Problem 12.1

a) $u^{'} = \dfrac{c}{\sqrt{\mu_r \varepsilon_r}} = \dfrac{3\times 10^8}{\sqrt{1\times 4}} = 1.5\times 10^8$ m/s

$a = 3\text{cm}$ and $b = 2\text{cm}$

$$f_c = \frac{u^{'}}{2}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2} \quad \Rightarrow \quad f_c = \frac{1.5\times 10^8}{2}\sqrt{(\frac{1}{0.03})^2 + (\frac{1}{0.02})^2} = 4.507\times 10^9 = 4.507\ \text{GHz}$$

b) $\beta^{'} = \omega\sqrt{\mu\varepsilon} = \dfrac{\omega}{c}\sqrt{\mu_r \varepsilon_r} = \dfrac{2\pi\times 20\times 10^9}{3\times 10^8}\sqrt{1\times 4} = 837.758$ rad/m

$$\beta = \beta\ \sqrt{1-(\frac{f_c}{f})^2} \quad \Rightarrow \quad \beta = 837.758\sqrt{1-(\frac{4.507\times 10^9}{20\times 10^9})^2}$$

$\beta = 837.758\times 0.9743 = 816.21$ rad/m

c) $u = u^{'}/\sqrt{1-(\dfrac{f_c}{f})^2} \quad \Rightarrow \quad u = \dfrac{1.5\times 10^8}{0.9743} = 1.5396\times 10^8$ m/s

[or $u = \dfrac{\omega}{\beta} = \dfrac{2\pi\times 20\times 10^9}{816.21} = 1.5396\times 10^8$ m/s]

Problem 12.5

Since the tunnel is air-filled, then $u' = c = 3\times10^8$ m/s

$$f_c = \frac{u'}{2}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2} \qquad \Rightarrow$$

$$f_c = \frac{3\times10^8}{2}\sqrt{(\frac{m}{8})^2 + (\frac{n}{16})^2} = \frac{3\times10^8}{2\times16}\sqrt{4m^2+n^2} = 9.375\times10^6\sqrt{4m^2+n^2}$$

The lowest possible value of f_c is clearly when $m = 0$ and $n = 1$ (they cannot be both zero).

$$\therefore f_c\big|_{\text{minimum}} = 9.375\times10^6\sqrt{0+1^2} = 9.375\times10^6 \text{ Hz}$$

a) Thus the tunnel will not pass a 1.5 MHz AM broadcast signal, because its frequency is below the lowest cutoff frequency of the waveguide.

b) The tunnel will pass a 120 MHz FM broadcast signal, because its frequency is higher the lowest cutoff frequency of the waveguide.

Problem 12.6

Since the waveguide is air-filled, then $u' = c = 3\times10^8$ m/s

Cutoff for TE_{03} equal 12GHz

$$f_c = \frac{u'}{2}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2} \quad \Rightarrow \quad 12\times10^9 = \frac{3\times10^8}{2}\sqrt{(\frac{0}{a})^2 + (\frac{3}{b})^2}$$

$$\sqrt{(\frac{3}{b})^2} = 80 \quad \Rightarrow \quad \frac{3}{b} = 80 \quad \Rightarrow \quad b = \frac{3}{80} = 3.750\,\text{cm}$$

Cutoff for TM_{11} equal 12GHz

$$f_c = \frac{u'}{2}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2} \quad \Rightarrow \quad 12\times10^9 = \frac{3\times10^8}{2}\sqrt{(\frac{1}{a})^2 + (\frac{1}{3.750\times10^{-2}})^2}$$

$$\sqrt{(\frac{1}{a})^2 + (\frac{1}{3.750\times10^{-2}})^2} = 80 \quad \Rightarrow \quad (\frac{1}{a})^2 + (\frac{1}{3.750\times10^{-2}})^2 = 6400$$

$$(\frac{1}{a})^2 + 711.111 = 6400 \quad \Rightarrow \quad (\frac{1}{a})^2 = 5688.89 \quad \Rightarrow \quad a = 1.326\,\text{cm}$$

Since $a < b$, the dominant mode is TE_{01} (if $a > b$, the dominant mode would be TE_{10}).

To calculate the cutoff frequency of the dominant mode:

$$f_c = \frac{u'}{2}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2} \quad \Rightarrow \quad f_c = \frac{3\times10^8}{2}\sqrt{(\frac{0}{1.326\times10^{-2}})^2 + (\frac{1}{3.750\times10^{-2}})^2}$$

$$f_c = \frac{3\times10^8}{2\times3.750\times10^{-2}} = 4\times10^9 = 4\text{GHz}$$

Thus, at 8GHz , the dominant mode will propagate in the waveguide because the operating frequency (8GHz) is higher than the cutoff frequency (4GHz) of the dominant mode.

Problem 12.8

$$\beta = 10\, rad/m \quad \text{and} \quad \beta' = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c} = \frac{2\pi \times 5\times 10^{9}}{3\times 10^{8}} = 104.72 \text{ rad/m}$$

$$\beta = \beta \sqrt{1-(\frac{f_c}{f})^2} \quad \Rightarrow \quad 10 = 104.72\sqrt{1-(\frac{f_c}{f})^2} \quad \Rightarrow \quad 0.09549 = \sqrt{1-(\frac{f_c}{f})^2}$$

$$1-(\frac{f_c}{f})^2 = 9.119\times 10^{-3} \quad \Rightarrow \quad \frac{f_c}{f} = 0.9954 \quad \Rightarrow$$

$$f_c = 0.9954 f = 0.9954\times 5\times 10^{9} = 4.9772\times 10^{9}\text{ Hz} = 4.9772 \text{ GHz}$$

b) The mode is TM_{21} $\Rightarrow$ m = 2 and n = 1.

