Problem 7.17

a) $\vec{B} = \mu_0 \dfrac{I}{2\pi\rho}\vec{a}_\phi$

$\rho \;\Rightarrow\; \sqrt{(-3)^2+(4)^2} = 5 \quad \& \quad \vec{a}_\phi \;\Rightarrow\; \dfrac{-4\vec{a}_x - 3\vec{a}_y}{\sqrt{(-4)^2+(-3)^2}} = \dfrac{-4\vec{a}_x - 3\vec{a}_y}{5}$ (unit vector in the direction of $\vec{B}$)

$$\vec{B} = 4\pi \times 10^{-7}\frac{2}{2\pi\times 5}\times\frac{-4\vec{a}_x - 3\vec{a}_y}{5} \quad \Rightarrow \quad \vec{B} = -64\times 10^{-9}\vec{a}_x - 48\times 10^{-9}\vec{a}_y \quad Wb/m^2$$

b) $\psi_m = \int_S \vec{B}.\vec{ds}$ using $\vec{ds} = d\rho dz\vec{a}_\phi$ which is normal to the square area.

$$\psi = \int_S \vec{B}.\vec{ds} \quad \Rightarrow \quad \int_{z=0}^{4}\int_{\rho=2}^{6}\left(\mu_0\frac{2}{2\pi\rho}\vec{a}_\phi\right).(d\rho dz\vec{a}_\phi)\Big|_{\phi=\pi/2} \quad \Rightarrow \quad \int_{z=0}^{4}\int_{\rho=2}^{6}\frac{\mu_0}{\pi\rho}d\rho dz$$

$$\therefore \quad \psi = \mu_0\frac{4}{\pi}\ln\frac{6}{2} = 1.3988\mu_0 = 1.758\times 10^{-6} = 1.758\ \mu Wb$$

Problem 7.25

a) The magnetic vector potential $\vec{A} = (2x^2y + yz)\vec{a}_x + (xy^2 - xz^3)\vec{a}_y - (6xyz - 2x^2y^2)\vec{a}_z \quad Wb/m$

$$\vec{B} = \nabla \times \vec{A} \quad \Rightarrow \quad \vec{B} = \vec{a}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \vec{a}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \vec{a}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\vec{B} = \vec{a}_x(-6xz + 4x^2y + 3xz^2) + \vec{a}_y(y + 6yz - 4xy^2) + \vec{a}_z(y^2 - z^3 - 2x^2 - z) \quad Wb/m^2$$

b) $\psi = \int_S \vec{B}.\vec{ds}$ using $\vec{ds} = dydz\vec{a}_x$ which is normal to the square area.

$$\psi = \int_S \vec{B}.\vec{ds} \quad \Rightarrow \quad \int_{z=0}^{2}\int_{y=0}^{2} \vec{B}.(dydz\vec{a}_x)\Big|_{x=1} \quad \Rightarrow \quad \int_{z=0}^{2}\int_{y=0}^{2}(-6xz + 4x^2y + 3xz^2)dydz\Big|_{x=1}$$

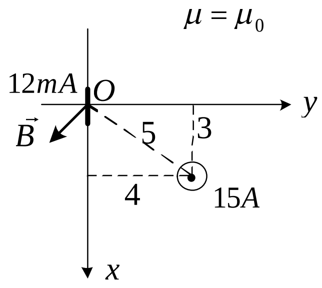
$$\Rightarrow \quad \int_{z=0}^{2}\int_{y=0}^{2}(-6z + 4y + 3z^2)dydz \quad \Rightarrow \quad \int_{z=0}^{2}(-12z + 8 + 6z^2)dz \quad \Rightarrow \quad -12\times\frac{2^2}{2} + 8\times 2 + 6\times\frac{2^3}{3}$$

$$\therefore \quad \psi = 8Wb$$

c) $\nabla\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \quad \Rightarrow \quad \nabla\vec{A} = 4xy + 2xy - 6xyz = 0$

$$\nabla\vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \quad \Rightarrow \quad \nabla\vec{B} = (-6z + 8xy + 3z^2) + (1 + 6z - 8xy) + (-3z^2 - 1) = 0$$

Problem 8.7



The magnetic field $\vec{B}$ *at the origin* O is given by:

$$\vec{B} = \mu_0 \frac{I}{2\pi\rho}\vec{a}_\phi$$

$$\rho \;\Rightarrow\; \sqrt{(3)^2 + (4)^2} = 5 \quad \& \quad \vec{a}_\phi \;\Rightarrow\; \frac{4\vec{a}_x - 3\vec{a}_y}{\sqrt{(4)^2 + (-3)^2}} = \frac{4\vec{a}_x - 3\vec{a}_y}{5} \quad \text{(unit vector in the direction of } \vec{B}\text{)}$$

$$\vec{B} = 4\pi \times 10^{-7}\frac{15}{2\pi \times 5} \times \frac{4\vec{a}_x - 3\vec{a}_y}{5} \quad \Rightarrow \quad \vec{B} = 480 \times 10^{-9}\vec{a}_x - 360 \times 10^{-9}\vec{a}_y \quad Wb/m^2$$

Using $\vec{F}_m = I\int_l \vec{dl} \times \vec{B}$ and integrating over the short current element of length 2cm.

