4.3

(a) $\vec{E} = \dfrac{Q_1 \vec{R_1}}{4\pi\epsilon_0 R_1^3} + \dfrac{Q_2 \vec{R_2}}{4\pi\epsilon_0 R_2^3}$

$$= \frac{Q_1(\vec{a}_x + 9\vec{a}_z)}{4\pi\epsilon_0(\sqrt{82})^3} + \frac{+4\times10^{-9}(3\vec{a}_x + 5\vec{a}_z)}{4\pi\epsilon_0(\sqrt{34})^3}$$

$$\therefore \frac{9Q_1}{(\sqrt{82})^3} + \frac{4\times10^{-9}\times5}{(\sqrt{34})^3} = 0$$

$$\therefore Q_1 = -\frac{(\sqrt{82})^3\times20\times10^{-9}}{9(\sqrt{34})^3} = -8.32\text{ nC}$$

b) $$\frac{Q_1}{(\sqrt{82})^3} + \frac{4\times10^{-9}\times3}{(\sqrt{34})^3} = 0$$

$$\therefore Q_1 = -\frac{(\sqrt{82})^3}{(\sqrt{34})^3}\times12\times10^{-9} = -44.95\text{ nC}$$

4.10 (a) Using $\vec{E}(0,0,h) = \dfrac{\rho_L a h}{2\epsilon_0[h^2+a^2]^{3/2}}\vec{a}_z$

(See example 4.4, pg. 117).

For this problem $\vec{E}(h,0,0) = \dfrac{\rho_L a h}{2\epsilon_0[h^2+a^2]^{3/2}}\vec{a}_x$

$$\vec{D}(h,0,0) = \frac{\rho_L a h}{2[h^2+a^2]^{3/2}}\vec{a}_x$$

$$\vec{D}(3,0,0) = \frac{(5\times10^{-6})(2)(3)}{2[3^2+2^2]^{3/2}}\vec{a}_x = 320\times10^{-9}\,\vec{a}_x \;\frac{C}{m^2}$$

$$= \vec{D_1}$$

b) $\vec{D}_2 = \vec{D}\Big|_{\text{due to the two charges only}}$

$$= \frac{Q\,(3\vec{a}_x + 3\vec{a}_y)}{4\pi\,(\sqrt{18})^3} + \frac{Q\,(3\vec{a}_x - 3\vec{a}_y)}{4\pi\,(\sqrt{18})^3}$$

$$\vec{D} = \vec{D}_1 + \vec{D}_2 = 320\times 10^{-9}\,\vec{a}_x + \frac{6Q\,\vec{a}_x}{4\pi(\sqrt{18})^3} = \vec{0}$$

$$\frac{6Q}{4\pi(\sqrt{18})^3} = -320\times 10^{-9}$$

$$\therefore\; Q = -\frac{320\times 4\pi\,(\sqrt{18})^3\times 10^{-9}}{6} = -51.18\,\mu C$$

4.12 $\quad \vec{E} = \vec{E}_Q + \vec{E}_\ell + \vec{E}_s$

$$\vec{E}_Q = \frac{100\times 10^{-12}\,(-3\vec{a}_x + 4\vec{a}_z)}{4\pi\epsilon_0\,(5)^3} = -21.6\times 10^{-3}\,\vec{a}_x + 28.8\times 10^{-3}\,\vec{a}_z$$

$$\vec{E}_\ell = \frac{2\times 10^{-9}}{2\pi\left(\frac{10^{-9}}{36\pi}\right)\sqrt{2}}\left(\frac{\vec{a}_y}{\sqrt{2}} + \frac{\vec{a}_z}{\sqrt{2}}\right)$$

$$= 18\,\vec{a}_y + 18\,\vec{a}_z$$

{Using $\vec{E} = \dfrac{\rho_\ell}{2\pi\epsilon_0\rho}\,\vec{a}_\rho$ adopted for this case}.

$$\vec{E}_s = -\vec{a}_z\,\frac{5\times 10^{-9}}{2\times\frac{10^{-9}}{36\pi}}$$

{Using $\vec{E} = \dfrac{\rho_s}{2\epsilon_0}\,\vec{a}_n$} (See eqn. 4.26)

$$= -90\pi\,\vec{a}_z = -282.74\,\vec{a}_z$$

$$\vec{E} = -0.022\,\vec{a}_x + 18\,\vec{a}_y - 264.71\,\vec{a}_z \qquad [\text{V/m}]$$

4.17

a) $\vec{D} = \epsilon_0 \vec{E} = \epsilon_0 (xy\,\vec{a}_x + x^2 \vec{a}_y)$

b) $\rho_v = \nabla \vec{D} = \epsilon_0 y$

4.23

$$\oint \vec{D} \cdot \vec{ds} = Q$$

for $\rho < 1 \Rightarrow \oint \vec{D} \cdot \vec{ds} = 0$

$$D\, 2\pi \rho L = 0$$

$$D = 0$$

$$\vec{D} = \vec{0} \text{ , } (\rho < 1)$$

for $1 < \rho < 2 \implies \oint \vec{D} \cdot \vec{ds} = Q$

$$D\, 2\pi \rho L = \int_{z=0}^{L} \int_{\phi=0}^{2\pi} \int_{\rho=1}^{\rho} (12\rho \times 10^{-9})\, \rho\, d\rho\, d\phi\, dz$$

$$D\, 2\pi \rho L = 12 \times 10^{-9} \times 2\pi \times L \times \left. \frac{\rho^3}{3} \right|_1^{\rho}$$

$$= 18 \pi L \times 10^{-9} \left[\rho^3 - 1 \right]$$

$$\vec{D} = \frac{4 \times 10^{-9} (\rho^3 - 1)}{\rho} \vec{a}_\rho \text{ , } (1 < \rho < 2)$$

for $\rho > 2 \implies D\, 2\pi \rho L = 8 \pi L \times 10^{-9} \left[2^3 - 1 \right]$

$$\vec{D} = \frac{28 \times 10^{-9}}{\rho} \vec{a}_\rho \text{ , } (\rho > 2)$$

4.24

a) $r = 2m$

$$\Psi = \oint \vec{D} \cdot \vec{ds} = Q$$

$$\Psi = \int_{r=1}^{2}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \left(\frac{10}{r^2} \times 10^{-3}\right) r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= 10^{-2}(2\pi)(1)(2) = 4\pi/100 = 0.04\pi \; [C]$$

$r = 6m$

$$\Psi = \oint \vec{D} \cdot \vec{ds} = Q = \int_{r=1}^{4}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \left(\frac{10 \times 10^{-3}}{r^2}\right) r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= 10^{-2}(2\pi)(3)(2) = 0.12\pi \; [C]$$

b) $r = 1 \, m$

$$\oint \vec{D} \cdot \vec{ds} = 0$$

$$D \, 4\pi r^2 \Big|_{r=1} = 0$$

$$D \, 4\pi = 0$$

$$\therefore \vec{D} = \vec{0} \quad [C/m^2]$$

$r = 5 \, m$

$$\oint \vec{D} \cdot \vec{ds} = \int_{r=1}^{4}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \left(\frac{10 \times 10^{-3}}{r^2}\right) r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$D \, 4\pi (5)^2 = 0.12\pi$$

$$\vec{D} = 0.0012 \, \vec{a_r} \quad [C/m^2]$$