Since it is a TM mode $\Rightarrow$ $H_z = 0$

For TM modes:

$$E_{xs} = -\frac{\gamma}{h^2}(\frac{m}{a})E_o \cos(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b})e^{-\gamma z}$$

where $h^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2 = (\frac{2\pi}{a})^2 + (\frac{\pi}{b})^2$

Since the waveguide is lossless, $\gamma = j\beta = j10$.

Also $E_o = 1$.

$$E_{xs} = -\frac{j10}{(\frac{2\pi}{a})^2 + (\frac{\pi}{b})^2}(\frac{2}{a})\cos(\frac{2\pi x}{a})\sin(\frac{\pi y}{b})e^{-j10z}$$

$$E_{xs} = \frac{20}{a}\frac{1}{(\frac{2\pi}{a})^2 + (\frac{\pi}{b})^2}\cos(\frac{2\pi x}{a})\sin(\frac{\pi y}{b})e^{-j10z}e^{-j90^o}$$

Notice that in this case there are infinite possible choices for a and b. In other words, they are not unique. Therefore, they will be kept in symbolic form.

Finally we get:

$$E_x = \frac{20}{a}\frac{1}{(\frac{2\pi}{a})^2 + (\frac{\pi}{b})^2}\cos(\frac{2\pi x}{a})\sin(\frac{\pi y}{b})\cos(\pi 10^{10} t - 10z - 90^o) \quad \text{V/m}$$

Problem 12.9

This problem has an error, because it is impossible to have $E_x = -10\sin(\frac{2\pi x}{a})\sin(\omega t - 150z)$. This means that the normal component of $\vec{E}$ is zero at the air-conductor boundaries located at $x = 0$ and $x = a$. Let us change the problem and assume that:

$$E_y = -10\sin(\frac{2\pi x}{a})\sin(\omega t - 150z)$$

a) Clearly $m = 2$ and $n = 0$. Therefore, the mode must be TE_{20}.

(it cannot be TM, because for TM modes, both m and n are not zero).

b) $f_c = \frac{u^{'}}{2}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2} \quad \Rightarrow \quad f_c = \frac{3\times 10^8}{2}\sqrt{(\frac{2}{0.012})^2 + (\frac{0}{0.012})^2} = 2.5\times 10^{10}\,\text{Hz} = 25\,\text{GHz}$

$$\lambda_c = \frac{u^{'}}{f_c} = \frac{3\times 10^8}{25\times 10^9} = 1.2\times 10^{-2}\,\text{m} = 1.2\,\text{cm}$$

c) $\beta = 150\, rad/m$

$$\beta^{'} = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c} = \frac{2\pi f}{3\times 10^8} = 2.0944\times 10^{-8} f$$

$$\beta = \beta\sqrt{1-(\frac{f_c}{f})^2} \quad \Rightarrow \quad 150 = 2.0944\times 10^{-8} f\sqrt{1-(\frac{25\times 10^9}{f})^2} \quad \Rightarrow$$

$$7.162\times 10^9 = \sqrt{f^2 - (25\times 10^9)^2} \quad \Rightarrow \quad 5.1294\times 10^{19} = f^2 - 6.25\times 10^{20}$$

$$f^2 = 6.25\times 10^{20} + 5.1294\times 10^{19} \quad \Rightarrow \quad f = 2.6006\times 10^{10} = 26.01\,\text{GHz}$$

d) $\gamma = \alpha + j\beta = 0 + j150 = j150$

Since the mode is TE, then:

$$\eta_{TE} = \frac{\eta^{'}}{\sqrt{1-(\frac{f_c}{f})^2}} = \frac{377}{\sqrt{1-(\frac{25}{26.01})^2}} = 1366.13\,\Omega$$

Problem 12.13

a) $E_y = 5\sin(\frac{2\pi x}{a})\cos(\frac{\pi y}{b})\sin(\omega t - 12z)$ V/m

$m = 2$ and $n = 1$.

The mode could be either TE or TM, but since we were told it is TE, then the operating mode is TE_{21}.

b) $\beta = 12 rad/m$ and $\beta^{'} = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c} = \frac{2\pi \times 6\times 10^9}{3\times 10^8} = 125.664$ rad/m

$$\beta = \beta\sqrt{1-(\frac{f_c}{f})^2} \quad \Rightarrow \quad 12 = 125.664\sqrt{1-(\frac{f_c}{f})^2} \quad \Rightarrow \quad 0.09549 = \sqrt{1-(\frac{f_c}{f})^2}$$

$$1-(\frac{f_c}{f})^2 = 9.119\times 10^{-3} \quad \Rightarrow \quad \frac{f_c}{f} = 0.9954 \quad \Rightarrow$$

$$f_c = 0.9954f = 0.9954\times 6\times 10^9 = 5.973\times 10^9 \text{ Hz} = 5.973 \text{ GHz}$$

c) $\eta_{TE} = \frac{\eta^{'}}{\sqrt{1-(\frac{f_c}{f})^2}} = \frac{377}{\sqrt{1-(\frac{5.973}{6})^2}} = 3978.41\Omega$

d) $H_x = -\eta_{TE} E_y$

$$H_x = -3978.41\times 5\sin(\frac{2\pi x}{a})\cos(\frac{\pi y}{b})\sin(\omega t - 12z)$$

$$H_x = -1.9892\times 10^4 \sin(\frac{2\pi x}{a})\cos(\frac{\pi y}{b})\sin(\omega t - 12z) \text{ A/m}$$