Deterministic Co-Existence Dynamic Channel Assignment Algorithms

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***Abstract*: The Co-Existence Dynamic Channel Assignment (CE-DCA) algorithms presented in [1] facilitate the coexistence of embedded autonomous underlay cellular systems. A key component of these algorithms is intelligent channel exclusion method. In addition to statistic method, a deterministic one was presented therein, whose performance is inferior to the former. This paper presents a collection of deterministic CE-DCA algorithms with improved performance. Among these are schemes that approach, or even exceed, the performance bound envisioned by that of the baseline DCA scheme, namely LP-DCA without channel exclusion.**

1. INTRODUCTION

The advantage of Dynamic Channel Assignment (DCA) lies in that every cell is free to choose any channel from the *universal set* of channels available to the network, the only constraint is being imposed through the interference from the cells within the frequency reuse distance. This capability provides capacity gain in addition to alleviating radio frequency planning. However, it nearly prevents any embedded autonomous microcellular system from finding available channels in real time. To accommodate their coexistence, we propose the *exclusion* of a subset of channels in each macrocell of the overlay system from the universal set of channels. Such exclusion should cause minimal performance degradation in the overlay system compared to conventional DCAs (without exclusion).

In [1], we presented the idea of Co-Existence DCA (CE-DCA) algorithms resulting from the exclusion as mentioned above. Formal requirements were defined to meet the needs of both systems. Statistic and deterministic methods were presented therein. The statistic method was shown to fulfill the requirements in the statistical sense whereas a deterministic scheme, namely, 6-Min, was derived to satisfy such requirements in the strict sense. However the advantage of statistic method is the simplicity and the possibility to be implemented autonomously by each macrocell. Deterministic schemes on the other hand should achieve superior performance with added complexity for coordination among the cells. Such coordination could be either distributed or centralized. While such expectation was not met by 6-Min, our continued investigations resulted in highly efficient deterministic schemes far superior in performance to that presented in [1]. We review the parameters governing the performance of CE-DCA algorithms and investigate their optimal ranges in section 2. The construction of a number of deterministic algorithms approaching the optimal performance, along with their reasons for conception, are presented in section 3. Their relative performances are compared in section 4.

2. CE-DCA AS AN OPTIMIZATION PROBLEM

We start by defining the design of CE-DCA as "the search for channel exclusion patterns to **minimize capacity loss in the overlay macrocellular system while maximizing the quantity of channels acquired by the underlay microcellular system**". Such an exclusion pattern should provide sufficient amount of channels to an underlay microcellular system wherever they are located, within the overlay macrocellular system. In other words, the set of channels available to each underlay microcellular system are the ones never utilized by its nearby overlay macrocells which may otherwise cause mutual interference.

The above objective can be achieved by excluding channel sets in those macrocells in the neighborhood. The intersection of those exclusion sets ought to be large enough to provide the required amount of channels to the underlay microcellular environment. The discussion from this point onward assumes cells with omni directional antenna placed at the centers of the cells. Let E_i be the set of channels excluded in macrocell i when executing DCA, $N_{\min}$ be the minimum number of channels required for an indoor mobile environment, then the requirement is,

$$\left| \bigcap_{i \in P} E_i \right| \geq N_{\min} \qquad (1)$$

where P is the set of cells that can cause mutual interference to the underlay microcellular system. The same logic and formulation extend straightforwardly to a complete macrocellular system which may accommodate multiple independent underlay microcellular systems.

