Problem 10.42

a) $\vec{\mathcal{P}} = \vec{E} \times \vec{H} \quad \Rightarrow \quad \vec{\mathcal{P}} = [\frac{V_o}{\rho \ln(b/a)}\cos(\omega t - \beta z)\vec{a}_\rho] \times [\frac{I_o}{2\pi\rho}\cos(\omega t - \beta z)\vec{a}_\phi]$

$$\vec{\mathcal{P}} = \frac{I_o V_o}{2\pi\rho^2 \ln(b/a)}\cos^2(\omega t - \beta z)\vec{a}_z$$

b) $\vec{\mathcal{P}} = \frac{I_o V_o}{2\pi\rho^2 \ln(b/a)}\cos^2(\omega t - \beta z)\vec{a}_z \quad \Rightarrow \quad \vec{\mathcal{P}}_{ave} = \frac{1}{2} \times \frac{I_o V_o}{2\pi\rho^2 \ln(b/a)}\vec{a}_z = \frac{I_o V_o}{4\pi\rho^2 \ln(b/a)}$

Where the average of $\cos^2(\omega t - \beta z)$ over time equals 1/2.

The average Poynting's Vector can also be obtained from:

$$\vec{\mathcal{P}}_{ave} = \frac{1}{2} \times \mathrm{Re}(\vec{E}_s \times \vec{H}_s^*) \quad \Rightarrow \quad \vec{\mathcal{P}}_{ave} = \frac{1}{2} \times \mathrm{Re}\{[\frac{V_o}{\rho \ln(b/a)} e^{j(\omega t - \beta z)}\vec{a}_\rho] \times [\frac{I_o}{2\pi\rho} e^{j(\omega t - \beta z)}\vec{a}_\phi]^*$$

$$\vec{\mathcal{P}}_{ave} = \frac{1}{2} \times \mathrm{Re}\{[\frac{V_o}{\rho \ln(b/a)} e^{j(\omega t - \beta z)}] \times [\frac{I_o}{2\pi\rho} e^{-j(\omega t - \beta z)}]\}\vec{a}_z$$

$$\vec{\mathcal{P}}_{ave} = \frac{1}{2} \times \mathrm{Re}[\frac{I_o V_o}{2\pi\rho^2 \ln(b/a)}\vec{a}_z] = \frac{I_o V_o}{4\pi\rho^2 \ln(b/a)}\vec{a}_z$$

The average power flow along the cable:

$$P_{ave} = \int_S \vec{\mathcal{P}}_{ave} . \vec{ds} \quad \Rightarrow \quad P_{ave} = \int_{\phi=0}^{2\pi} \int_{\rho=a}^{b} \frac{I_o V_o}{4\pi\rho^2 \ln(b/a)}\vec{a}_z . (\rho d\rho d\phi \vec{a}_z)$$

$$P_{ave} = \int_{\phi=0}^{2\pi} \int_{\rho=a}^{b} \frac{I_o V_o}{4\pi\rho \ln(b/a)} d\rho d\phi \quad \Rightarrow \quad P_{ave} = \int_{\phi=0}^{2\pi} \frac{I_o V_o \ln(b/a)}{4\pi \ln(b/a)} d\phi \quad \Rightarrow \quad P_{ave} = \frac{2\pi I_o V_o}{4\pi}$$

$$\therefore \quad P_{ave} = \frac{1}{2} I_o V_o$$

Which is a well-known result in circuit theory, valid for sinusoidal steady-state.

