Problem 13.2

$$E_{\theta s} = \eta H_{\phi s} \qquad \Rightarrow \qquad E_{\theta s} = 377 \times \frac{jI_0 \beta dl}{4\pi r} \sin\theta e^{-j\beta r}$$

$$\therefore |E_{\theta s}| = 377 \times \frac{I_0 \beta dl}{4\pi r} |\sin\theta|$$

$$\omega = 2\pi 10^7 \qquad \Rightarrow \qquad \beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{u} = \frac{\omega}{c} = \frac{2\pi 10^7}{3\times 10^8} = 0.20944\,\text{rad/m}$$

$$|E_{\theta s}| = 377 \times \frac{10\times 0.20944 \times 0.20}{4\pi \times 100} |\sin 90^o| = 0.1257\,\text{V/m}$$

Problem 13.4

$$H_{\phi s} = \frac{jI_0 \beta dl}{4\pi r} \sin\theta e^{-j\beta r}$$

$$E_{\theta s} = \eta H_{\phi s}$$

$$\omega = 2\pi \times 300 \times 10^6 \quad \Rightarrow \quad \beta = \frac{\omega}{c} = \frac{2\pi \times 300 \times 10^6}{3 \times 10^8} = 6.2832 \,\text{rad/m}$$

At $(10, 30^o, 90^o)$, we have:

$$H_{\phi s} = \frac{jI_0 \beta dl}{4\pi r} \sin\theta e^{-j\beta r} \quad \Rightarrow \quad H_{\phi s} = \frac{j\,2 \times 6.2832 \times 0.005}{4\pi \times 10} \sin 30^o e^{-j\,6.2832 \times 10}$$

$$H_{\phi s} = j\,2.5 \times 10^{-4} e^{-j62.832} \quad \Rightarrow$$

$$H_{\phi s} = 2.5 \times 10^{-4} e^{-j62.832 + j\frac{\pi}{2}} = 2.5 \times 10^{-4} e^{-j\,61.261} = 2.5 \times 10^{-4} e^{-j\,3510.01^o} = 2.5 \times 10^{-4} e^{-j\,270^o} \ \text{A/m}$$

$$E_{\theta s} = \eta H_{\phi s} \quad \Rightarrow \quad E_{\theta s} = 377 \times 2.5 \times 10^{-4} e^{-j\,270^o} = 9.425 \times 10^{-2} e^{-j\,270^o} V\ /m$$

$$\therefore \vec{H}_s = 2.5 \times 10^{-4} e^{-j\,270^o} \vec{a}_\phi \ \ \text{A/m}$$

$$\therefore \vec{E}_s = 9.425 \times 10^{-2} e^{-j\,270^o} \vec{a}_\theta \quad V\ /m$$

Problem 13.14

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^6} = 100\ m$$

The antenna length is $6m$, which is less than $\frac{\lambda}{10} = \frac{100}{10} = 10\text{m}$. Thus we can use a Hertzian dipole model.

$$R_{rad} = 80\pi^2(\frac{dl}{\lambda})^2 \quad \Rightarrow \quad R_{rad} = 80\pi^2(\frac{6}{100})^2 = 2.84\ \Omega$$

Problem 13.26

$$P_{rad} = 40\pi^2 (\frac{dl}{\lambda})^2 I_o^2$$

$$P_{avg} = \frac{1}{2}\eta \left| H_{\phi s} \right|^2 = \frac{1}{2}\eta \left| \frac{jI_0 \beta dl}{4\pi r} \sin\theta e^{-i\beta r} \right|^2$$

$$P_{avg} = \frac{\eta I_o^2 \beta^2 dl^2}{32\pi^2 r^2} \sin^2\theta = \frac{\eta I_o^2 (\frac{2\pi}{\lambda})^2 dl^2}{32\pi^2 r^2} \sin^2\theta = P_{avg} = \frac{\eta I_o^2 \beta^2 dl^2}{32\pi^2 r^2} \sin^2\theta = \frac{\eta I_o^2 (\frac{dl}{\lambda})^2}{8r^2} \sin^2\theta$$

$$P_{avg} = \frac{120\pi (\frac{dl}{\lambda})^2 I_o^2}{8r^2} \sin^2\theta = \frac{3\times 40\pi^2 (\frac{dl}{\lambda})^2 I_o^2}{8\pi r^2} \sin^2\theta$$

$$P_{avg} = \frac{3\times P_{rad}}{8\pi r^2} \sin^2\theta = \frac{3\sin^2\theta}{8\pi r^2} P_{rad} = \frac{1.5\sin^2\theta}{4\pi r^2} P_{rad}$$