In addition to the requirement expressed in equation (1), we introduce further criteria that refine our strategies of the channel exclusions. First, it is intuitively obvious that one should minimize the size of exclusion set per cell, $|E_i|$. Second, the sizes of intersections of exclusion channel sets, $\left| \bigcap_{i \in C} E_i \right|$ for $|C| \leq |P|$, also plays an important role in that the smaller they can be kept, the higher capacity macrocells will retain. Moreover, let us recall one of the fundamental differences between DCA and FCA concerning channel usage. With FCA, the minimal set of cells, Ω, that can utilize all the channels in U form a frequency reuse cluster; whereas with DCA every cell is allowed to use the entire U. We call Ω the *universal cluster*, thus $|\Omega|$ is typically 7 in FCA and 1 in DCA. The "universal cluster" in the context of CE-DCA, is the "minimal set of clustered cells with no common exclusion channels", *i.e.*,

$$\bigcap_{i \in \Omega} E_i = \phi \qquad (2)$$

where ϕ is the null set. The requirement posed by equation (1) dictates that $|\Omega| > |P|$. It is obvious $|\Omega|$ should be kept as small as possible and definitely smaller than that of FCA.

Having identified the above parameters, namely, $N_{\min}$, $|E_i|$, $\left|\bigcap_{i \in C} E_i\right|$ for $|C| \leq |P|$, and $|\Omega|$, that govern the performance of CE-DCA algorithms, it is quite intuitive to see an increase in the above parameter values would have a negative effect on overlay macrocellular systems. However, for a given $N_{\min}$, both $|E_i|$ and $\left|\bigcap_{i \in C} E_i\right|$ for $|C| \leq |P|$ should be sufficiently large to satisfy the requirements of an underlay microcellular system. On the other hand, $|\Omega|$, whose minimum value is $|P|+1$, is really independent of $N_{\min}$. Finally, the channel exclusion patterns that minimize the above parameter values while satisfying the requirement in equation (1) may result into different $\left|\bigcup_i E_i\right|$, called the *channel span* requirement. By definition, $\left|\bigcup_i E_i\right|$ must not exceed $|U|$. Thus, channel span is a factor governing the maximum achievable $N_{\min}$, for a given $|U|$.

Our initial formulation of the problem is based on the parameters as identified above. As will be seen from the simulation study in the following section, there are additional factors governing the performance of CE-CDA algorithms. In particular, there are two algorithms, characterized by exactly the same values of the above parameters, providing drastically different performance. Our continued investigations reveal that they differ in a new factor, called the *co-channel-exclusion* cell layout, defined as "the pattern a particular channel being excluded in the cell layout". Co-channel-exclusion cells are the set of cells with identical E_i. It appears that the more compact this layout is the better the CE-DCA performance.

Without loss of generality, our studies are based on a regular hexagonal cellular topology with 2-cell buffering. In the case of FCA, this layout and constraint correspond to a conventional 7 cell frequency reuse cluster. As illustrated in Figure1, a microcellular system may be located at three types of locations, labeled as a, b, and c. The number of channels available to the microcells with each CE-DCA depends on the type of location they are, while a minimum of $N_{\min}$ channels must be guaranteed by the CE-DCA in the worst case location (*i.e.*, at c). Thus we focus on $|P|=3$ in this paper and extensions to layout with different $|P|$ can be made similarly.

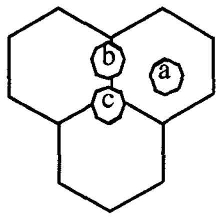

Figure1 Three types of locations (a,b, and c) where microcellular environments may exist under a hexagonal macrocellular overlay layout

3. EXCLUSION SCHEMES FOR CE-DCA

Let us begin by examining the range of parameter $|\Omega|$ in which tradeoffs can be made. At one extreme, if $|E_i| = N_{\min}$, every cell would exclude the same set of channels with a size of $N_{\min}$. We call this the "common exclusion" in which case $|\Omega| = \infty$. On the other extreme, we should be able to design exclusion patterns where every cluster of 3 cells has a common exclusion set of size $N_{\min}$ while each group of 4 or more cells are able to utilize the universal set of channels. In this case $|\Omega| = 4$ which is the minimum value attainable following the basic requirements.

