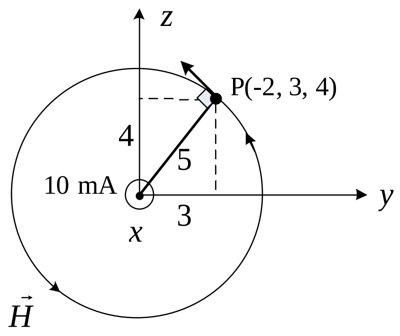
Problem 7.3



$$\vec{H} = \frac{I}{2\pi\rho}\vec{a}_\phi$$

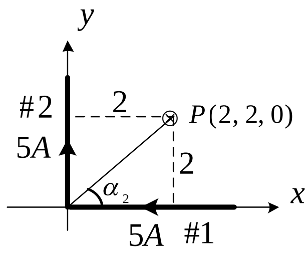
$\rho \Rightarrow \sqrt{3^2 + 4^2} = 5$ (ρ is replace by the distance from the conductor to the observation point P)

$\vec{a}_\phi \Rightarrow \dfrac{-4\vec{a}_y + 3\vec{a}_z}{\sqrt{4^2 + 3^2}} = \dfrac{-4\vec{a}_y + 3\vec{a}_z}{5}$ ($\vec{a}_\phi$ is replace by a unit vector in the direction of $\vec{H}$ at P)

$$\vec{H} = \frac{I}{2\pi\rho}\vec{a}_\phi \quad \Rightarrow \quad \vec{H}(-2, 3, 4) = \frac{10\times 10^{-3}}{2\pi\times 5}\times\frac{-4\vec{a}_y + 3\vec{a}_z}{5}$$

$$\therefore \quad \vec{H}(-2, 3, 4) = -254.7\vec{a}_y + 191.0\vec{a}_z \ \ \mu A / m$$

Problem 7.6



a) Using $\vec{H} = \vec{a}_\phi \dfrac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)$ at P = P(2, 2, 0)

For line segment #1: $\rho \Rightarrow 2$, $\alpha_2 = 45^o$, $\alpha_1 = 180^o$, $\vec{a}_\phi \Rightarrow -\vec{a}_z$

$$\therefore \vec{H}_1(2, 2, 0) = (-\vec{a}_z)\frac{5}{4\pi 2}(\cos 45^o - \cos 180^o) = -\frac{5}{8\pi}(0.7071+1)\vec{a}_z = -0.3396\vec{a}_z$$

For line segment #2: $\rho \Rightarrow 2$, $\alpha_2 = 0^o$, $\alpha_1 = 135^o$, $\vec{a}_\phi \Rightarrow -\vec{a}_z$

$$\therefore \vec{H}_2(2, 2, 0) = (-\vec{a}_z)\frac{5}{4\pi 2}(\cos 0^o - \cos 135^o) = -\frac{5}{8\pi}(1+0.7071)\vec{a}_z = -0.3396\vec{a}_z$$

$$\therefore \vec{H}(2, 2, 0) = \vec{H}_1(2, 2, 0) + \vec{H}_2(2, 2, 0) = -0.6792\vec{a}_z \ A/m$$

b) At P = P(0, -2, 0)

For line segment #1: $\rho \Rightarrow 2$, $\alpha_2 = 90^o$, $\alpha_1 = 180^o$, $\vec{a}_\phi \Rightarrow +\vec{a}_z$

$$\therefore \vec{H}_1(0, -2, 0) = \vec{a}_z \frac{5}{4\pi 2}(\cos 90^o - \cos 180^o) = \frac{5}{8\pi}(0+1)\vec{a}_z = 0.1989\vec{a}_z$$

For line segment #2: $\rho \Rightarrow 0$, $\alpha_2 = 0^o$, $\alpha_1 = 0^o$, $\vec{a}_\phi \Rightarrow \vec{?}$

$$\vec{H}_2(0, -2, 0) = \vec{?}\frac{5}{4\pi \times 0}(\cos 0^o - \cos 0^o) = \frac{\vec{0}}{0}$$

(the expression gives $\dfrac{\vec{0}}{0}$. However, it can be shown that in this case $\vec{H}_2(0, -2, 0) = \vec{0}$).

$$\therefore \vec{H}(0, -2, 0) = \vec{H}_1(0, -2, 0) = 0.1989\vec{a}_z$$

c) At P = P(0, 0, 2)

