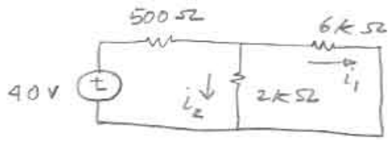
7.1

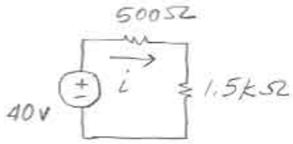
a) The switch remains closed for a long time before it was opened at $t = 0$. Therefore, the circuit is D.C. for $t < 0$.

$\therefore$ Inductor is replaced by a short circuit:



$(t < 0)$

$\Downarrow$



$$i = \frac{40}{2000} = 20\ mA$$

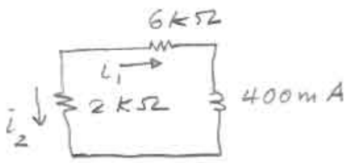
Using CDR in the 1st circuit:

$$i_1 = \frac{2}{8} \times i = \frac{1}{4} \times 20mA = 5\ mA\ ,\ t < 0$$

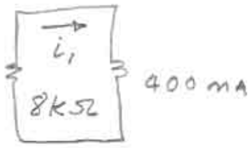
$$i_2 = \frac{6}{8} \times i = \frac{3}{4} \times 20mA = 15mA\ ,\ t < 0$$

$$\therefore\ i_1(0^-) = 5\ mA \neq i_2(0^-) = 15mA$$

b) For $t > 0$ $\Rightarrow$



$\therefore$



$i_1(0^-) = i_1(0^+) = 5mA$ (because current through an inductor is continuous)

$$\therefore\ i_2(0^+) = -i_1(0^+) = -5mA$$

c) $i_1(t) = i_1(0)\, e^{-t/\tau}$, $t \geq 0$

$i_1(0) = 5\ mA$, $\tau = \frac{L}{R} = \frac{0.4}{8000} = 0.05 \times 10^{-3}$ s

$\therefore i_1(t) = 5\, e^{-t/5\times10^{-5}}\ mA = 5\, e^{-20000t}\ mA$, $t \geq 0$

d) $i_2(t) = -i_1(t) = -5\, e^{-20000t}\ mA$, $t > 0$

e) The current through a resistor can be discontinuous (i.e. it can change instantaneously). But the current through the inductor cannot change instantaneously (i.e. it is always continuous).

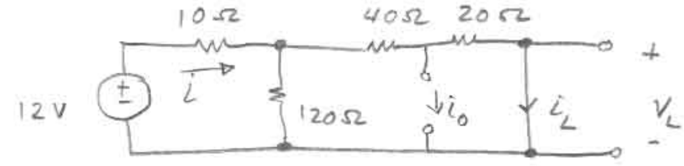
The switching results in $i_1(0^-) = i_1(0^+)$,

and $i_2(0^+) = -i_1(0^+) \Rightarrow i_2(0^+) = -i_1(0^-)$

$\neq i_2(0^-)$

7.3

a) For $t < 0$, the circuit is D.C. because the switch remains open for a long time. Thus,

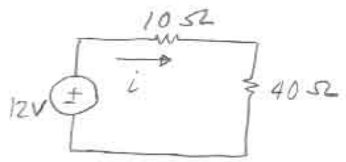


$t < 0$

$i_o(t) = 0$ for $t < 0$.

$\therefore i_o(0^-) = 0$

b)



$(t < 0)$

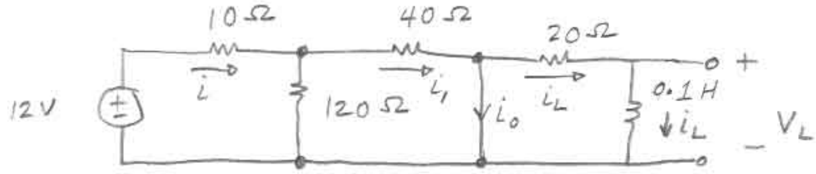
$$i(t) = \frac{12}{50} = 240\,mA, \quad t < 0$$

$$\therefore i_L(t) = \frac{120}{180} \times 240\,mA \quad \text{(CDR is used)}$$

$$= 160\,mA, \quad t < 0$$

$$\therefore i_L(0^-) = 160\,mA$$

c) For $t > 0$



$(t > 0)$

$40 // 120\Omega \Rightarrow 30\Omega$

$\therefore \; i(t) = \frac{12}{10+30} = \frac{12}{40} = 0.3\text{ A}, \; t>0$

CDR $\Rightarrow i_1(t) = \frac{120}{160} \times 0.3 = \frac{3}{4} \times 0.3 = 0.225\text{ A}, \; t>0.$

