KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

SUMMER SESSION 2007/2008

EE 340 (01) MAJOR EXAM I

TIME: 10:30 -11:45 A.M.

DATE: SATURDAY 26-JULY-2008

LOCATION: IN CLASS

Student's Name:........KEY..........................

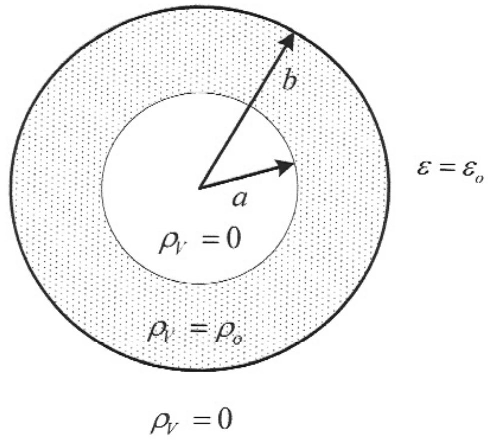
Student's I.D. Number: ....

	Maximum Score	Score
Problem 1	**25**	
Problem 2	**25**	
Problem 3	**25**	
Problem 4	**25**	
Total	**100**	

Problem 1 [25 points]

Consider the following *spherical* volume charge density distribution, which exists in free space:

$$\rho_V = \begin{cases} \rho_o & \text{for} \quad b > r > a \\ 0 & \text{otherwise} \end{cases}$$



Where ρ_o is some constant. Use Gauss's law to derive an expression for the resulting $\vec{E}$ field in the regions:

a) $r < a$

b) $b > r > a$

c) $r > b$

a) $\oint_S \vec{D} \cdot \vec{ds} = Q$, $\therefore D 4\pi r^2 = 0$

$\therefore \vec{E} = \vec{0}$

b) $D 4\pi r^2 = \frac{4\pi}{3}(r^3 - a^3)\rho_o$

$\therefore \vec{E} = \frac{\rho_o (r^3 - a^3)}{3 \epsilon_o r^2} \vec{a_r}$

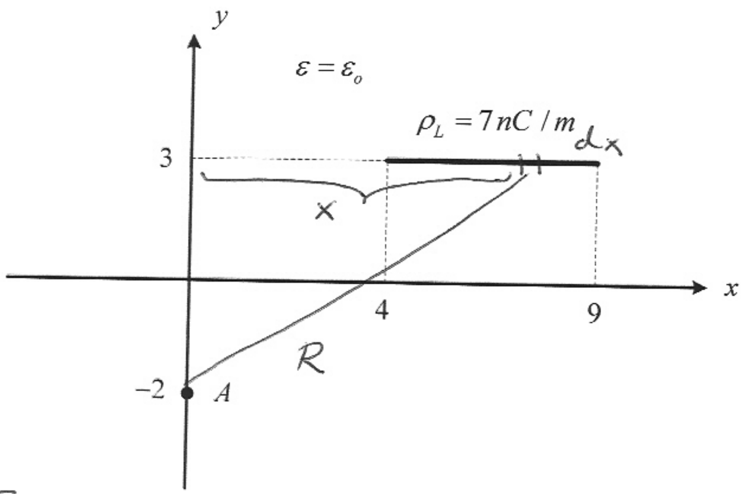
c) $D 4\pi r^2 = \frac{4\pi}{3}(b^3 - a^3)\rho_o$

$\therefore \vec{E} = \frac{\rho_o (b^3 - a^3)}{3 \epsilon_o r} \vec{a_r}$

Problem 2 [25 points]

As shown in the diagram, a line charge with the uniform charge density $\rho_L = 7\ nC/m$ exists in free space. The line charge exists entirely in the $x-y$ plane and extends from $x = 4$ to $x = 9$. Calculate the resulting electrostatic potential at point A, located at (0, -2, 0).

[Hint: $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x+\sqrt{x^2+a^2})$]



