Problem 4.3

a) The *magnitude* of the force acting on Q_1 due to Q_2 equals:

$$F_{21} = \frac{Q_2 Q_1}{4\pi\varepsilon_0 R^2} = \frac{(10\times10^{-9})\times(10\times10^{-9})}{4\pi\times(10^{-9}/36\pi)\times(0.1)^2}$$

$$\therefore F_{21} = 9\times10^{-5} = 90\,\mu N$$

$\therefore F_{31} = 9\times10^{-5} = 90\,\mu N$ (magnitude of the force acting on Q_1 due to Q_3)

Since there is 60^o angle between these two forces, then the magnitude of the total force acting on Q_1:

$$F_1 = F_{21}\cos 30^o + F_{31}\cos 30^o = 2F_{21}\cos 30^o$$

$$F_1 = 2\times9\times10^{-5}\times0.866 = 1.5588\times10^{-4}$$

$$\therefore F_1 = 155.88\,\mu N$$

$$\therefore F_1 = F_2 = F_3 = 155.88\,\mu N$$

[Notice that although the magnitude of each force is the same, the directions of these forces are different, which means that the vectors $\vec{F}_1 \neq \vec{F}_2 \neq \vec{F}_3$].

b) The electric field at the center of the triangle is obviously zero, because the electric fields due to each charge cancel each other.

Problem 4.4

a) $\vec{E}(5,0,6)=\vec{E}_1(5,0,6)+\vec{E}_2(5,0,6)$

$$\vec{E}(5,0,6)=\frac{Q_1\vec{R}_1}{4\pi\varepsilon_0 R_1^{\,3}}+\frac{Q_2\vec{R}_2}{4\pi\varepsilon_0 R_2^{\,3}}$$

$$\vec{E}(5,0,6)=\frac{Q_1[(5-4)\vec{a}_x+(6+3)\vec{a}_z]}{\frac{10^{-9}}{9}[1^2+9^2]^{3/2}}+\frac{4\times 10^{-9}\times[(5-2)\vec{a}_x+(6-1)\vec{a}_z]}{\frac{10^{-9}}{9}[3^2+5^2]^{3/2}}$$

$$\vec{E}(5,0,6)=\frac{Q_1(\vec{a}_x+9\vec{a}_z)\times 9\times 10^9}{742.54}+\frac{36\times[3\vec{a}_x+5\vec{a}_z]}{198.25}$$

$$\vec{E}(5,0,6)=Q_1(\vec{a}_x+9\vec{a}_z)\times 1.2121\times 10^7+0.18159\times[3\vec{a}_x+5\vec{a}_z]$$

$$\vec{E}(5,0,6)=(1.2121\times 10^7 Q_1+3\times 0.18159)\vec{a}_x+(9\times 1.2121\times 10^7 Q_1+5\times 0.18159)\vec{a}_z$$

$$E_z=0 \quad\Rightarrow\quad (9\times 1.2121\times 10^7 Q_1+5\times 0.18159)=0$$

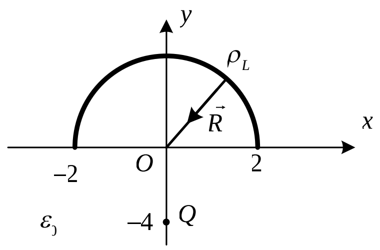
$$\therefore\quad Q_1=-8.323\times 10^{-9}=-8.323\,nC$$

b) This means $F_x=qE_x=0$

$$E_x=0 \quad\Rightarrow\quad (1.2121\times 10^7 Q_1+3\times 0.18159)=0$$

$$\therefore\quad Q_1=-4.494\times 10^{-8}=-44.94\,nC$$

Problem 4.5



The electric field at the origin, $\vec{E}(0,0,0)$ is due to both Q and the uniform line charge density ρ_L distributed over the semi-circle.

$$\vec{E}(0,0,0) = E_Q(0,0,0) + E_L(0,0,0)$$

$$E_Q(0,0,0) = \frac{Q}{4\pi\varepsilon_0(4)^2}\vec{a}_y = \frac{Q}{64\pi\varepsilon_0}\vec{a}_y$$

$$E_L(0,0,0) = \int_L \frac{\rho_L dl\vec{R}}{4\pi\varepsilon_0 R^3}$$

Where: $\vec{R} = -2\vec{a}_\rho$ and $\rho_L = \frac{\text{Charge}}{\text{length of semicircle}} = \frac{10nC}{2\pi} = 1.592nC$

$dl = \rho d\phi = 2d\phi$ because ρ has a fixed value of 2 over the semicircular path.

$$\therefore \quad E_L(0,0,0) = \int_L \frac{\rho_L(2d\phi)(-2\vec{a}_\rho)}{4\pi\varepsilon_0(2)^3} = \int_{\phi=0}^{\pi} \frac{\rho_L(2d\phi)(-2\vec{a}_\rho)}{4\pi\varepsilon_0(2)^3} = -\frac{\rho_L}{8\pi\varepsilon_0}\int_{\phi=0}^{\pi}\vec{a}_\rho d\phi$$

[notice that $\vec{a}_\rho = \vec{a}_\rho(\phi)$. Thus we cannot take $\vec{a}_\rho$ out of the integral, because it depends on the integration variable ϕ]. Converting $\vec{a}_\rho$ to rectangular coordinates:

$$E_L(0,0,0) = -\frac{\rho_L}{8\pi\varepsilon_0}\int_{\phi=0}^{\pi}(\cos\phi\vec{a}_x + \sin\phi\vec{a}_y)d\phi$$

