EE 340 HW #8 Solution.
3rd Edition

7.21

a) $\vec{B} = \mu_0 \dfrac{I}{2\pi\rho} \vec{a}_\phi$, at $P(-3,4,7)$, we have:

$$\vec{B} = 4\pi 10^{-7} \frac{2}{2\pi\sqrt{3^2+4^2}} \vec{a}_\phi = \frac{8\times 10^{-7}}{10} \vec{a}_\phi = 80\, \vec{a}_\phi \left[\frac{pWb}{m^2}\right]$$

b) $\Psi = \int \vec{B} \cdot \vec{ds}$

$$= \int_{z=0}^{4}\int_{\rho=2}^{6} \frac{\mu_0 I}{2\pi\rho}\, d\rho\, dz = (4)\, \frac{\mu_0 I}{2\pi} \int_{\rho=2}^{6} \frac{d\rho}{\rho}$$

$$= \frac{2\mu_0 I}{\pi} \ln \frac{6}{2} = \frac{2\mu_0 I}{\pi} \ln 3 = 1.76\ [\mu Wb]$$

7.26

a) Find $\nabla \times \vec{A}$ and $\nabla \cdot \vec{A}$, then make a decision.

$$\nabla \times \vec{A} = \vec{a}_x + e^{-x}\, \vec{a}_y - \cos ax\, \vec{a}_z$$

$$\nabla \cdot \vec{A} = -ay \sin ax$$

$\vec{A}$ cannot represent an electrostatic or magnetostatic fields.

b) $\nabla \times \vec{B} = \vec{0}$

$\nabla \cdot \vec{B} = 0$

$\vec{B}$ can represent both an electrostatic and a magnetostatic field.

c) $\nabla \times \vec{C} = 2r\cos\theta\, \vec{a_r} - 3r\sin\theta\, \vec{a_\theta}$

$\nabla \cdot \vec{C} = 0$

$\vec{C}$ can represent a magneto-static field, but cannot represent an electrostatic field.

<u>7.28</u>

a) $\vec{B} = \nabla \times \vec{A} = \left(-6xz + 4x^2y + 3xz^2\right)\vec{a_x} + \left(y + 6yz - 4xy^2\right)\vec{a_y} + \left(y^2 - z^3 - 2x^2 - z\right)\vec{a_z}$

b) $\psi = \int_{z=0}^{2}\int_{y=0}^{2}\left(-6xz + 4x^2y + 3xz^2\right) dy\, dz \Big|_{x=1} = \int \vec{B}\cdot\vec{ds}$

$= \int_0^2\int_0^2\left(-6z + 4y + 3z^2\right) dy\, dz = 8 \;[Wb]$

or

$\psi = \oint \vec{A}\cdot\vec{dl} = \int_{y=0}^{2}(xy^2 - xz^3)\,dy \Big|_{\substack{x=1\\ z=0}}$

$+ \int_{z=0}^{2} -(6xyz - 2x^2y^2)\,dz \Big|_{\substack{x=1\\ y=2}} + \int_{y=2}^{0}(xy^2 - xz^3)\,dy \Big|_{\substack{x=1\\ z=2}}$

$+ \int_{z=2}^{0} -(6xyz - 2x^2y^2)\,dz \Big|_{\substack{x=1\\ y=0}}$

$= \frac{8}{3} - 8 + \left(16 - \frac{8}{3}\right) + 0 = 8\,\text{Wb}.$

c) $\nabla \cdot \vec{B} = 4xy + 2xy - 6xy = 0$

$\nabla \cdot \vec{A} = -6z + 8xy + 3z^2 + 1 + 6z - 8xy - 3z^2 - 1 = 0$

7.33

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

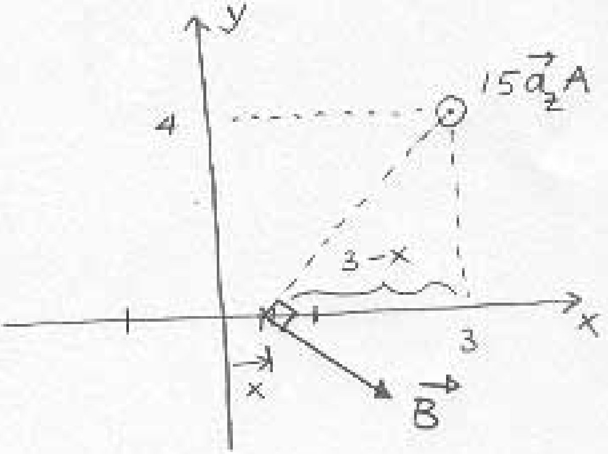
$$\vec{J} = -\frac{1}{\mu_0} \nabla^2 \vec{A} = -\frac{\vec{a}_z}{\mu_0} \nabla^2\left(\frac{10}{\rho^2}\right) \qquad \left[\begin{matrix}\text{since } \vec{a}_z \\ \text{is uniform}\end{matrix}\right].$$

