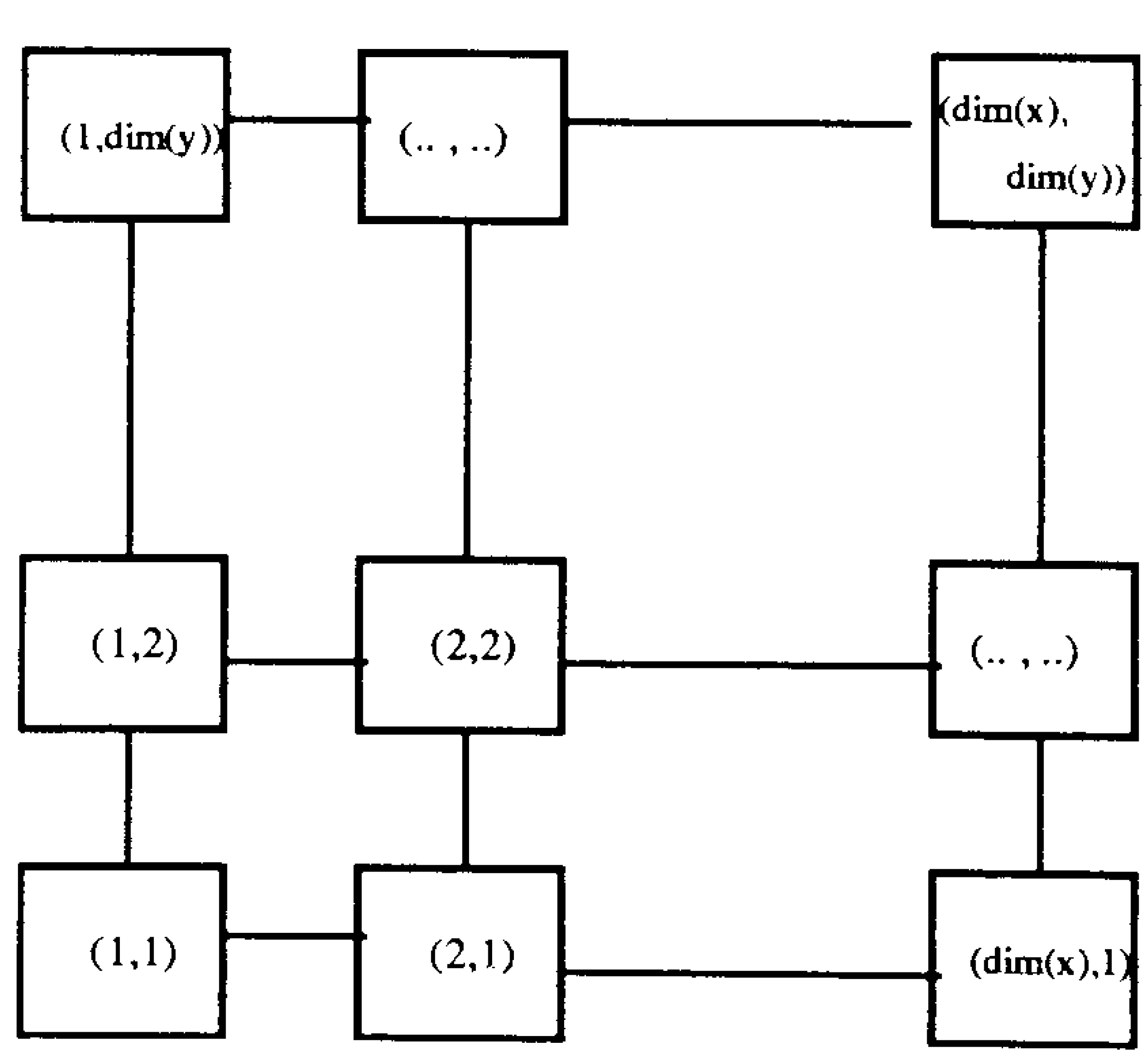

Figure 1. The 2-D grid topology used for the implementation of both the EFD-BPM and the RS-BPM. The number shown indicates the position of each processor in terms of the 2-D grid

the parallel computer code. For every propagational step the processors exchange only the local mesh points at the border of each computational block. We have implemented both of the explicit methods on a single processor (serial computation) of the transputer in addition to the parallel implementations, in order to study the gain in efficiency by using the transputer array. All computer codes were written in FORTRAN with a double-precision accuracy under PARIX operating environment software.[11]

