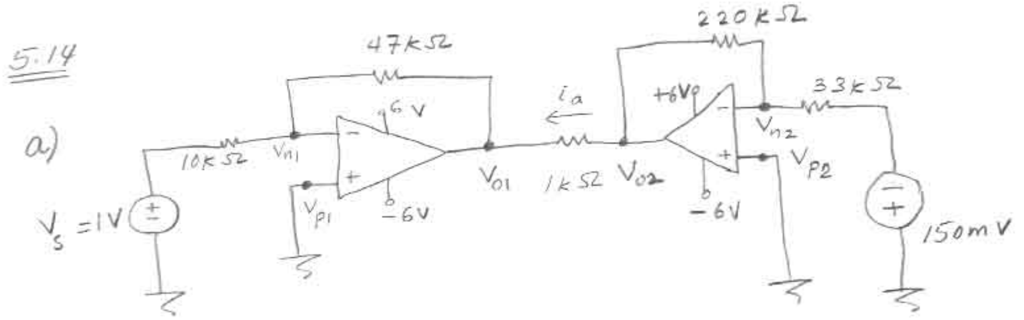
5.14

a)



$V_{p1} = 0 \Rightarrow V_{n1} = 0$,

KCL at $V_{n1} \Rightarrow \dfrac{0-1}{10^4} + \dfrac{0 - V_{01}}{47\times10^3} = 0 \Rightarrow V_{01} = -4.7V$

$V_{p2} = 0 \Rightarrow V_{n2} = 0$,

KCL at $V_{n2} \Rightarrow \dfrac{0 + 150\times10^{-3}}{33\times10^3} + \dfrac{0 - V_{02}}{220\times10^3} = 0 \Rightarrow V_{02} = 1\ V$

$$i_a = \frac{V_{02} - V_{01}}{1K} = \frac{1+4.7}{1000} = 5.7mA$$

b) For $i_a = 0 \Rightarrow V_{01} = V_{02} = 1$, thus we need to adjust V_s until $V_{01} = 1\ V$.

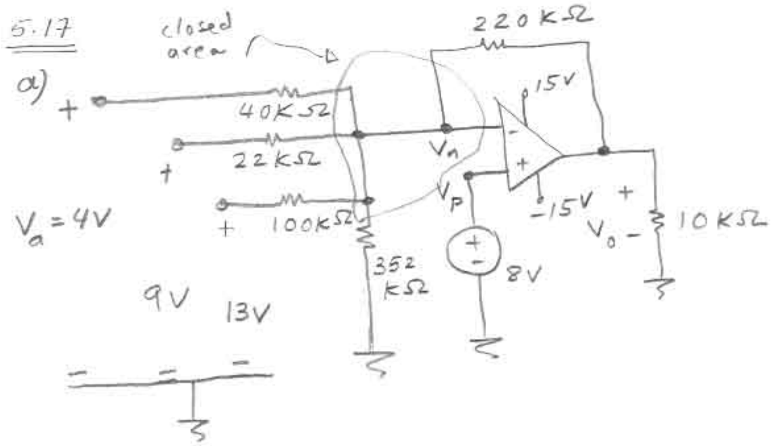
$$\frac{0 - V_s}{10000} + \frac{0-1}{47000} = 0 \quad (\text{KCL at } V_{n1})$$

$\Downarrow$

$$V_s = -0.213\ V$$

5.17

a)