$\therefore \; i_1(0^+) = 0.225\text{ A}$

$i_0(0^+) = i_1(0^+) - i_L(0^+)$

$= i_1(0^+) - i_L(0^-)$

$= 225\text{ mA} - 160\text{ mA} = 65\text{ mA}$

d) $i_L(0^+) = i_L(0^-) = 160\text{ mA}$

e) $i_L(t) = i_L(0)\, e^{-t/\tau}$, $\quad \tau = \frac{L}{R} = \frac{0.1}{20} = 5\text{ ms}$

$i_L(0) = 160\text{ mA}$

$\therefore \; i_L(t) = 160\, e^{-t/5\times10^{-3}}\text{ mA} = 160\, e^{-200t}\text{ mA}, \; t \geq 0$

$i_o(t) = i_1(t) - i_L(t) = (225 - 160\, e^{-200t})\text{ mA}, \; t>0.$

$\therefore \; i_o(\infty) = 225\text{ mA}$

[or since $i_L(\infty) = 0$

$\therefore \; i_o(\infty) = i_1(\infty) - i_L(\infty) = 225\text{ mA} - 0 = 225\text{ mA}$]

f) $i_L(\infty) = 0$

g) $i_L(t) = 160\, e^{-200t}\ mA \quad , t \geq 0$

h) $v_L(0^-) = 0.$

i) $v_L(t) = L \dfrac{di_L}{dt} = 0.1 \dfrac{d}{dt}\left[160\ e^{-200t}\ mA\right], t > 0$

$$= 0.1(-200)(160)\ e^{-200t}\ mV$$

$$= -3200\ e^{-200t}\ mV$$

$$= -3.2\ e^{-200t}\ V \quad , t > 0$$

$\therefore\ v_L(0^+) = -3.2\ V$

$$\left[\begin{aligned} \text{or} : \quad V_L(0^+) &= -20\, i_L(0^+) \\ &= -20(160m) = -3.2V \end{aligned}\right]$$

j) $V_L(\infty) = 0$, because $i_L(t)$ becomes zero (i.e. constant current) as $t \to \infty$.

You can also make the same conclusion from the voltage expression in part i).

k) $v_L(t) = -3.2\ e^{-200t}\ V, \ t > 0.$ (see part i)

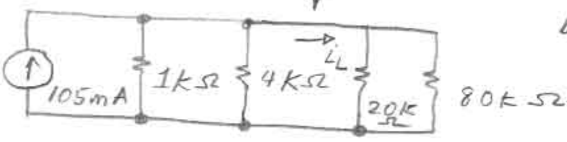
l) $i_1(t) = (225 - 160\, e^{-200t})\ mA, \ t \geq 0$ (see part e).

7.18

a) The two swiches remain closed for a long time before they are opened at $t = 0$.

$\therefore$ The circuit is D.C. for $t < 0$

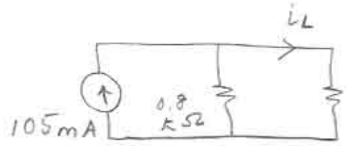
inductor is replaced by a short



$t < 0$

$1k\Omega \,//\, 4k\Omega \Rightarrow 0.8\,k\Omega$

$20k\Omega \,//\, 80k\Omega \Rightarrow 16\,k\Omega$



$\Rightarrow i_L(t) = \dfrac{0.8}{16.8} \times 105 \text{ mA}$

$t < 0$

$= 5 \text{ mA}$

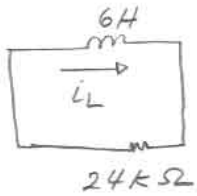
$\therefore i_L(0^-) = 5 \text{ mA}$

The initial energy stored in the 6 H inductor is $W_L(0^-) = \frac{1}{2} L\, i_L^2(0^-) = \frac{1}{2} \times 6 \times (5 \times 10^{-3})^2$