$$V_A = \int_l \frac{\rho_l\, dl}{4\pi\epsilon R}$$

$$dl = dx$$

$$R = \sqrt{x^2+25}$$

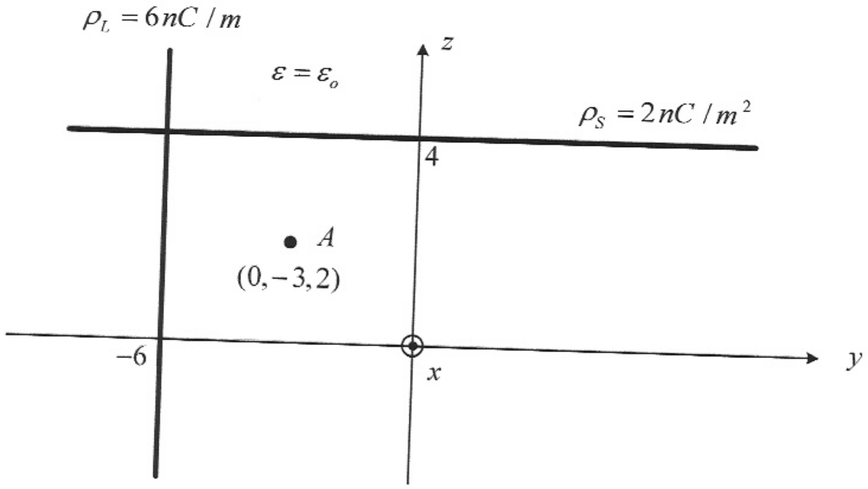
$$\therefore V_A = \int_4^9 \frac{7\times10^{-9}\, dx}{4\pi\epsilon_0\sqrt{x^2+25}}$$

$$= \frac{7\times10^{-9}}{(10^{-9}/9)} \int_4^9 \frac{dx}{\sqrt{x^2+25}} = 63\left[\ln\left(x+\sqrt{x^2+25}\right)\right]_4^9$$

$$= 63 \ln \frac{9+\sqrt{106}}{4+\sqrt{41}} = 38.922\ V$$

Problem 3 [25 points]

An *infinitely* long line with a uniform density $\rho_L = 6nC/m$ is placed at $x = 0, y = -6$ in free space. In addition, an infinitely large sheet of charge with a uniform density $\rho_S = 2nC/m^2$ is placed at $z = 4$. Calculate the resulting electric field $\vec{E}$ at point A, located at $(0,-3,2)$.



$$\vec{E}_s = \frac{\rho_s}{2\epsilon}\vec{a}_n \Rightarrow \vec{E}_s = \frac{2\times10^{-9}}{2\times\frac{10^{-9}}{36\pi}}(-\vec{a}_z) = -36\pi\vec{a}_z$$

$$\vec{E}_\ell = \frac{\rho_\ell}{2\pi\epsilon\rho}\vec{a}_\rho \Rightarrow \vec{E}_\ell = \frac{6\times10^{-9}}{2\pi\left(\frac{10^{-9}}{36\pi}\right)(3)}(\vec{a}_y) = 36\vec{a}_y$$

$$\therefore \vec{E} = \vec{E}_s + \vec{E}_\ell = 36\vec{a}_y - 36\pi\vec{a}_z \quad [V/m].$$

Problem 4 [25 points]

Consider the two semi-infinite media, medium 1 ($y > 0$) and medium 2 ($y < 0$). The permittivities of the two media are $\varepsilon_1 = 4\varepsilon_o$ and $\varepsilon_2 = 6\varepsilon_o$, respectively. The plane boundary ($y = 0$) that separates the two media is *charge-free*. The electric flux density in medium 1 is given by $\vec{D}_1 = 16\vec{a}_x + (2-y)\vec{a}_y - 8\vec{a}_z \quad [nC/m^2]$.

a) Find the electric flux density $\vec{D}_2$ *at the boundary.*

b) Calculate the potential difference V_{AB}, where points A and B are respectively located at $(1,+2,3)$ and $(1,+5,3)$.

a) $D_{2n} = D_{1n} = 2 - y \Big|_{y=0} = 2$

$$\vec{E}_{1t} = \vec{E}_{2t} \Rightarrow \frac{\vec{D}_{1t}}{\epsilon_1} = \frac{\vec{D}_{2t}}{\epsilon_2} \Rightarrow \vec{D}_{2t} = \frac{\epsilon_2}{\epsilon_1}\vec{D}_{1t}$$

$$\therefore \vec{D}_{2t} = \frac{6\epsilon_o}{4\epsilon_o}\left[16\vec{a}_x - 8\vec{a}_z\right] = 24\vec{a}_x - 12\vec{a}_z$$

$$\therefore \vec{D}_2 = 24\vec{a}_x + 2\vec{a}_y - 12\vec{a}_z \quad [nC/m^2]$$

b) $$\vec{E}_1 = \frac{\vec{D}_1}{\epsilon_1} = \frac{\left[16\vec{a}_x + (2-y)\vec{a}_y - 8\vec{a}_z\right]\times 10^{-9}}{4\epsilon_o}$$

$$= \frac{10^{-9}}{4\left(\frac{10^{-9}}{36\pi}\right)}\left[16\vec{a}_x + (2-y)\vec{a}_y - 8\vec{a}_z\right]$$

$$= 9\pi\left[16\vec{a}_x + (2-y)\vec{a}_y - 8\vec{a}_z\right]$$

$$V_{AB} = -\int_A^B \vec{E}_1 \cdot \vec{dl} = -\int_{y=2}^{5} E_{1y}\,dy = -9\pi\int_2^5 (2-y)\,dy$$

$$= 40.5\pi \text{ V} = 127.23 \text{ V}$$