3.2. *The Connection Machine*

The implementation of the two explicit methods on the CM is totally different from those on the transputer array. Unlike the transputer array, where the links between processors are chosen explicitly by the user, the CM achieves the parallel mechanism globally rather than locally. The CM computer code resides in the central control unit (the front-end computer) and the data, to be computed in parallel, are distributed to the private memory of each processor of the CM. Then the central control unit of the machine broadcasts a system of identical serial instructions to all processors, where these instructions are executed simultaneously by all processors on all local data of each processor (see Figure 2).[10,12] In the case that the data array is larger than the CM resources (number of processors) then the system will create a virtual processing mechanism, which means that each physical processor simulates a number of virtual processors, to accommodate all the data, by subdividing its local memory. The CM computer programs are similar to ordinary serial programs in the sense that the instructions are executed serially; however, some computations are done concurrently. The communications between processors inside the machine and between the front end and the CM are all transparent to the user. We have implemented both of the explicit methods on the CM using half (8k) and full (16k) size of the CM. In these implementations CM-FORTRAN-2.1.1 with double-precision accuracy has been used.[12]

3.3. *Speed*

All computer codes have been tested to analyse the rib waveguide shown in Figure 3 (see Section 4). To concentrate on the efficiency issue of the parallel machines, we have set the number of mesh points in both directions to be equal ($M_x = M_y$) with uniform grid spacing. The best way to compare between the two explicit methods and between the performance of the two machines is by computing the total CPU time per propagational step for different numbers of transverse mesh points and by changing the number of processors as well. Figure 4 (in log–log scale) shows the total CPU time of both the second-order EFD-BPM and the second-order RS-