Exclusion scheme	*Optimum/efficient parameters*
Common	$|E_i|$
2-Min	$|E_i|, \left|\bigcap_{i \in C} E_i\right|$ for $C \leq |\Omega|$
6-Min	$|\Omega|$
3-Min	$\left|\bigcup_i E_i\right|, |\Omega|, |E_i|$
FPP	$\left|\bigcup_i E_i\right|, |\Omega|,$ *cochannel exclusion cell layout*
Inverse-FCA	$|\Omega|, |E_i|, \left|\bigcup_i E_i\right|,$ *cochannel exclusion cell layout*

Table1 Summary of deterministic schemes with the parameters being optimized or enhanced.

The exclusion pattern providing globally optimum performance is the target of an ongoing effort. In this subsection, we present six deterministic CE-DCAs which attempt to optimize some of the parameters identified in the previous section: *Common, 2-Min, 6-Min, 3-Min, FPP and Inverse-FCA Exclusion* CE-DCAs. While the Inverse-FCA exclusion scheme does not satisfy the basic requirement in (1), it is presented as a reference case. Table1 summarizes the parameters being optimized or enhanced in each of the algorithm. Each algorithm along with its design motivation is presented in the following.

CE-DCA with Common Exclusion:

By definition, equation (1) dictates that $|E_i|$ be at least $N_{\min}$. The simplest method is to exclude a unique set with $N_{\min}$ channels from all the macrocells so that the set is available to serve the indoor systems at any location. However, as we have discussed before, $|\Omega| = \infty$ in this case, thus it's spectral utilization is inefficient. The CE-DCA performance in the outdoor system is simply that of the baseline conventional DCA algorithm with the truncated U. This capacity loss may not be acceptable by the outdoor service provider especially if some of the indoor systems are operated by independent parties.

CE-DCA with 2-Min Exclusion:

With the minimum possible value of $|E_i|$, *i.e.*, $N_{\min}$, common exclusion is inefficient with regard to all other key parameters. It leads us to conjecture that the optimum strategy for CE-DCA would be one with $|E_i|=kN_{\min}$, where $k > 1$. We find that the next smallest integer value of *k, i.e.*, 2, ensures a proper tessellation of exclusion sets among macrocells while satisfying equation (1) and results in better $|\Omega|$. As illustrated in Figure 2, this algorithm requires three mutually exclusive subsets say a, b, and c of size $N_{\min}$. Each macrocell is to be excluded from using two of the three subsets of channels. With this exclusion pattern, every cluster of 6 cells such as 'A' satisfy equation (2). Thus, $|\Omega| = 6$ which is close to the maximum value, 7 we would consider, since $|\Omega| = 7$ in conventional FCA. Moreover, clusters of 4 cells with certain orientation also satisfy equation (2). This CE-DCA leads to an excellent performance as shown later.

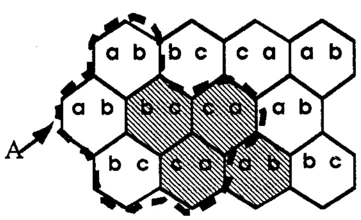

Figure 2 Exclusion set assignment for 2-Min.

CE-DCA with 6-Min Exclusion:

The 2-Min Exclusion scheme provides us with a great value for $|E_i|$, but not $|\Omega|$. To optimize $|\Omega|$, we design an exclusion pattern where every cluster of 3 cells has a common exclusion set of size $N_{\min}$, while each group of 4 or more cells are able to utilize the universal set of channels; thus $|\Omega| = 4$. The size of exclusion set per cell, $|E_i|$ for this case is found to be $6N_{\min}$. As shown in Figure 3, for every three mutually adjacent cells to have a common exclusion set c_i of size $N_{\min}$, the three cells should all exclude c_i; whereas for each 4-cell cluster to have no common exclusion set, each corner of a hexagon has to be assigned a different common exclusion set c_i, $i=1,\ldots,6$, and the cell with those six corners has to exclude the union of these six sets. Note that six sets c_i associated with each macrocell must be mutually exclusive or else it will result in $|\Omega| > 4$. We name this strategy the "6-Min" exclusion.

