5.3

$$I = \int \vec{J} \cdot \vec{ds} = \int_{\rho=0}^{a}\int_{\phi=0}^{2\pi} 10 e^{-(1-\frac{\rho}{a})} \vec{a}_z \cdot (\vec{a}_z \rho \, d\rho \, d\phi)$$

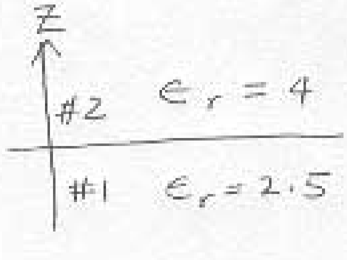
$$= 2\pi \int_{\rho=0}^{a} 10 e^{-(1-\frac{\rho}{a})} \rho \, d\rho$$

$$= 20\pi e^{-1} \int_{\rho=0}^{a} \rho e^{\rho/a} \, d\rho$$ [use integration by parts]

$$= 20\pi e^{-1} \left[\rho (a e^{\rho/a}) \Big|_0^a - \int_0^a a e^{\rho/a} \, d\rho \right]$$

$$= 20\pi e^{-1} \left[a^2 e - \left(a^2 e^{\rho/a} \Big|_0^a \right) \right] = 20\pi e^{-1} \left[a^2 \right]$$

$$= 20\pi a^2 e^{-1}$$



5.27

a) $\vec{E}_{1t} = -30\vec{a}_x + 50\vec{a}_y$

$= \vec{E}_{2t}$

$$\vec{E}_{1n} = 70\vec{a}_z = \frac{\vec{D}_{1n}}{\epsilon_1} = \frac{\vec{D}_{2n}}{\epsilon_1} = \frac{\epsilon_2 \vec{E}_{2n}}{\epsilon_1} = \frac{\epsilon_{2r}\vec{E}_{2n}}{\epsilon_{1r}}$$

$$\therefore \vec{E}_{2n} = 70\vec{a}_z \left(\frac{\epsilon_{1r}}{\epsilon_{2r}} \right) = 70\vec{a}_z \left(\frac{2.5}{4} \right)$$

$$= \frac{175}{4} \vec{a}_z$$

$$\vec{E}_2 = -30\vec{a}_z + 50\vec{a}_y + \frac{175}{4}\vec{a}_z$$

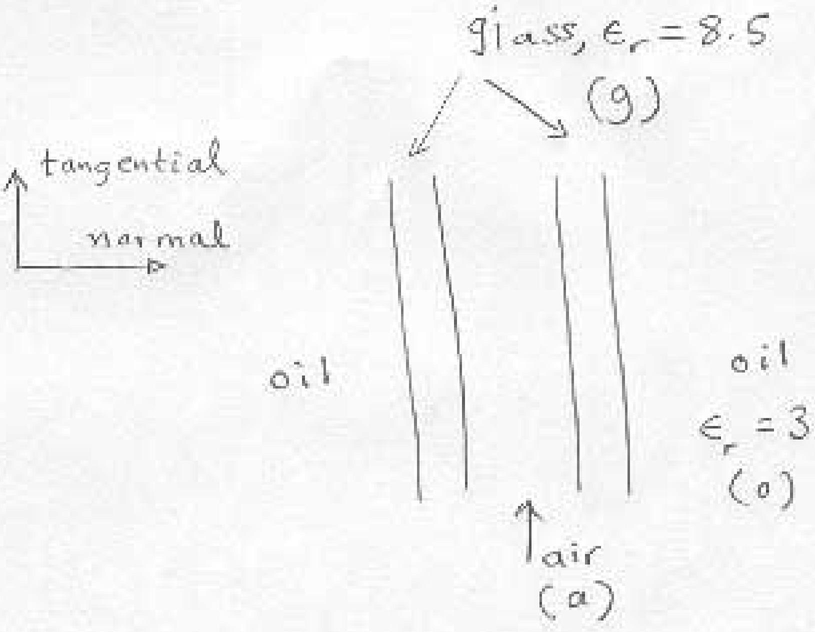
$$\vec{D}_2 = \epsilon_2 \vec{E}_2 = 4\epsilon_0\left(-30\vec{a}_z + 50\vec{a}_y + \frac{175}{4}\vec{a}_z\right)$$

$$= -120\epsilon_0 \vec{a}_z + 200\,\epsilon_0 \vec{a}_y + 175\epsilon_0 \vec{a}_z$$

c) $\tan\alpha = \dfrac{E_{1t}}{E_{1n}} = \sqrt{900+2500}\,/\,70 = 0.833$

$\alpha = 39.79°$

5.31



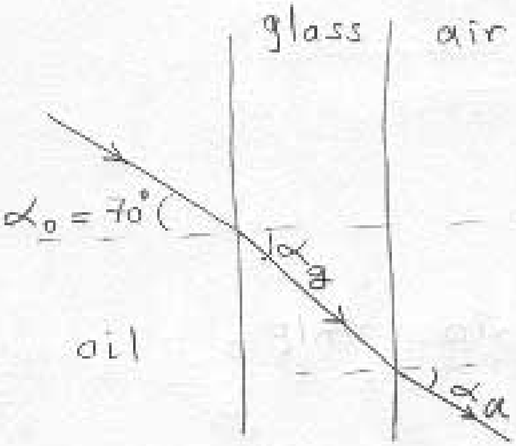
a) $E_{tg} = E_{to} = E_{ta}$

$D_{no} = D_{ng} = D_{na}$

$E_{ng} = \dfrac{3\,E_{no}}{8.5} = \dfrac{3\times 2000}{8.5} = 705.9$ V/m

$E_{na} = \dfrac{3E_{no}}{(1)} = 3\times 2000 = 6000$ V/m

b) $\tan\alpha_o = \dfrac{E_{to}}{E_{no}}$



$\tan 70° = \dfrac{E_{to}}{2000}$

$\therefore E_{to} = 2000(\tan 70°)$

$= 5494.95$ V/m

$\therefore E_{tg} = E_{ta} = E_{to} = 5494.95$

$D_{ng} = D_{na} = D_{no}$

$E_{ng} = \dfrac{3\,E_{no}}{8.5} = 705.9$

$E_{na} = \dfrac{3\,E_{no}}{(1)} = 3 \times 2000 = 6000$

$\therefore$ in glass $\left(E_{tg} = 5494.95,\ E_{ng} = 705.9\right)$

$\therefore \tan\alpha_g = \dfrac{E_{tg}}{E_{ng}} = \dfrac{5494.95}{705.9} = 7.784,\quad \alpha_g = 82.68^\circ$

In air $\left(E_{ta} = 5494.95,\ E_{na} = 6000\right)$

$\therefore \alpha_a = \tan^{-1}\left(\dfrac{5494.95}{6000}\right) = 42.48^\circ$

5.32 a) $D_n = \rho_s = \epsilon_0\left(\sqrt{225+64}\right) = 17\epsilon_0 \ \frac{C}{m^2}$

Actually $\rho_s = \pm 17\epsilon_0$ and the actual sign cannot be determined, because we do not have enough information.

b) $\vec{D} = +\rho_s \vec{a}_y$

$= +20 \times 10^{-9}\, \vec{a}_y$