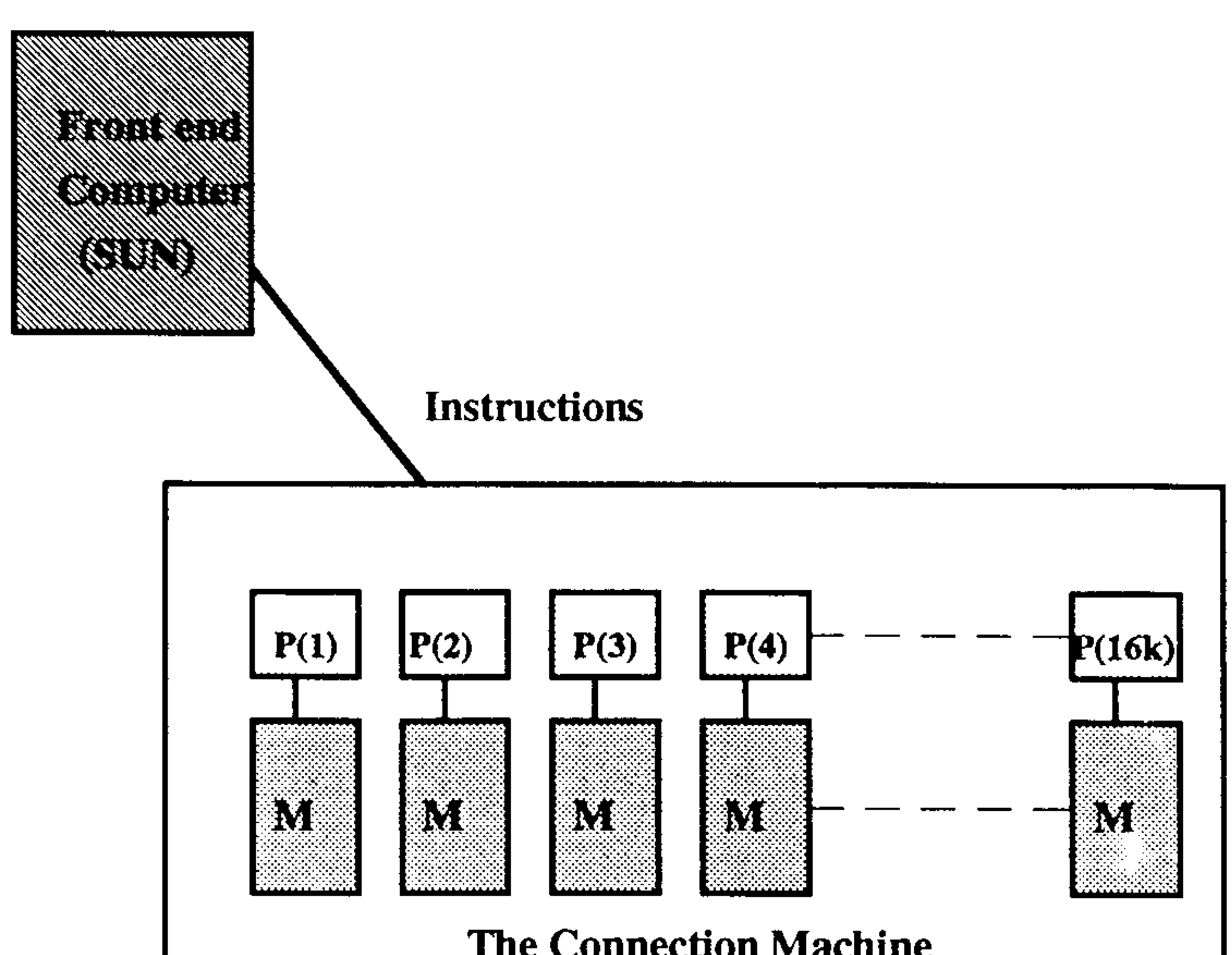

Figure 2. The Connection Machine. P = processor and M = memory

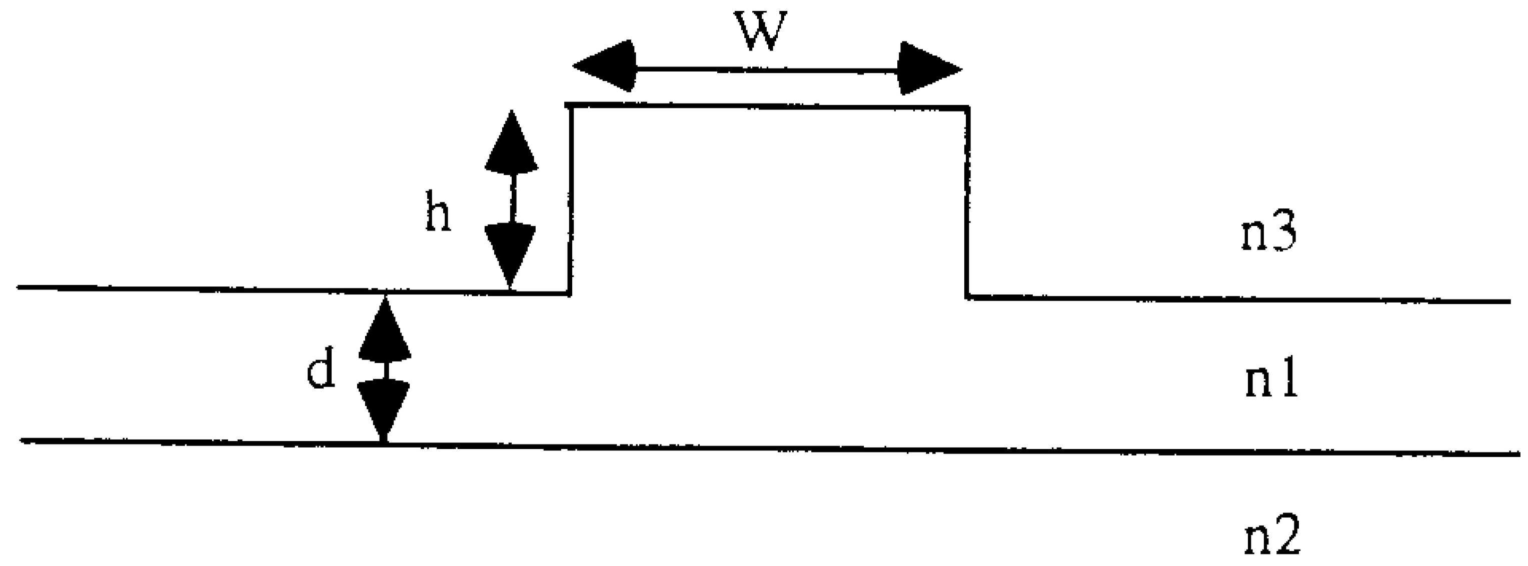

Figure 3. The rib waveguide that is used for testing both of the explicit methods

BPM per propagational step versus the number of mesh points in one of the transverse direction x. The figure shows both the serial and the transputer results in addition to the CM results for comparison. Some of these results have been reported in Reference 7, but the CM results are different because the CM computer programs have been compiled with the updated version of CM-FORTRAN. In the figure, 'serial' means a single processor and '2 × 2' means that four processors of the transputer array have been used in the computations; other notation can be understood accordingly. 'CM-8k' and 'CM-16k' means that half and full CM resources have been used, respectively. It can be seen from the figure that for a fixed M_x the speed of both methods increases as the number of processors of the transputer increases. On the other hand, generally the CM speed is faster than the best performance of the transputer array. For example at $M_x = 400$, the CM-16k is faster than the 64 processors of the transputer array by 11 times for the EFD-BPM and 6·8 times for the RS-BPM. We can notice in Figure 4 that the serial plot does not contain results exceeding $M_x = 240$ for the EFD-BPM and $M_x = 250$ for the RS-BPM, owing to memory limitations of the computer. To assess the performance of the parallel implementations, we define the following terms:

$$\text{Speed-up} = \frac{\text{Serial speed}}{\text{Parallel speed}} \tag{6}$$

and

$$\text{Efficiency} = \frac{\text{Speed-up}}{\text{Number of processors}} \tag{7}$$