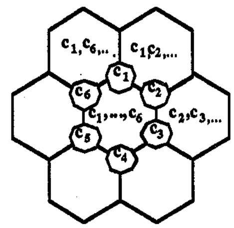

Figure 3 Exclusion with minimum $|\Omega|$.

In Figure 4, we illustrate an exclusion pattern satisfying these conditions. To maintain the minimum value of $|\Omega|$, it requires 18 mutually exclusive channel sets of size $N_{\min}$ within U. Hence this 6-Min exclusion requires the size of U large enough to satisfy the condition that $|U| \geq 18N_{\min}$.

In comparison to the 2-Min Exclusion which optimizes $|E_i|$ but sacrifices $|\Omega|$, the 6-Min Exclusion seems to optimize $|\Omega|$ at the expenses of $|E_i|$. It will be interesting to compare their performances in our latter sections.

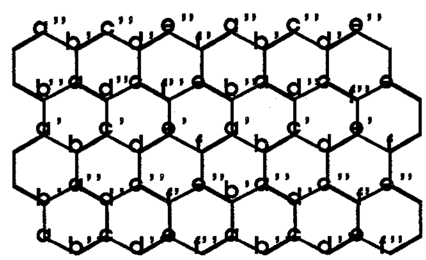

Figure 4 *6-Min* exclusion pattern achieved with 18 channel sets {a,...,f,a',...,f',a'',...,f''} of size $N_{\min}$. An exclusion channel assigned to a corner is to be excluded from all three cells sharing the corner.

CE-DCA with 3-Min Exclusion:

In the quest of reducing the sizes of $|E_i|$ and $\left|\bigcup_i E_i\right|$ as required by the 6-Min Exclusion, we further designed an exclusion pattern that excludes just 3 $N_{\min}$ channels per cell, *i.e.*, $|E_i| = 3\,N_{\min}$, and requires only 4 mutually exclusive channel sets of size $N_{\min}$ within U, *i.e.*, $\left|\bigcup_i E_i\right| = 4$ as shown in Figure 5. We call this pattern "3-Min" Exclusion. This design can accommodate a much larger value of $N_{\min}$ for a given universal channel set size, compared to the 6-Min Exclusion; specifically, it only requires $|U| > 4\ N_{\min}$. However, although all the clusters of 4 cells and 5 cells with certain shapes and orientations are allowed access to all the channels in U, not every clusters of 5 cells has that property (such as the shaded one). Any cluster of 6 cells such as A is able to use U. Thus $|\Omega| = 6$, just like in 2-Min Exclusion. With greater $|E_i|$ than the latter, we expect this scheme to be outperformed by the 2-Min Exclusion.

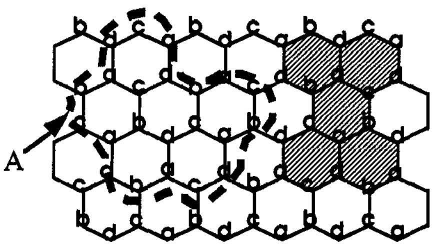

Figure 5 3-Min exclusion pattern achieved with 4 channel sets {a, b, c, d} of size $N_{\min}$.

Finite Projective Plane (FPP) Exclusion:

Among the exclusion schemes presented above, 6-Min is the only one that minimizes $|\Omega|$. Nevertheless, its performance lags behind other schemes as will be seen in the next section. An additional drawback of 6-Min is that it requires a large span of the channel set. In search of techniques to improve the performance and reduce this span, we come up with a Finite Projective Plane (FPP) based exclusion scheme which not only cuts down the span from $18N_{\min}$ to $14\ N_{\min}$ but also improves the performance significantly. The FPP Exclusion is similar to 6-Min in that $|E_i| = 6\ N_{\min}$, yet it is very different in its selection of E_i. The method of exclusion set selection in this scheme stems from the *theory of finite projective planes* [10,11]. The definition of FPP is as follows.