$= 75 \mu J$

For $t > 0$ $\Rightarrow$

$t > 0$



$(t > 0)$

$$i_L(t) = i_L(0)\, e^{-t/\tau} \quad , \quad \tau = \frac{L}{R} = \frac{6}{24000} = 0.25\,ms$$

$$\therefore \frac{1}{\tau} = 4000\ s^{-1}$$

$$i_L(0) = 5\ mA$$

$$\therefore\ i_L(t) = 5\, e^{-4000t}\ mA \ , \quad t \geq 0$$

The current through the $4k\Omega$ resistor is

$$i_R(t) = i_L(t) = 5\, e^{-4000t}\ mA \ , \quad t \geq 0.$$

$$P_{4k\Omega}(t) = i_R^2(t)\, R = \left(5 \times 10^{-3} e^{-4000t}\right)^2 \times 4000$$

$$= 25 \times 10^{-6} \times 4000\, e^{-8000t}$$

$$= 100\, e^{-8000t}\ mW \ , \quad t \geq 0$$

$\therefore$ Energy dissipated in the $4k\Omega$ resistor after $t = 0$ is:

$$W_{4k\Omega}(t) = \int_0^t P_{4k\Omega}(t)\, dt = \int_0^t 0.1\, e^{-8000t}\, dt$$

$$= \frac{0.1\, e^{-8000t}}{-8000}\Bigg|_0^t = \frac{0.1}{8000}\left(1 - e^{-8000t}\right)$$

$= 12.5\left(1 - e^{-8000t}\right) \mu J$

To find the instant of time t at which

$W_{4k\Omega} = 0.1 \times W_L(0^-) \Rightarrow$

$12.5\left(1 - e^{-8000t}\right) = 0.1 \times 75$

$1 - e^{-8000t} = 0.6 \Rightarrow e^{-8000t} = 0.4$

$-8000t = \ln 0.4 \Rightarrow t = \dfrac{\ln 0.4}{-8000} = 114.54 \mu s$

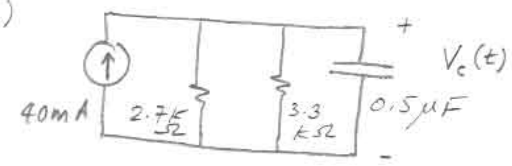
b) At $t = 114.54 \mu s$, the energy dissipated by the $4k\Omega$ & $20k\Omega$ resistors is 6 times larger than the energy dissipated by the $4k\Omega$ resistor.

$\therefore$ % of energy is 60%.

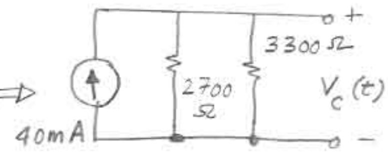
This conclusion is valid be cause the same current passes through the $4k\Omega$ & $20k\Omega$ resistors.

7.22

a)



$t < 0$
(Circuit is D.C.)

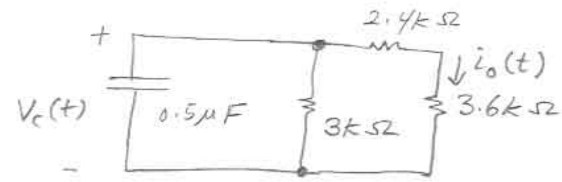


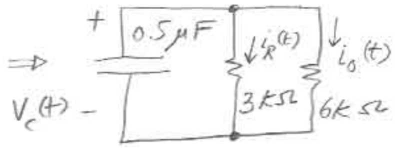
$(t < 0)$

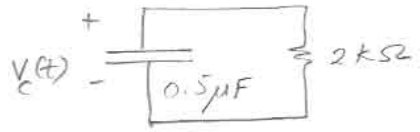
$$V_c(t) = (40\,m) \times (2700 // 3300)$$