The efficiency calculation, defined by equation (7), is only for the transputer array results, where it is unrealistic to do that for the Connection Machine results because the processors are different from those of the transputer array. Figure 5 shows the speed-up for both of the explicit methods. The figure shows that the speed-up for both algorithms, for a fixed number of mesh points, increases as the number of processors increases. At $M_x = 240$, the speed-up factors for the EFD-BPM when using the full transputer size (8 by 8) and the full CM resources (CM-16k) are around 54 and 547·3 respectively, while the speed-up of the RS-BPM when using 8 by 8 of the transputer array and the CM-16k is around 60 and 369 respectively. The other performance indicator of the transputer, the efficiency, is shown in Figure 6 for both of the explicit methods. It can be seen from the figure, for both methods, that for a fixed number of processors the efficiency increases as the number of mesh points increases. For $M_x = 240$ the percentage efficiencies of the EFD-BPM and RS-BPM are around 98% and 100%, respectively, when using four processors. On the other hand and at the same number of mesh points ($M_x = 240$) but when using 64 processors, the percentage efficiencies of the EFD-BPM and the RS-BPM are around 84% and 94% respectively. It can be also seen from Figure 6 that the efficiency of the RS-BPM is always higher than that of the EFD-BPM. The reason for this is that the ratio of the computational time to the communicational time between processors for the RS-BPM is higher than that of the EFD-BPM. Finally, the comparison between the speed of the two parallel methods shows that the parallel EFD-BPM is always faster than the parallel RS-BPM; for example when using 16k of the CM at $M_x = 1000$ the speed of the EFD-BPM is around 6·8 times faster than the RS-BPM.