Definition: A FPP of order-q consists of q^2+q+1 points and that many lines. Any line is incident with $q+1$ points and any point is incident with $q+1$ lines.

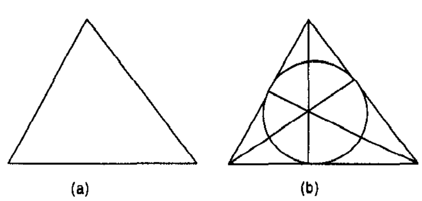

Figure 6 Examples of finite projective planes (a) q=1 (b) q=2

The simplest example of FPP is the FPP of order 1, which is represented by the 3 lines and 3 points of a triangle. Figure 6 shows examples of FPP of orders 1 and 2. The 7 lines and 7 points in the FPP of order 2 includes a circle that represent the 7[th] line. FPP has been used to design new FCA schemes with improved frequency reuse efficiency [12]. We apply it to CE-DCA which leads to an efficient co-exclusion channel cell layout.

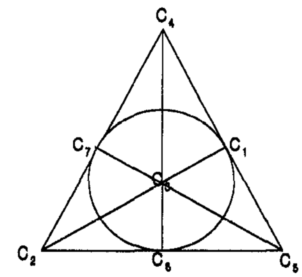

Figure 7 Mapping of exclusion channel sets to FPP of order 2

The exclusion problem with optimal $|\Omega|$ (=4) can be mapped onto a FPP of order 2 in the following manner: Consider 7 distinct sets of channels $\{c_1, ..., c_7\}$ and map them onto 7 points in the FPP, as illustrated in

Figure 7. Form 7 subsets (lines) of 3 channel sets such that any channel set (point), c_i, i=1,...,7 is in 3 of the 7 subsets (lines). These 7 subsets can be assigned to a cluster of 7 cells such that alternative triplets of cells get a commonly excluded channel set, i.e., c_1, c_6, and c_7, as shown in Figure 8. The pattern is repeated with a 7 cell re-exclusion distance. Note that only three triplet will achieve common exclusion due to the fact that all lines of the FPP of order 2 pass through no more than three points. Correspondingly there are only three exclusion channel sets assigned to any cell. However, the remaining triplets of cells can be provided with a commonly excluded channel set by repeating the process with another set of 7 channel sets. The exclusion pattern following this approach is shown in Figure 9.

To our amazement, this scheme is found to have **excellent traffic performance** on overlay macrocellular system compared to 6-Min. It is seen these two methods differ in their co-exclusion channel layout (Figure 10). The kind of co-exclusion channel cell layout of FPP exclusion seem to result in a better channel dynamics leading to more efficient channel reuse distances.

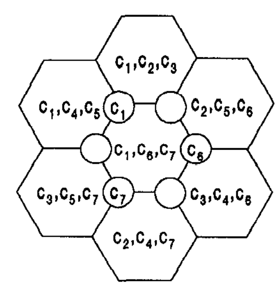

Figure 8 Exclusion of 7 set of channels, {c1 ,..., c7} in a cluster of 7 cells to provide one common channel set exclusion for alternative triplets of cells. The procedure is to be repeated with a different set of 7 channel sets to provide common exclusions for the remaining triplets.

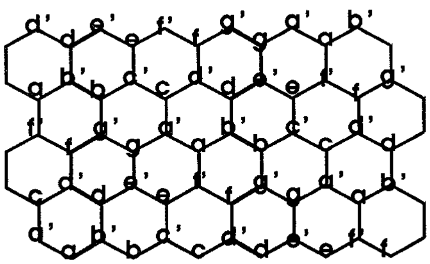

Figure 9 FPP exclusion achieved with 14 channel sets {a ,..., g, a',..., g'} of size N_{min}.