$$= -\frac{\vec{a}_z}{\mu_0} 10 \left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\, \partial\left(\tfrac{1}{\rho^2}\right)/\partial\rho\right)\right]$$

$$= -\frac{10\vec{a}_z}{\mu_0} \frac{1}{\rho} \frac{\partial}{\partial\rho}\left[\rho\left(-2\rho^{-3}\right)\right]$$

$$= \frac{20\vec{a}_z}{\mu_0} \frac{1}{\rho}\left[-2\rho^{-3}\right] = -\frac{40\vec{a}_z}{\mu_0 \rho^4} \quad \left[A/m^2\right]$$

8.7



$$\vec{B} = \frac{\mu_0 I}{2\pi\rho}\vec{a}_\phi$$

$$= \frac{\mu_0 15}{2\pi\sqrt{4^2+(3-x)^2}}\left[\frac{4\vec{a}_x - (3-x)\vec{a}_y}{\sqrt{4^2+(3-x)^2}}\right]$$

$$= \frac{15\mu_0\left[4\vec{a}_x - (3-x)\vec{a}_y\right]}{2\pi\left[4^2+(3-x)^2\right]}$$

$$\vec{F} = I\int d\vec{l} \times \vec{B} = 12\times10^{-3}\int_{-10^{-2}}^{10^{-2}}\left(\vec{a}_x\, dx \times \vec{B}\right)$$

$$= \frac{12\times10^{-3}}{2\pi}\int_{-0.01}^{0.01}\frac{-15\mu_0(3-x)\,\vec{a}_z\,dx}{16+(3-x)^2} = -\frac{15\mu_0\times12\times10^{-3}}{2\pi}\vec{a}_z\int_{-0.01}^{0.01}\frac{(3-x)\,dx}{16+(3-x)^2}$$

$$= \frac{-15\mu_0\times12\times10^{-3}}{2\pi}\vec{a}_z\int_{-0.01}^{0.01}\left(-\frac{1}{2}\right)\frac{d\left[16+(3-x)^2\right]}{\left[16+(3-x)^2\right]}$$

$$= +\frac{180 \times 10^{-3}}{2\pi} \mu_0 \vec{a}_z \times \frac{1}{2} \ln \left| 16 + (3-x)^2 \right| \Big|_{-0.01}^{0.01}$$

$$= +\frac{90 \times 10^{-3}}{2\pi} \mu_0 \vec{a}_z \ln \left(\frac{16 + 2.99^2}{16 + 3.01^2} \right)$$

$$= +\frac{90 \times 10^{-3}}{2\pi} \times 4\pi 10^{-7} \vec{a}_z \ln 0.9952$$

$$= \vec{a}_z \left(\frac{90 \times 10^{-3} \times 4\pi 10^{-7}}{2\pi} \right) \left(-4.8 \times 10^{-3} \right)$$

$$= -86.4 \times 10^{-12} \vec{a}_z = -86.4 \vec{a}_z \ [PN]$$

Alternative method.

$$\vec{F} = I \oint \vec{dl} \times \vec{B}$$

, since the 12mA line is very short compared to the distance to the 15A line, the $\vec{B}$ field along the 12mA line can be assumed uniform.

$$= I \vec{l} \times \vec{B}$$

$\vec{l}$

$$= 12 \times 10^{-3} (0.02 \vec{a}_x)$$

$$\times \boxed{\frac{\mu_0 15}{2\pi \sqrt{25}} \left[\frac{4\vec{a}_x - 3\vec{a}_y}{\sqrt{25}} \right]}$$

$\vec{B}$ at the origin

$$= 12 \times 10^{-3} \times 0.02 \times \frac{15 \mu_0}{2\pi (25)} (-3\vec{a}_z)$$

$$= -86.4 \vec{a}_z \ [PN].$$