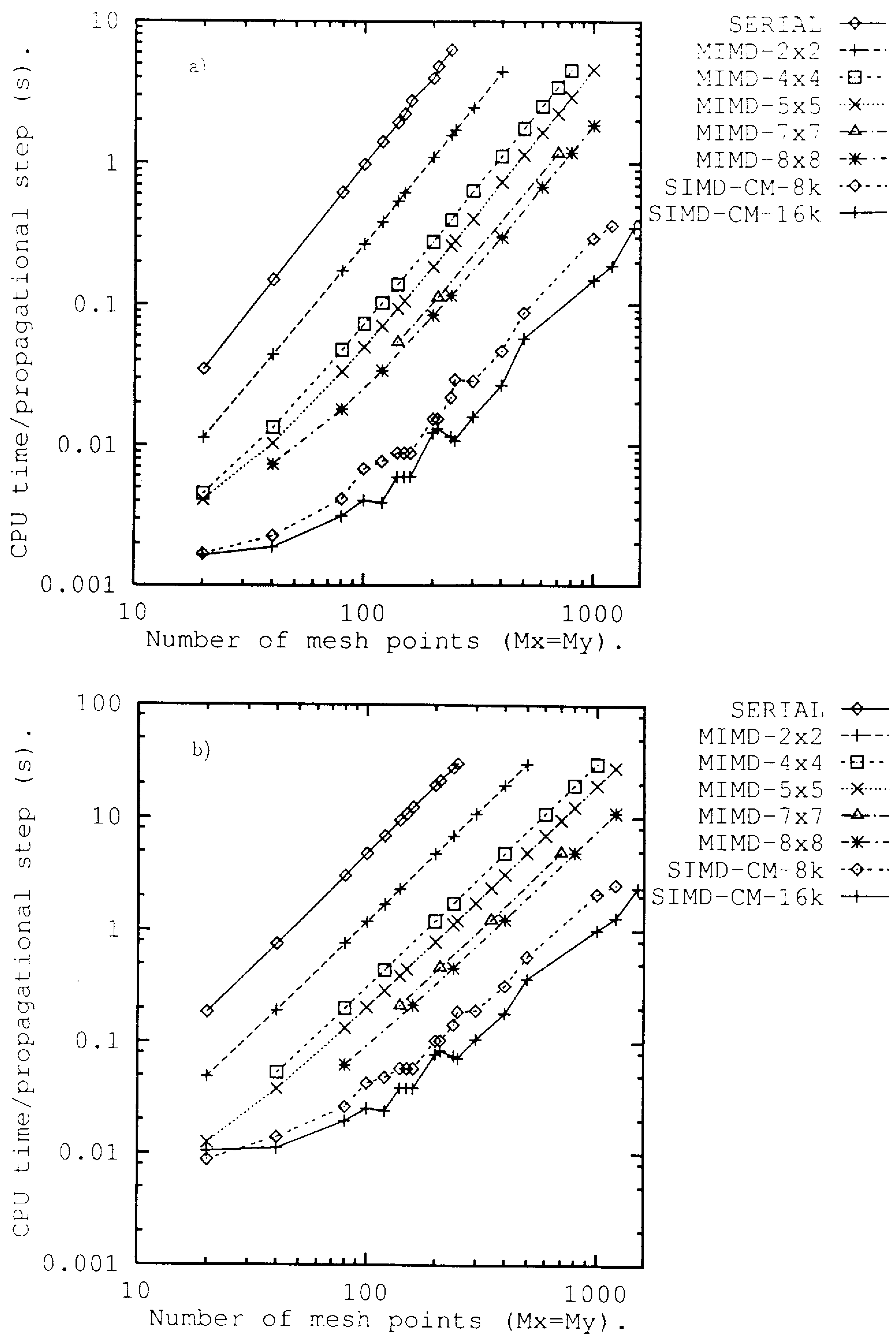

Figure 4. Comparison between the speed of the serial and the parallel implementations of the explicit methods using the transputer array (MIMD) and the Connection Machine (SIMD-CM). (a) The EFD-BPM; (b) the RS-BPM

4. ACCURACY ANALYSIS

In this section we show the results of the accuracy analysis of testing the two parallel explicit methods explained in Sections 2 and 3. Although the accuracy of the serial EFD-BPM has been verified by Reference 5, in order to compare between the two parallel algorithms we have tested both of the parallel explicit methods to analyse three well-known different rib waveguides (Figure