Both $\vec{a}_x$ and $\vec{a}_y$ are uniform vectors and thus they are independent of ϕ. Therefore, we can take them both out of the integral, giving:

$$E_L(0,0,0) = -\vec{a}_x\frac{\rho_L}{8\pi\varepsilon_0}\int_{\phi=0}^{\pi}\cos\phi d\phi - \vec{a}_y\frac{\rho_L}{8\pi\varepsilon_0}\int_{\phi=0}^{\pi}\sin\phi d\phi$$

$$E_L(0,0,0) = -\vec{a}_x\frac{\rho_L}{8\pi\varepsilon_0}\sin\phi\Big|_0^{\pi} + \vec{a}_y\frac{\rho_L}{8\pi\varepsilon_0}\cos\phi\Big|_0^{\pi} = -\frac{\rho_L}{4\pi\varepsilon_0}\vec{a}_y$$

$$\vec{E}(0,0,0) = E_Q(0,0,0) + E_L(0,0,0) \quad \Rightarrow \quad \vec{E}(0,0,0) = \frac{Q}{64\pi\varepsilon_0}\vec{a}_y - \frac{\rho_L}{4\pi\varepsilon_0}\vec{a}_y$$

$$\vec{E}(0,0,0) = 0 \quad \Rightarrow \quad \frac{Q}{64\pi\varepsilon_0} - \frac{\rho_L}{4\pi\varepsilon_0} = 0 \quad \Rightarrow \quad Q = 16\rho_L = 25.47nC$$

Problem 4.12

$$\vec{E}(1,1,1) = \vec{E}_Q(1,1,1) + \vec{E}_L(1,1,1) + \vec{E}_S(1,1,1)$$

$$\vec{E}_Q(1,1,1) = \frac{100\times10^{-12}[(1-4)\vec{a}_x + (1-1)\vec{a}_y + (1+3)\vec{a}_z]}{(10^{-9}/9)[9+16]^{3/2}} = -0.0216\vec{a}_x + 0.0288\vec{a}_z$$

To calculate $\vec{E}_L(1,1,1)$, use $\vec{E}_L = \dfrac{\rho_L}{2\pi\varepsilon_0\rho}\vec{a}_\rho$

Replace ρ by $\sqrt{(1)^2+(1)^2} = \sqrt{2}$ (which is the <u>perpendicular</u> distance between the observation point $(1,1,1)$ and the line charge).

Replace $\vec{a}_\rho$ by $\dfrac{\vec{a}_y + \vec{a}_z}{\sqrt{2}}$ (which a <u>unit</u> vector <u>perpendicular</u> to the to the line charge).

$$\therefore \quad \vec{E}_L(1,1,1) = \frac{2\times10^{-9}}{2\pi\varepsilon_0\sqrt{2}}\times\frac{\vec{a}_y + \vec{a}_z}{\sqrt{2}} = 18\vec{a}_y + 18\vec{a}_z$$

To calculate $\vec{E}_S(1,1,1)$, use $\vec{E}_S = \dfrac{\rho_S}{2\varepsilon_0}\vec{a}_N$

Replace $\vec{a}_N$ by $-\vec{a}_z$ (which a <u>unit</u> vector <u>perpendicular</u> to the surface charge and pointing downwards, where the observation point is).

$$\therefore \quad \vec{E}_S(1,1,1) = \frac{5\times10^{-9}}{2\varepsilon_0}(-\vec{a}_z) = -282.49\vec{a}_z$$

$$\vec{E}(1,1,1) = \vec{E}_Q(1,1,1) + \vec{E}_L(1,1,1) + \vec{E}_S(1,1,1) \quad \Rightarrow \quad \vec{E}(1,1,1) = -0.0216\vec{a}_x + 18\vec{a}_y - 264.46\vec{a}_z \quad \text{V/m}$$

Problem 4.14

a) $\rho_{S1} = 10\times10^{-6}$, $\rho_{S2} = -20\times10^{-6}$, $\rho_{S3} = 30\times10^{-6}$

To calculate $\vec{E}_S(5,-1,4)$, use $\vec{E}_S = \dfrac{\rho_S}{2\varepsilon_0}\vec{a}_N$

For ρ_{S1} replace $\vec{a}_N$ by $+\vec{a}_x$ (normal to surface and towards the observation point)

For ρ_{S2} replace $\vec{a}_N$ by $+\vec{a}_y$

For ρ_{S3} replace $\vec{a}_N$ by $-\vec{a}_z$

$$\vec{E}_S(5,-1,4) = \frac{10\times10^{-6}}{2\varepsilon_0}\vec{a}_x + \frac{-20\times10^{-6}}{2\varepsilon_0}\vec{a}_y + \frac{30\times10^{-6}}{2\varepsilon_0}(-\vec{a}_z)$$

$$\therefore\quad \vec{E}_S(5,-1,4) = 5.650\times10^{5}\vec{a}_x - 1.130\times10^{6}\vec{a}_y - 1.695\times10^{6}\vec{a}_z \quad \text{V/m}$$

Problem 4.16

$$\vec{E}_L(0,0,0)=\frac{10\times10^{-9}}{2\pi\varepsilon_0(2)}\times(-\vec{a}_y)+\frac{-10\times10^{-9}}{2\pi\varepsilon_0(2)}\times(+\vec{a}_y)=-180\vec{a}_y \qquad \text{V/m}$$