For line segment #1: $\rho \Rightarrow 2$, $\alpha_2 = 90^o$, $\alpha_1 = 180^o$, $\vec{a}_\phi \Rightarrow +\vec{a}_y$

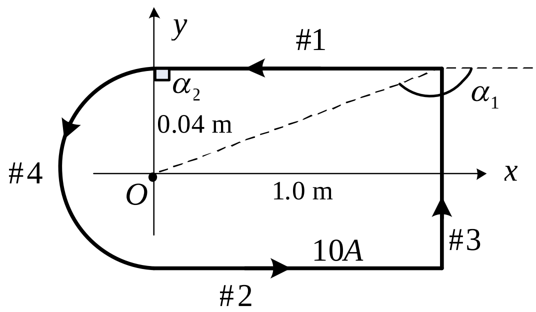
$$\therefore \vec{H}_1(0, 0, 2) = \vec{a}_y \frac{5}{4\pi 2}(\cos 90^o - \cos 180^o) = \frac{5}{8\pi}(0+1)\vec{a}_y = 0.1989\vec{a}_y$$

For line segment #2: $\rho \Rightarrow 2$, $\alpha_2 = 0^o$, $\alpha_1 = 90^o$, $\vec{a}_\phi \Rightarrow +\vec{a}_x$

$$\therefore \vec{H}_2(0, 0, 2) = \vec{a}_x \frac{5}{4\pi 2}(\cos 0^o - \cos 90^o) = \frac{5}{8\pi}(1+0)\vec{a}_x = 0.1989\vec{a}_x$$

$$\therefore \vec{H}(0, 0, 2) = 0.1989\vec{a}_x + 0.1989\vec{a}_y$$

Problem 7.11



$$\vec{H} = \vec{a}_\phi \frac{10}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)$$

For line #1 $\quad \vec{a}_\phi \Rightarrow \vec{a}_z$, $\quad \rho = 0.04$, $\alpha_2 = 90^o$, $(180^o - \alpha_1) = \tan^{-1}(\frac{0.04}{1}) = 2.291^o \Rightarrow \alpha_1 = 177.71^o$

$$\vec{H}_1 = \vec{H}_2 = \vec{a}_z \frac{10}{4\pi(0.04)}(\cos 90^o - \cos 177.71^o) = 198.78\vec{a}_z$$

For line #3 $\quad \vec{a}_\phi \Rightarrow \vec{a}_z$, $\quad \rho = 1.0$, $\alpha_2 = \tan^{-1}(\frac{1}{0.04}) = 87.709^o$, $\quad \alpha_1 = 180^o - \alpha_2 = 92.291^o$

$$\vec{H}_3 = \vec{a}_z \frac{10}{4\pi(1)}(\cos 87.709^o - \cos 92.291^o) = 0.0636\vec{a}_z$$

($\vec{H}_3$ is a relatively weak field, because wire 3 is short and it is also far from the point of observation O)

For line # 4, it is known that the magnetic field intensity *at the center* of a circular loop equals the current divided by the diameter of the loop. Thus for the semi-circular loop, we have:

$$\vec{H}_4 = \vec{a}_z \frac{10}{2(0.04)} \times \frac{1}{2} = 62.5\vec{a}_z$$

$$\therefore \vec{H} = \vec{H}_1 + \vec{H}_2 + \vec{H}_3 + \vec{H}_4 = 2\times 198.78\vec{a}_z + 0.0636\vec{a}_z + 62.5\vec{a}_z = 460.12\vec{a}_z \quad \text{A/m}$$

Problem 7.14

a) $\vec{H} = y^2 z\vec{a}_x + 2(x+1)yz\vec{a}_y - (x+1)z^2\vec{a}_z$

Use Ampere's law in differential form $\nabla\times\vec{H} = \vec{J}$ to find the current density $\vec{J}$

$$\vec{J} = \nabla\times\vec{H} = \vec{a}_x\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) + \vec{a}_y\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) + \vec{a}_z\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$$\vec{J} = \vec{a}_x[0 - 2(x+1)y] + \vec{a}_y(y^2 + z^2) + \vec{a}_z(2yz - 2yz)$$

$$\vec{J} = -2(x+1)y\vec{a}_x + (y^2 + z^2)\vec{a}_y$$

$$\vec{J}(1, 0, -3) = -2(1+1)\times 0\times\vec{a}_x + [0^2 + (-3)^2]\vec{a}_y = 9\vec{a}_y \qquad A/m^2$$