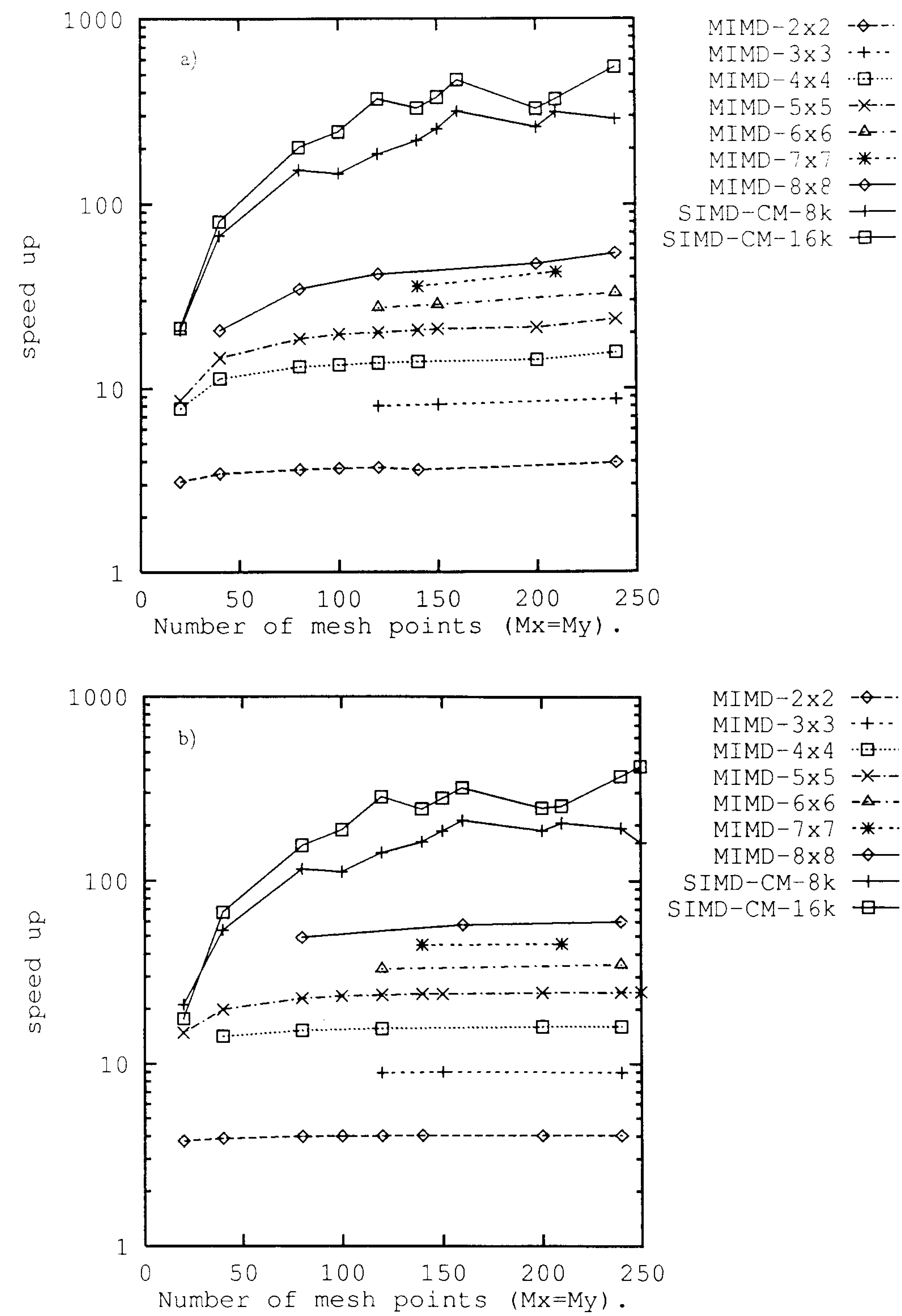

Figure 5. Speed-up of both of the parallel explicit methods using the MIMD and the SIMD computers. (a) The EFD-BPM; (b) the RS-BPM

3). The parameters of the three structures are shown in Table I, where the operating wavelength of all structures is $\lambda = 1{\cdot}55$ μm. The transverse computational window size of structure 1 is small because it is a strong guide, whereas structure 3 is a very weak guide; thus the window size must be large to minimize the influence of the boundary. The first check involves the computation of the fundamental mode indices (N_{eff}) for the three structures. The second test is the computation of the coupling lengths (L_c) of directional couplers consisting of two core rib waveguides (see