Problem 10.50

For normal incidence, the transmission coefficient is given by $\tau = \dfrac{2\eta_2}{\eta_2 + \eta_1}$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} \quad \Rightarrow \quad \tau = \frac{2\sqrt{\dfrac{\mu_2}{\varepsilon_2}}}{\sqrt{\dfrac{\mu_2}{\varepsilon_2}} + \sqrt{\dfrac{\mu_1}{\varepsilon_1}}} \quad \Rightarrow \quad \tau = \frac{2 \times 377\sqrt{\dfrac{\mu_{r2}}{\varepsilon_{r2}}}}{377\sqrt{\dfrac{\mu_{r2}}{\varepsilon_{r2}}} + 377\sqrt{\dfrac{\mu_{r1}}{\varepsilon_{r1}}}}$$

$$\tau = \frac{2 \times \sqrt{\dfrac{\mu_{r2}}{\varepsilon_{r2}}}}{\sqrt{\dfrac{\mu_{r2}}{\varepsilon_{r2}}} + \sqrt{\dfrac{\mu_{r1}}{\varepsilon_{r1}}}} \quad \Rightarrow \quad \tau = \frac{2 \times \sqrt{\dfrac{9}{1}}}{\sqrt{\dfrac{9}{1}} + \sqrt{\dfrac{1}{1}}} = \frac{2 \times 3}{3+1} = \frac{6}{4} = 1.5$$

$$E_{to} = \tau E_{io} \quad \Rightarrow \quad E_{to} = 1.5 \times 2 = 3 \text{ V/m}$$

Thus the transmitted average power flow per unit area is:

$$\mathcal{P}_{2\text{ave}} = \frac{1}{2}\frac{|E_{to}|^2}{\eta_2} = \frac{1}{2}\frac{3^2}{377\sqrt{\dfrac{9}{1}}} = \frac{1}{2} \times \frac{3}{377} = 3.980 \times 10^{-3} \text{ W/m}^2$$

Problem 10.52

a) $\vec{E}_i = 10\cos(\omega t - z)\vec{a}_y$ V/m

$$\beta_1 = \beta_{air} = 1 \qquad \Rightarrow \quad \lambda_{air} = \frac{2\pi}{\beta_{air}} = \frac{2\pi}{1} = 6.283 \text{ m}$$

$$u_{air} = \frac{\omega}{\beta_{air}} = c \qquad \Rightarrow \quad \frac{\omega}{1} = 3\times10^8 \qquad \Rightarrow \quad \omega = 3\times10^8 \text{ rad/s}$$

(the value of ω is valid in both air and the dielectric, because ω does not depend on the medium).

$$u_2 = \frac{\omega}{\beta_2} = \frac{c}{\sqrt{\mu_{2r}\varepsilon_{2r}}} \qquad \Rightarrow \quad \frac{3\times10^8}{\beta_2} = \frac{3\times10^8}{\sqrt{1\times3}} \qquad \Rightarrow \quad \beta_2 = \sqrt{3} = 1.732$$

$$\lambda_2 = \frac{2\pi}{\beta_1} = \frac{2\pi}{1.732} = 3.628 \text{ m}$$

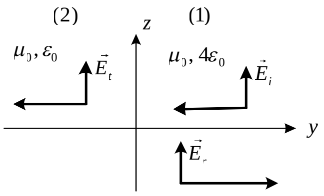
b) $\vec{H}_i = \dfrac{10}{377}\cos(3\times10^8\, t - z)(-\vec{a}_x) = -0.0265\cos(3\times10^8\, t - z)\vec{a}_x$ V/m

c) $$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\dfrac{\mu_{r2}}{\varepsilon_{r2}}} - \sqrt{\dfrac{\mu_{r1}}{\varepsilon_{r1}}}}{\sqrt{\dfrac{\mu_{r2}}{\varepsilon_{r2}}} + \sqrt{\dfrac{\mu_{r1}}{\varepsilon_{r1}}}} = \frac{\sqrt{\dfrac{1}{3}} - \sqrt{\dfrac{1}{1}}}{\sqrt{\dfrac{1}{3}} + \sqrt{\dfrac{1}{1}}} = \frac{0.57735 - 1}{0.57735 + 1} = -0.268$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2\sqrt{\dfrac{\mu_{r2}}{\varepsilon_{r2}}}}{\sqrt{\dfrac{\mu_{r2}}{\varepsilon_{r2}}} + \sqrt{\dfrac{\mu_{r1}}{\varepsilon_{r1}}}} = \frac{2\times\sqrt{\dfrac{1}{3}}}{\sqrt{\dfrac{1}{3}} + \sqrt{\dfrac{1}{1}}} = \frac{2\times0.57735}{0.57735+1} = 0.732$$