$$= 40\,m \times 1485 = 59.4\ V, \quad t < 0$$

For $t > 0$







$$\therefore\ v_c(t) = v_c(0)\, e^{-t/\tau}$$

$$\tau = RC = 0.5\times10^{-6}\,(2000)$$
$$= 10^{-3}\ s$$

$$v_c(0^-) = 59.4 = v_c(0)$$

$$\therefore\ v_c(t) = 59.4\, e^{-1000t}, \quad t \geq 0.$$

$$\therefore\ i_o(t) = \frac{v_c(t)}{6000} = \frac{59.4\, e^{-1000t}}{6000} = 9.9\, e^{-1000t}\ mA, \quad t \geq 0$$

b) $i_R(t) = \dfrac{v_c(t)}{3000} = \dfrac{59.4\,e^{-1000t}}{3000}$

$= 19.8\,e^{-1000t}$ mA , $t \geq 0$.

$\therefore P_{3k}(t) = i_R^2(t)\,R = \left(19.8\times10^{-3}e^{-1000t}\right)^2 \times 3000$

$= 1.176\,e^{-2000t}$ W

$w_{3k\Omega}(t) = \displaystyle\int_0^t p_{3k\Omega}(t)\,dt = \int_0^t 1.176\,e^{-2000t}\,dt$

$= \dfrac{1.176\,e^{-2000t}}{-2000}\Big|_0^t = \dfrac{1.176}{2000}\left(1-e^{-2000t}\right)$

$\therefore w_{3k\Omega}(500\mu s) = \dfrac{1.176}{2000}\left(1-e^{-2000\times500\times10^{-6}}\right)$

$= 371.7\ \mu J$

Initial energy stored in the capacitor:

$W_c(0) = \frac{1}{2}C\,V_c^2(0) = \frac{1}{2}(500\times10^{-6})(59.4)^2$

$= 882.1\ \mu J$

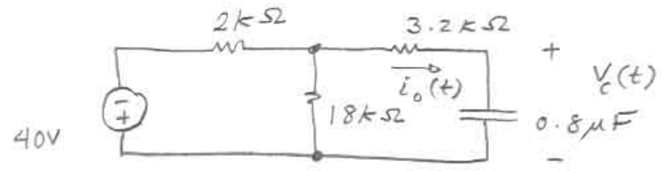
% of energy dissipated after 500μs =

$\dfrac{371.7\,\mu J}{882.1\,\mu J}\times 100\,\% = 42.1\,\%$

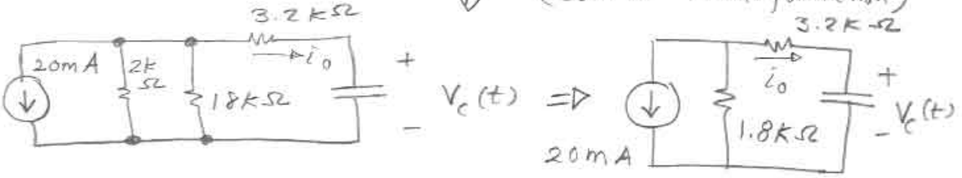
7.53

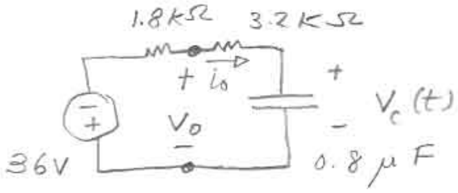
For $t < 0$, the voltage accross C is zero (why?).

For $t > 0$



⇓ (Source Transformation)





$$\therefore\ i_o(0^+) = \frac{-36 - V_C(0^+)}{5K} = \frac{-36-0}{5000} = -7.2\,mA$$

b) $i_o(\infty) = 0$. (Capacitor behaves as an open circuit as $t \to \infty$).

c) $\tau = RC = 5000 \times 0.8 \times 10^{-6} = 4\,ms$

d) $v_c(t) = (-36)(1 - e^{-t/\tau}) = -36(1 - e^{-250t}),\ t \geq 0$

$$\therefore\ i_o(t) = \frac{-36 - v_c(t)}{5000} = \frac{-36\,e^{-250t}}{5000}$$

$$= -7.2\,e^{-250t}\ mA,\ t \geq 0$$

e) $36 + 1800\,i_o + V_o = 0 \Rightarrow v_o(t) = -36 + 12.96\,e^{-250t},\ t \geq 0$