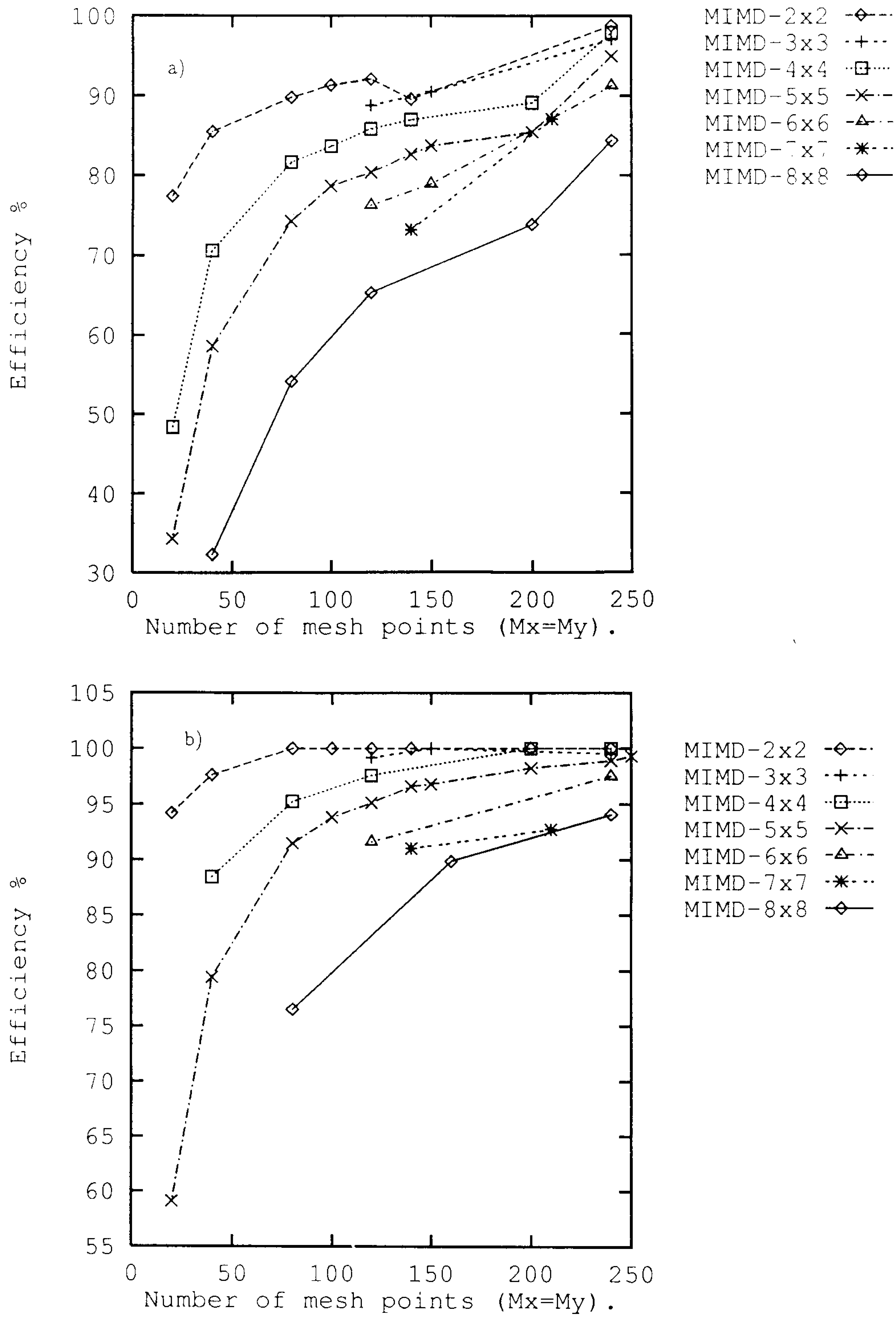

Figure 6. The efficiency in percentage of both of the parallel explicit methods of using the MIMD computer. (a) The EFD-BPM; (b) the RS-BPM

Figure 3) with a separation gap of 2 μm. In the calculations we have set the internal interfaces of layers of different refractive indices to be half-way between two adjacent mesh points, the reference index n_0 to be the substrate refractive index n_2 and zero boundary conditions at the edges of the computational window.

The power spectral method[13] has been used to compute the mode indices of the three structures from the BPM fields. This method uses the numerical correlation function, between the input

Table I. The rib waveguide parameters used in the computations (see Figure 3)

Structure	n_1	n_2	n_3	w (μm)	d (μm)	h (μm)	x-window (μm)	y-window (μm)
1	3·44	3·34	1·0	2·0	0·2	1·1	8·0	8·0
2	3·44	3·36	1·0	3·0	0·9	0·1	33·0	5·0
3	3·44	3·435	1·0	4·0	3·5	2·5	130·0	33·0

field and the BPM computed field, to locate the peaks in the Fourier space. First the correlation function is multiplied by a Hanning window function and then Fourier transformation is performed with respect to the axial distance z. The position of the power spectral peaks can determine, by applying a line-shape fitting technique, the parabolic propagation constant from which the Helmholtz propagation constant can be computed. The full description of the power spectral method can be found in Reference 13.

In the following analysis, the input field is set to be a gaussian field centred in the middle of the guiding layer to excite the fundamental mode of the structure. For the computation of the coupling length of the directional coupler, the even and the odd modes indices (N_e and N_o) have to be computed ($L_c = \lambda/2(N_e - N_o)$). In order to excite the even mode, a sum of two equal-amplitude gaussian fields centred in the middle of each arm of the coupler has been launched as an input. The same has been done to excite the odd mode of the coupler except that the two gaussian fields have opposite sign. The longitudinal distance z considered in this analysis is around 0·5 mm which involves 65,536 steps when Δz is very small.

Tables II, III and IV show the fundamental mode indices N_{eff} and the coupling lengths L_c of structures 1, 2 and 3 respectively, computed by both of the parallel EFD-BPM and RS-BPM. Table II, shows values of N_{eff} and L_c for various transverse mesh sizes Δx and Δy of both algorithms, as well as for various step sizes Δz for the RS-BPM. The EFD-BPM is not stable for values of Δz larger than those given in Table II. As expected for any finite-difference method, the results converge as the grid spacing decreases; the convergence of the two methods is clearly demonstrated in Table II. In order for the RS-BPM to converge, Δz should be decreased as the transverse mesh sizes are decreased. It can be observed from Table II that both methods produce similar results around almost the same step size, thus we can conclude that the parallel EFD-BPM is more efficient than the parallel RS-BPM since it is faster. The results of Tables III and IV indicate similar conclusions to those of Table II; however, we have noticed that the values of the mode index and the coupling length produced by the EFD-BPM for structure 2, are larger than the values of the RS-BPM. To test this further, we have decreased Δx to 0·05 for the EFD-BPM and calculated the new values of N_{eff} and L_c which are 3·395641865 and 0·8299 mm respectively. These are in good agreement with the values of the RS-BPM.

To validate the results of the parallel explicit methods, Table V contains published results of References 1 and 14 of the same structures for comparisons. It can be observed that the results of the EFD-BPM and the RS-BPM are as accurate as the results of Table V and very close to those in Reference 14. It has been pointed out by Feit and Fleck,[1] and can be seen from Table V, that the coupling lengths of the FFT-BPM are always shorter than any other method. In addition to the discussion given in Reference 1, we suggest that Δz should be decreased further in order to arrive at better results.