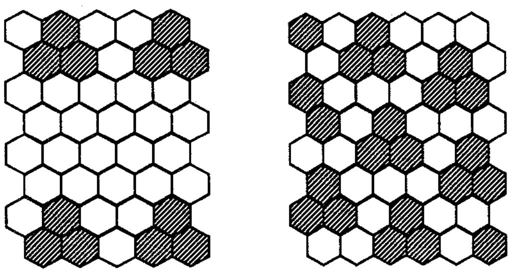

Figure 10 Co-exclusion channel cell pattern (shaded) in 6-Min (left) and FPP (right) exclusion schemes.

Random exclusion:

Random exclusion as presented in [1] preserves one of the advantages that conventional DCA s offer, namely, no frequency planning in any form. We randomly pick an exclusion set of size $|E_i|$ for each of the cells. Also, this scheme according to our measures, outperform some of the deterministic schemes. Nevertheless, the later developments of deterministic schemes presented in this paper outperform the random exclusion schemes drastically.

4. PERFORMANCE

The relative performance of different CE-DCA algorithms have to be evaluated with two aspects - the performance in indoor microcellular systems, measured by the amount of spectrum made available to them by each scheme, and that in outdoor macrocellular system, measured by the capacity change due to the exclusion. The overall performance is determined by the combination of the two aspects. Following subsections present the assessment of the two different aspects of performance as well as the combined effect.

4.1 Performance on Indoor Microcellular systems

Except in the case of "random" exclusion, the parameters such as N_{min} and $|\Omega|$ are either predefined or can be computed deterministically. For random exclusion, the $|E_i|$ is predefined but the resulting N_{min} and $|\Omega|$ would have random sizes with certain statistics. The distributions of exclusion set sizes, for random exclusion, were derived in [1]. Exclusion set sizes with deterministic schemes are directly proportional to $|E_i|$. The exclusion set sizes of deterministic schemes are summarized in Table 2.

SCHEME	AVERAGE COMMON EXCLUSION SET SIZE FOR DIFFERENT CLUSTER SIZES, C AS MULTIPLES OF $\|E_i\|$				
	$C=2$	$C=3$	$C=4$	$C=5$	$C\geq 6$
Common	1	1	1	1	1
Inverse-FCA	0	0	0	0	0
FPP	1/3	1/6	0	0	0
3-Min	2/3	1/3	0	3/15	0
2-Min	2/3	1/2	1/3	1/15	0
6-Min	1/3	1/6	0	0	0

Table 2 Common exclusion set sizes for deterministic exclusion schemes

Note that the number of common exclusions (or the expected value) per single, pair, and tiplet of cells are different. Hence an indoor microcellular system can scavenge different amount of spectrum depending on its position within the macrocellular environment as discussed in [1].

4.2 Performance on outdoor macrocellular system

The throughput performance of CE-DCA algorithm for the outdoor macrocellular system suffer certain amount of degradation compared to pure DCA in some of the exclusion schemes. But two of the deterministic algorithms namely, FPP and Inverse FCA show performance superior to that of baseline DCA namely, LP-DCA. The performance with $|U|=56/|E_i|=18$ and $|U|=420/|E_i|=60$ are shown in Figure 11. Note that for "inverse FCA" exclusion the maximum of $|E_i|$ with $|U|=56$ is 8 and hence is not included in the set of plots. Nevertheless it is found that the inverse FCA *performance* exceeds that of pure LP-DCA as is shown in the plot on right. FPP exclusion is found to give performance close to that of pure LP-DCA for smaller U and is superior to the latter with larger U. These results indicate the possibility to use channel exclusion as a strategy to improve performance of distributed DCA algorithms such as LP-DCA on macrocellular systems.