b) For the given area $y = 1,\ 0 \le x \le 1,\ 0 \le z \le 1 \quad \Rightarrow \quad \vec{ds} = \vec{a}_y\,dxdz$ (normal to area)

$$I = \int_S \vec{J}.\vec{ds} \quad \Rightarrow \quad I = \int_{z=0}^{1}\int_{x=0}^{1} \vec{J}.\vec{a}_y\,dxdz = \int_{z=0}^{1}\int_{x=0}^{1} (y^2 + z^2)\Big|_{y=1} dxdz$$

$$I = \int_{z=0}^{1}\int_{x=0}^{1} (z^2 + 1)dxdz = \int_{z=0}^{1} (z^2 + 1)dz = \frac{4}{3}A$$

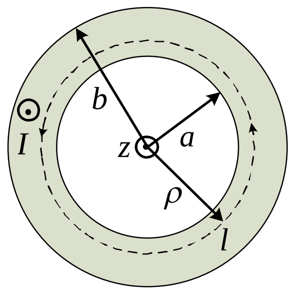
Problem 7.18

a) The close line integral of $\vec{H}$ equals the current enclosed.

$$\oint_l \vec{H}.\vec{dl} = I$$

b) Clearly $\vec{H}$ points in the ϕ direction.

Using the closed circular path $l \Rightarrow \vec{dl} = \rho d\phi \vec{a}_\phi$



For $\rho < a$

$$\oint_l \vec{H}.\vec{dl} = I \quad \Rightarrow \quad \oint_l \vec{H}.\rho d\phi \vec{a}_\phi = 0 \quad \Rightarrow \quad \int_{\phi=0}^{2\pi} H \rho d\phi = 0 \quad \Rightarrow \quad H(2\pi\rho) = 0 \quad \Rightarrow \quad H = \frac{0}{2\pi\rho} = 0$$

For $a < \rho < b$

$$\oint_l \vec{H}.\vec{dl} = I \quad \Rightarrow \quad H(2\pi\rho) = I \times \frac{\pi(\rho^2 - a^2)}{\pi(b^2 - a^2)} \quad \Rightarrow \quad H = \frac{I(\rho^2 - a^2)}{2\pi\rho(b^2 - a^2)}$$

For $\rho > b$

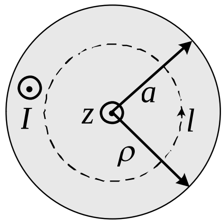
$$\oint_l \vec{H}.\vec{dl} = I \quad \Rightarrow \quad H(2\pi\rho) = I \quad \Rightarrow \quad H = \frac{I}{2\pi\rho}$$

Writing the field in vector form and summarizing:

$$\vec{H} = \begin{cases} \vec{0} & , \rho < a \\ \dfrac{I(\rho^2 - a^2)}{2\pi\rho(b^2 - a^2)} \vec{a}_\phi & , a < \rho < b \\ \dfrac{I}{2\pi\rho} \vec{a}_\phi & , \rho > b \end{cases}$$

Problem 7.20

a)



For $\rho < a$ (within the conductor)

$$\oint_l \vec{H}.\vec{dl} = I \quad \Rightarrow \quad H(2\pi\rho) = I \times \frac{\pi\rho^2}{\pi a^2} \quad \Rightarrow \quad H = \frac{I\rho}{2\pi a^2} \quad \Rightarrow \quad \vec{H} = \frac{I\rho}{2\pi a^2}\vec{a}_\phi$$

For $\rho > a$ (outside the conductor)

$$\oint_l \vec{H}.\vec{dl} = I \quad \Rightarrow \quad H(2\pi\rho) = I \quad \Rightarrow \quad H = \frac{I}{2\pi\rho} \quad \Rightarrow \quad \vec{H} = \frac{I}{2\pi\rho}\vec{a}_\phi$$

b)

At (0, 1 cm, 0), use $\vec{H} = \frac{I\rho}{2\pi a^2}\vec{a}_\phi \quad \Rightarrow \quad \vec{H} = \frac{3(0.01)}{2\pi(0.02)^2}\vec{a}_\phi = 11.937\vec{a}_\phi \ A/m$

At (0, 4 cm, 0), use $\vec{H} = \frac{I}{2\pi\rho}\vec{a}_\phi \quad \Rightarrow \quad \vec{H} = \frac{3}{2\pi(0.04)}\vec{a}_\phi = 11.937\vec{a}_\phi \ A/m$