Notice that the source of $\vec{B}$ is relatively far from the short current element ($\rho = 500cm >> 2cm$).

Thus we expect that $\vec{B}$ changes very little over the short current element of length 2cm. In other words, the value of $\vec{B}$ is almost the same at all point of this current element.

Under this condition, we can assume that $\vec{B}$ is uniform over the short current element.

Thus $\vec{B} = 480 \times 10^{-9}\vec{a}_x - 360 \times 10^{-9}\vec{a}_y \quad Wb/m^2$ at all points of the element.

Under this assumption, integration over the length of the element becomes multiplication leading to:

$$\vec{F}_m = I\int_l \vec{dl} \times \vec{B} \quad \overset{Uniform\ \vec{B}}{\Rightarrow} \quad \vec{F}_m = I\left(\int_l \vec{dl}\right) \times \vec{B} \quad \Rightarrow \quad \vec{F}_m = I\ (\vec{l}\,) \times \vec{B} \quad \Rightarrow$$

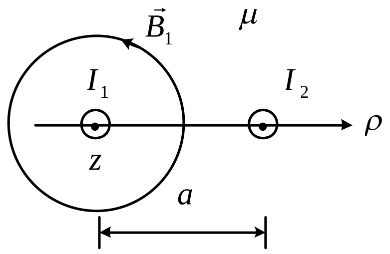
$$\vec{F}_m = 12 \times 10^{-3}(0.02\vec{a}_x) \times \vec{B} = 24 \times 10^{-5}\vec{a}_x \times \vec{B}$$

$$\vec{F}_m = 24 \times 10^{-5}\vec{a}_x \times (480 \times 10^{-9}\vec{a}_x - 360 \times 10^{-9}\vec{a}_y)$$

$$\vec{F}_m = -86.4 \times 10^{-12}\vec{a}_z = -86.4\vec{a}_z \quad pN$$

Problem 8.9

Consider the two parallel infinitely long straight current filaments carrying currents I_1 and I_2 in the z direction (see figure). Let us develop an expression for the force *per unit length* acting on I_2 due to I_1, namely $\vec{F}_{12}^{'}$.



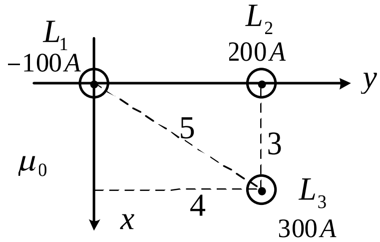
The magnetic field due to I_1 is given by $\vec{B}_1 = \frac{\mu I_1}{2\pi\rho}\vec{a}_\phi$

$$\vec{F}_{12} = I_2\int_{l_2} \vec{dl} \times \vec{B}_1 \quad \Rightarrow \quad \vec{F}_{12} = I_2\int_{l_2} dz\vec{a}_z \times \frac{\mu I_1}{2\pi\rho}\vec{a}_\phi \quad \xRightarrow{\text{for length } h \text{ of } I_2} \quad \vec{F}_{12} = I_2\int_{z=0}^{h} dz\vec{a}_z \times \frac{\mu I_1}{2\pi\rho}\vec{a}_\phi \Bigg|_{\rho=a}$$

$$\vec{F}_{12} = I_2\int_{z=0}^{h} dz\vec{a}_z \times \frac{\mu I_1}{2\pi a}\vec{a}_\phi = I_2\int_{z=0}^{h} \frac{\mu I_1}{2\pi a}(-\vec{a}_\rho)dz = -I_2\frac{\mu h I_1}{2\pi a}\vec{a}_\rho$$

$$\therefore \quad \vec{F}_{12}^{'} = \frac{\vec{F}_{12}}{h} = -\frac{\mu I_1 I_2}{2\pi a}\vec{a}_\rho$$

(the force is attractive if the currents are in the same direction and repulsive if they have opposite directions). We can use this result to easily solve all parts of this problem.