4.3 Overall performance

The overall performance of CE-DCA algorithms is represented by the joint measure of nominal carried traffic for the macrocells at a blocking of 1% and the expected number of available channels per microcellular system. Derived from the results in 4.1 and 4.2 with various $|E_i|$, Figure 12 illustrates the relative performances of CE-DCA algorithms with representative ratios of micro-to-macro cells radii, a. Since the spectrum available to the microcellular system under random exclusion is a random variable, we include its $\pm3\sigma$ bars in the plots to indicate the uncertainty ranges.

The value of a in practice are in the range of [0.1 - 0.3]. Within this range of a and for smaller values, common exclusion performs worse compared to other CE-DCAs. 3-Min becomes worst around a=0.3. The FPP and Inverse FCA schemes stand out in there performance. However, Inverse FCA exclusion does not provide channels for microcellular systems popping up on boundaries. Hence FPP is deemed to be superior to any other schemes presented herein. Random exclusion was found in [1] to render best compromise between capacity and simplicity. The randomness nature bodes well with no frequency planning. Here we see all of the new deterministic algorithms except 3-Min provide better performance than Random CE-DCA. 6-Min performs considerably worse compared to other CE-DCAs. 3-Min becomes worst around a=0.3. 2-Min starts to outperform Random around there. The FPP and Inverse FCA schemes stand out in there performance. However, Inverse FCA exclusion does not provide channels for microcellular systems popping up on boundaries. Hence we consider FPP to be superior to any other schemes presented herein.

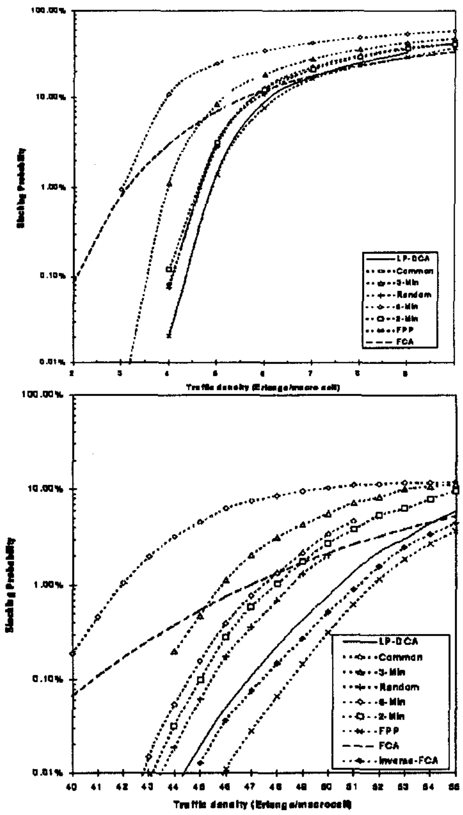

Figure 11 Performance of CE-DCA algorithms with different settings: $|U|=56/|E_i|=18$ (left) and $|U|=420/|E_i|=60$(right).