(check: $1 + \Gamma = 1 - 0.268 = 0.732 = \tau$)

Problem 10.53



a) $\dfrac{\omega}{\beta_1} = \dfrac{c}{\sqrt{\mu_{1r}\varepsilon_{1r}}} \quad \Rightarrow \quad \dfrac{10^8}{\beta_1} = \dfrac{3\times 10^8}{\sqrt{1\times 4}} \quad \Rightarrow \quad \beta_1 = 0.667 \text{ rad/m}$

$\therefore \quad \vec{E}_i = 5\cos(10^8 t + 0.667y)\vec{a}_z \text{ V/m}$

$$\Gamma = \frac{\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} - \sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}}{\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} + \sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}} = \frac{1-\sqrt{\frac{1}{4}}}{1+\sqrt{\frac{1}{4}}} = \frac{1-0.5}{1+0.5} = 0.3333$$

$\therefore \quad \vec{E}_r = \Gamma \times 5\cos(10^8 t - 0.6667y)\vec{a}_z = 0.3333\times 5\cos(10^8 t - 0.6667y)\vec{a}_z = 1.6665\cos(10^8 t - 0.6667y)\vec{a}_z$ V/

$\vec{E}_1 = \vec{E}_i + \vec{E}_r \quad \Rightarrow \quad \vec{E}_1 = 5\cos(10^8 t + 0.6667y)\vec{a}_z + 1.6665\cos(10^8 t - 0.6667y)\vec{a}_z \text{ V/m}$

b) For the incident wave:

$$\vec{\mathcal{P}}_{\text{iave}} = \frac{1}{2}\frac{|E_{io}|^2}{\eta_1}(-\vec{a}_y) = -\frac{1}{2}\frac{5^2}{377\sqrt{\frac{1}{4}}}\vec{a}_y = -66.313\times 10^{-3}\vec{a}_y \text{ W/m}^2$$

For the reflected wave: $\vec{\mathcal{P}}_{\text{rave}} = \dfrac{1}{2}\dfrac{|E_{ro}|^2}{\eta_1}(+\vec{a}_y) = \dfrac{1}{2}\dfrac{(1.6665)^2}{377\sqrt{\frac{1}{4}}}\vec{a}_y = 7.367\times 10^{-3}\vec{a}_y \text{ W/m}^2$

$\vec{\mathcal{P}}_{\text{1ave}} = \vec{\mathcal{P}}_{\text{iave}} + \vec{\mathcal{P}}_{\text{rave}} = -66.313\times 10^{-3}\vec{a}_y + 7.367\times 10^{-3}\vec{a}_y = -58.95\times 10^{-3}\vec{a}_y \text{ W/m}^2$

(which means that the *net average power* flow is in the $-y$ direction).

c) For the transmitted wave:

$$\tau = \frac{2\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}}}{\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} + \sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}} = \frac{2\times 1}{1+\sqrt{\frac{1}{4}}} = \frac{2}{1+0.5} = 1.3333 \quad \Rightarrow \quad E_{to} = \tau E_{io} \quad \Rightarrow \quad E_{to} = 5\times 1.3333 = 6.6667 \text{ V/m}$$

$$\vec{\mathcal{P}}_{\text{2ave}} = \vec{\mathcal{P}}_{\text{tave}} = \frac{1}{2}\frac{|E_{to}|^2}{\eta_2}(-\vec{a}_y) = -\frac{1}{2}\times\frac{6.6667^2}{377}\vec{a}_y = -58.95\times 10^{-3}\vec{a}_y \text{ W/m}^2$$

$\vec{\mathcal{P}}_{\text{1ave}} = \vec{\mathcal{P}}_{\text{2ave}} = -58.95\times 10^{-3}\vec{a}_y \text{ W/m}^2$

Thus the average Poynting's vector in both regions is the same.