KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

SECOND SEMESTER 2007/2008

EE 340 (04) MAJOR EXAM I

TIME: 11:00 -11:50 A.M.

DATE: MONDAY 31-MARCH-2008

LOCATION: IN CLASS

Student's Name:........................ KEY ....

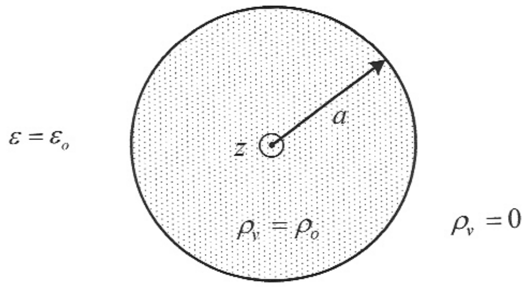
Student's I.D. Number: ....

	Maximum Score	**Score**
Problem 1	**25**	
Problem 2	**25**	
Problem 3	**25**	
Problem 4	**25**	
Total	**100**	

Problem 1 [25 points]

Consider the following volume charge density distribution, which exists in free space:

$$\rho_V = \begin{cases} \rho_o & \text{for} \quad \rho < a \\ 0 & \text{for} \quad \rho > a \end{cases}$$



Where ρ_o is some constant. Use Gauss's law to derive an expression for the resulting $\vec{E}$ field in the regions:

a) $\rho < a$

b) $\rho > a$

$$\oint \vec{D} \cdot \vec{ds} = Q$$

a) $$D \, 2\pi \rho L = \pi \rho^2 L \rho_o$$

$$\vec{E} = \frac{\rho_o \rho}{2 \epsilon_o} \vec{a}_\rho$$

b) $$D \, 2\pi \rho L = \pi a^2 L \rho_o$$

$$\vec{E} = \frac{\rho_o a^2}{2 \epsilon_o \rho} \vec{a}_\rho$$

Problem 2 [25 points]

The electrostatic field $\vec{E} = x^2\vec{a}_x + (1-y)\vec{a}_y + 2z\vec{a}_z$ [V/m] exists in free space. Calculate the corresponding:

a) Electrostatic potential difference V_{AB}, where the rectangular coordinates of points A and B are respectively given by (0, 0, 0) and (0, 10, 8).

b) Volume charge density at point P, whose rectangular coordinates are (2, 1, 7).

a) $$V_{AB} = -\int_A^B \vec{E} \cdot \vec{d\ell}$$

$$= -\int_0^{10} (1-y)\,dy - \int_0^{8} 2z\,dz$$

$$= 40 - 64 = -24 \text{ V}$$

b) $$\vec{D} = \epsilon_0 \vec{E} = \epsilon_0 \left[x^2 \vec{a_x} + (1-y)\vec{a_y} + 2z\vec{a_z} \right]$$

$$\rho_V = \nabla \vec{D}$$

$$= \epsilon_0 (2x+1)$$

$$\rho_V \Big|_P = 5\epsilon_0 = 44 \frac{\mu C}{m^3}$$

Problem 3 [25 points]

Consider two infinitely large sheets. Sheet # 1, which carries the uniform surface $12[nC/m^2]$, is placed in the $y-z$ plane. Sheet #2 carries the uniform surface $-18[nC/m^2]$ and placed in the $x-z$ plane. Assuming free space throughout, calculate the resulting $\vec{D}$ field at the observation point P, whose rectangular coordinates are (-3, 3, 3).

$$\vec{D} = \frac{\rho_s}{2}\vec{a}_n$$

$$\vec{D}_1 = \frac{12n}{2}(-\vec{a}_x) = -6 \times 10^{-9}\vec{a}_x$$

$$\vec{D}_2 = \frac{18n}{2}(-\vec{a}_y) = -9 \times 10^{-9}\vec{a}_y$$

$$\vec{D} = \vec{D}_1 + \vec{D}_2 = \left(-6\vec{a}_x - 9\vec{a}_y\right)\frac{nC}{m^2}$$

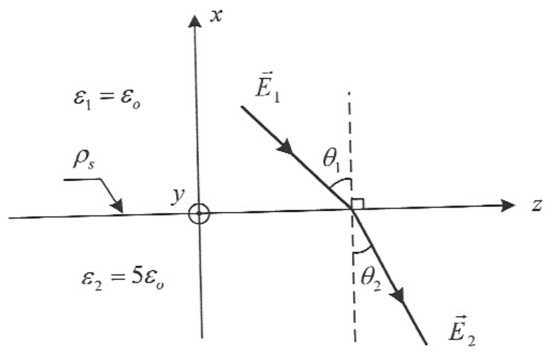
Problem 4 [25 points]

Consider the two semi-infinite media, medium 1 ($x > 0$) and medium 2 ($x < 0$). The permittivities of the two media are $\varepsilon_1 = \varepsilon_o$ and $\varepsilon_2 = 5\varepsilon_o$, respectively. The plane boundary ($x = 0$) contains a *uniform* surface charge with density $\rho_s = 60 \times 10^{-12} [C/m^2]$. The electrostatic field in medium 1 is given by $\vec{E}_1 = (x-6)\vec{a}_x + 10\vec{a}_z \quad [V/m]$. Find:

a) The electric field $\vec{E}_1$ (the electric field in medium 1) *at the boundary.*

b) The electric field $\vec{E}_2$ (the electric field in medium 2) *at the boundary.*

c) The angle θ_2 [i.e. the angle that $\vec{E}_2$ found in part a) makes with respect to the normal].



a) $\vec{E}_1 = -6\vec{a}_x + 10\vec{a}_z$ at the boundary.

b) $E_{1t} = 10 = E_{2t}$

$D_{1n} - D_{2n} = \rho_s \Rightarrow \epsilon_1 E_{1n} - \epsilon_2 E_{2n} = \rho_s$

$\epsilon_o E_{1n} - 5\epsilon_o E_{2n} = \rho_s \Rightarrow \epsilon_o(-6) - 5\epsilon_o E_{2n} = 60 \times 10^{-12}$

$E_{2n} = -2.557$

$\therefore \vec{E}_2 = -2.557\vec{a}_x + 10\vec{a}_z$

c) $\theta_2 = \tan^{-1}\left(\frac{10}{2.557}\right) = 75.66^\circ$.