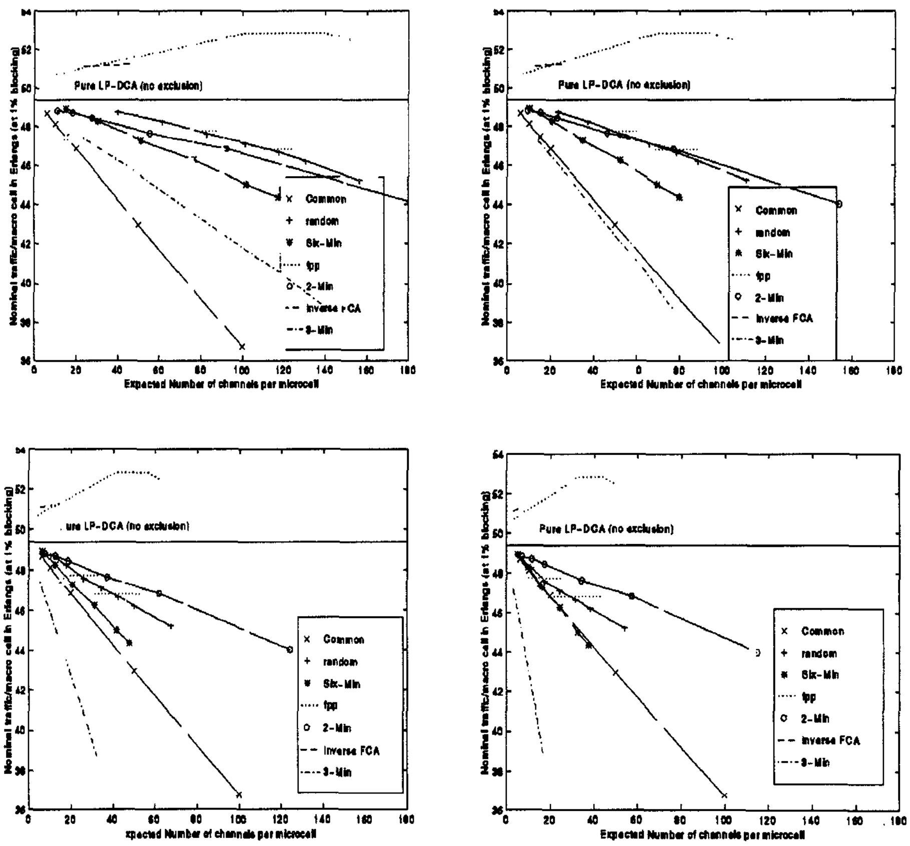

Figure 12 Nominal traffic (at a blocking of 1%) in macrocell versus expected number of channels per microcell at $|U|$=420. From left to right and top to bottom the performances are for a=0.1, 0.3, 0.5, and 0.6 respectively. The horizontal bars indicate $\pm3\sigma$ range.

5. CONCLUSION

Many microcellular systems, mostly indoor, operate within the same spectrum of outdoor macrocellular network *autonomously.* Their operations are based on the assumption that there are relatively stationary channel sets never used by the local macrocells. This assumption holds true when the macrocellular system operates under FCA. However, with the realization of the DCA advantages such as its capacity gain and ease of frequency planning, the macrocellular systems are turning away from FCA to DCA, and the available stationary channel sets at the locality of autonomous indoor systems will no longer exist.

Co-Existence DCA (CE-DCA) was proposed in [1] to satisfy the conflicting needs of both systems. While random CE-DCA preserves the advantage of not requiring global frequency planning, to exploit the potential advantages of more coordinated exclusions, we presented a number of novel deterministic solutions in this paper and evaluated their performance in terms of the joint macro/micro cells capacity. Key factors governing the relative performance of CE-DCA algorithms in macrocellular systems are: the size of the exclusion set per macrocell and the size of the universal cluster. The former is largely dictated by the minimally required number of available channels in the microcells. The latter, on the other hand, varies with the type of the exclusion algorithms. The quantity of spectrum acquired by a microcellular system is dictated by the type of algorithm as well as the relative position of the microcellular system within the macrocellular environment. We quantified the performance of CE-DCAs in microcells by the expected number of channels available to a typical microcellular system.

Among the new deterministic algorithms presented in this paper, CE-DCA with 3-Min exclusion results into the worst performance, except when the microcells are very small. If the microcellular radius is greater than 30% of that of the macrocells, CE-DCA with 2-Min exclusion outperforms random CE-DCA as well as all but FPP and Inverse FCA.CE-DCA with Inverse-FCA exclusion provides the smallest universal cluster size and pretty good performance, with the caveat that its expected number of channels available in microcells is very limited. Overall, FPP appears to be the best deterministic CE-DCA algorithm. It seems to provide microcells, channel availability together with a capacity "increase" for the macrocells. This interesting phenomenon is currently under further investigation.

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