a) $\vec{F}_{12}^{'} = \frac{\mu_0(100)(200)}{2\pi\times 4}(+\vec{a}_y) = 10^{-3}\vec{a}_y \ N/m$ (repulsive)

b) $\vec{F}_{21}^{'} = \frac{\mu_0(100)(200)}{2\pi\times 4}(-\vec{a}_y) = -10^{-3}\vec{a}_y \ N/m$ (repulsive)

c) $\vec{F}_{13}^{'} = \frac{\mu_0(100)(300)}{2\pi\times 5}\times\frac{(3\vec{a}_x + 4\vec{a}_y)}{\sqrt{9+16}} = (0.72\vec{a}_x + 0.96\vec{a}_y)\times 10^{-3} \ N/m$ (repulsive)

d) $\vec{F}_{23}^{'} = \frac{\mu_0(200)(300)}{2\pi\times 3}(-\vec{a}_x) = -\vec{a}_x \ 4\times 10^{-3} \ N/m$ (attractive)

$$\vec{F}_3^{'} = \vec{F}_{13}^{'} + \vec{F}_{23}^{'} = (0.72\vec{a}_x + 0.96\vec{a}_y)\times 10^{-3} - \vec{a}_x \ 4\times 10^{-3} = (-3.28\vec{a}_x + 0.96\vec{a}_y)\times 10^{-3} \ N/m$$

Problem 8.22

$$\vec{H}_1 = (24\vec{a}_x - 30\vec{a}_y + 40\vec{a}_z)\times 10^3 \quad \Rightarrow \quad \vec{H}_{1n} = 40\times 10^3\vec{a}_z \quad \& \quad \vec{H}_{1t} = (24\vec{a}_x - 30\vec{a}_y)\times 10^3$$

$$\vec{B}_{1n} = \vec{B}_{2n} \quad \Rightarrow \quad \mu_1\vec{H}_{1n} = \vec{B}_{2n}$$

$$\therefore \quad \vec{B}_{2n} = 50\mu_0\vec{H}_{1n} = 50\mu_0 \times 40\times 10^3\vec{a}_z = 2.5133\vec{a}_z$$

$$(\vec{H}_1 - \vec{H}_2)\times \vec{a}_{n12} = \vec{K} \quad \Rightarrow$$

$$(24000\vec{a}_x - 30000\vec{a}_y + 40000\vec{a}_z - H_{2x}\vec{a}_x - H_{2y}\vec{a}_y - H_{2z}\vec{a}_z)\times(-\vec{a}_z) = 6000\vec{a}_x$$

$$24000\vec{a}_y + 30000\vec{a}_x - H_{2x}\vec{a}_y + H_{2y}\vec{a}_x = 6000\vec{a}_x$$

$$H_{2x} = 24\times 10^3 \quad \& \quad 30000 + H_{2y} = 6000 \quad \Rightarrow \quad H_{2y} = -24\times 10^3$$

$$\vec{H}_{2t} = (24\vec{a}_x - 24\vec{a}_y)\times 10^3$$

$$\vec{B}_{2t} = 100\mu_0 \times \vec{H}_{2t} = 100\mu_0 \times (24\vec{a}_x - 24\vec{a}_y)\times 10^3$$

$$\vec{B}_{2t} = (3.016\vec{a}_x - 3.016\vec{a}_y)$$

$$\therefore \quad \vec{B}_2 = \vec{B}_{2t} + \vec{B}_{2n} = 3.016\vec{a}_x - 3.016\vec{a}_y + 2.5133\vec{a}_z \quad Wb/m^2$$

Problem 8.28

$$\vec{B}_1 = \mu_0(22\vec{a}_\rho + 45\vec{a}_\phi)\ Wb/m^2$$

$\vec{B}_{1n} = 22\mu_0\vec{a}_\rho$ & $\vec{B}_{1t} = 45\mu_0\vec{a}_\phi$ (the normal to the cylindrical surface points in the ρ direction)

$$\vec{B}_{2n} = \vec{B}_{1n} = 22\mu_0\vec{a}_\rho = 2.765\times10^{-5}\quad Wb/m^2$$

$\vec{H}_{2t} = \vec{H}_{1t}$ because there is no surface current.

$$\vec{H}_{2t} = \vec{H}_{1t} \quad\Rightarrow\quad \frac{\vec{B}_{2t}}{\mu_2} = \frac{\vec{B}_{1t}}{\mu_1} \quad\Rightarrow\quad \vec{B}_{2t} = \mu_2 \times \frac{\vec{B}_{1t}}{\mu_1} = \mu_0 \times \frac{\vec{B}_{1t}}{800\mu_0} = \frac{\vec{B}_{1t}}{800}$$

$$\vec{B}_{2t} = \frac{45\mu_0\vec{a}_\phi}{800} = 7.069\times10^{-3}\vec{a}_\phi \quad Wb/m^2$$

$$\vec{B}_2 = \vec{B}_{2n} + \vec{B}_{2t} = 2.765\times10^{-5}\vec{a}_\rho + 7.069\times10^{-8}\vec{a}_\phi \quad Wb/m^2$$