5. CONCLUSION

It has been demonstrated in this work that implementing the finite-difference explicit versions of the BPM on a supercomputer results in a large speed-up of the execution of these algorithms in comparison to the serial execution. These methods are very well suited to the parallel environment because they inherit the locality of spatial points, which reduces the communication overhead between parallel processors. On the other hand, the computation of the field at any given spatial point using the ADI-BPM requires information from all parts of the problem, which is very expensive in terms of parallel computing. The implementations of the EFD-BPM and the RS-BPM on the transputer array showed that around 90–100 per cent in the efficiency gain could

Table II. The fundamental mode indices and the coupling lengths of structure 1, computed using the parallel EFD-BPM and RS-BPM

Δx (μm)	Δy (μm)	Δz (μm)	N_{eff} EFD-BPM	N_{eff} RS-BPM	L_c (mm) EFD-BPM	L_c (mm) RS-BPM
0·2	0·1	0·25	—	3·390792451	—	285·45
		0·125	—	3·392857134	—	300·27
		0·0625	—	3·393297053	—	303·45
		0·035	3·393440335	—	304·52	—
		0·03125	—	3·393403764	—	304·28
		0·015625	—	3·393430250	—	304·52
0·1	0·1	0·25	—	3·388004414	—	$n_e < n_o$
		0·125	—	3·391420992	—	293·56
		0·0625	—	3·392132577	—	315·68
		0·03125	—	3·392304294	—	321·44
		0·025	3·392362259	—	323·46	—
		0·015625	—	3·392346865	—	322·78
		0·0078125	—	3·392357486	—	323·19
0·1	0·05	0·125	—	3·382249421	—	—
		0·0625	—	3·389903808	—	299·00
		0·03125	—	3·391376826	—	326·32
		0·015625	—	3·391718884	—	333·05
		0·0124	3·391831397	—	335·50	—
		0·0078125	—	3·391802926	—	334·63
0·05	0·05	0·0625	—	3·388289549	—	256·88
		0·03125	—	3·390725915	—	305·00
		0·015625	—	3·391287509	—	331·34
		0·008	3·391472234	—	340·66	—
		0·0078125	—	3·391425235	—	338·28
		0·00390625	—	3·391459065	—	340·21
0·025	0·025	0·03125	—	3·375348807	—	24·08
		0·015625	—	3·388199074	—	214·33
		0·0078125	—	3·390513402	—	308·15
		0·00390625	—	3·391051924	—	335·208
		0·002	3·391228915	—	358·80	—

Table III. The fundamental mode indices and the coupling lengths of structure 2, computed using the parallel EFD-BPM and RS-BPM

Δx (μm)	Δy (μm)	Δz (μm)	N_{eff} EFD-BPM	N_{eff} RS-BPM	L_c (mm) EFD-BPM	L_c (mm) RS-BPM
0·1	0·05	0·125	—	3·385005863	—	0·84325
		0·0625	—	3·393453043	—	0·82987
		0·03125	—	3·395147688	—	0·83523
		0·015625	—	3·395536853	—	0·83702
		0·0124	3·395678682	—	0·91000	—
		0·0078125	—	3·395632373	—	0·83731

be achieved. On the other hand, the implementations of the same methods on the Connection Machine have produced even faster parallel computer codes compared to the best performance of the transputer array implementations. Comparisons between the two parallel explicit methods have indicated that the EFD-BPM is several times faster than the RS-BPM per propagational step. The accuracy of the two methods is confirmed by analysing three-dimensional rib waveguides and directional couplers, and the results have been compared with other serial techniques. It has been concluded that the EFD-BPM is more efficient than the RS-BPM, since the latter converges

Table IV. The fundamental mode indices and the coupling lengths of structure 3, computed using the parallel EFD-BPM and RS-BPM

Δx (μm)	Δy (μm)	Δz (μm)	N_{eff} EFD-BPM	N_{eff} RS-BPM	L_c (mm) EFD-BPM	L_c (mm) RS-BPM
0·1	0·05	0·125	—	3·4364715439	—	1·40587
		0·0625	—	3·436979326	—	1·81475
		0·03125	—	3·437084489	—	1·24674
		0·012	3·437155943	—	1·25277	—

Table V. The fundamental mode indices and the coupling lengths in References 1 and 14 used for comparisons

	N_{eff}				L_c (mm)			
Structure	FD(1)[14]	FD(2)[14]	WAVE[14]	PBM-FFT[1]	FD(1)[14]	FD(2)[14]	WAVE[14]	PBM-FFT[1]
1	3·390617745	3·391291712	3·390449	3·3913	357	341	$N_e < N_o$	65·1
2	3·39516625	3·395429873	3·394888	3·3960	0·797	0·811	0·827	0·71
3	3·436842513	3·436863500	3·436724	3·4365	1·273	1·347	1·968	0·93

at a similar step size to that of the EFD-BPM. Finally, the solution of the parabolic equation, discussed in this work, is common for many large major mathematical applications where the same implementations of the parallel explicit techniques could be used to speed up their execution time.

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