$V_p = 8V \Rightarrow V_n = 8V$

KCL around closed area $\Rightarrow$

$$\frac{8-V_o}{220K} + \frac{8-0}{352K} + \frac{8-13}{100K} + \frac{8-9}{22K} + \frac{8-4}{40k} = 0$$

$$\frac{8-V_o}{220} + \frac{8}{352} - \frac{5}{100} - \frac{1}{22} + \frac{4}{40} = 0$$

$$\frac{V_o-8}{220} = 2.7273 \times 10^{-2} \Rightarrow V_o = 14V$$

b)

$$\frac{8-V_o}{220K} + \frac{8-0}{352K} + \frac{8-13}{100K} + \frac{8-9}{22K} + \frac{8-V_a}{40K} = 0$$

$$\frac{8-V_o}{220} + \frac{8}{352} - \frac{5}{100} - \frac{1}{22} + \frac{8-V_a}{40} = 0$$

$$\frac{8-V_o}{220} - 7.2727 \times 10^{-2} + \frac{8-V_a}{40} = 0$$

$$V_o = 220\left(-7.2727 \times 10^{-2} + \frac{8-V_a}{40}\right) + 8$$

$$V_o = -16 + 44 - 5.5V_a + 8 = 36 - 5.5V_a$$

$$-15 \le V_o \le 15$$ (condition for the Op Amp. to operate in the linear region).

$$\therefore \quad -15 \le 36 - 5.5V_a \le 15$$

$$-51 \le -5.5V_a \le -21$$

$$\frac{-51}{-5.5} \ge V_a \ge \frac{-21}{-5.5}$$

$$\therefore \quad 9.273 \ge V_a \ge 3.818$$

<u>6.3</u>

$$v_L(t) = \begin{cases} 3e^{-4t} \text{ mV}, & 0 \le t \le 2 \\ -3e^{-4(t-2)} \text{ mV}, & 2 < t < \infty \end{cases}$$

$$i_L(t) = \frac{1}{L}\int_{t_0}^{t} v_L(t)\,dt + i_L(t_0) \qquad (1)$$

we know that $i_L(0) = 0$ (given).

$$\therefore \quad i_L(t) = \frac{1}{L}\int_0^t v_L(t) + i_L(0) = \frac{1}{L}\int_0^t v_L(t)\,dt \qquad (2)$$

For $0 \le t \le 2 \Rightarrow$

$$i_L(t) = \frac{1}{2.5\times 10^{-3}}\int_0^t 3e^{-4t}\times 10^{-3}\,dt$$

$$= 400(3)(10^{-3})\frac{e^{-4t}}{-4}\Big|_0^t = -0.3\,e^{-4t}\Big|_0^t$$ (4)

$$= -0.3\left[e^{-4t}-1\right] = 0.3 - 0.3\,e^{-4t} \quad A \qquad (3)$$

From (3) $\Rightarrow i_L(2) = 0.3 - 0.3e^{-8} = 0.3\ A$

Using (1) $\Rightarrow$

$$i_L(t) = \frac{1}{L}\int_2^t v_L(t) + i_L(2) \quad , \text{ for } t>2$$

$$= \frac{1}{2.5\times10^{-3}}\int_2^t \left(-3e^{-4(t-2)}\right)\times10^{-3}\,dt + 0.3$$

$$= 400(-3\times10^{-3})\int_2^t e^{-4(t-2)}\,dt + 0.3$$

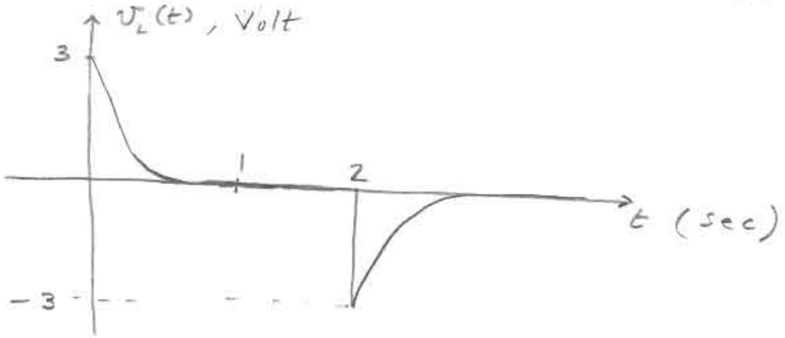
$$= -1.2\,\frac{e^{-4(t-2)}}{-4}\Big|_2^t + 0.3$$

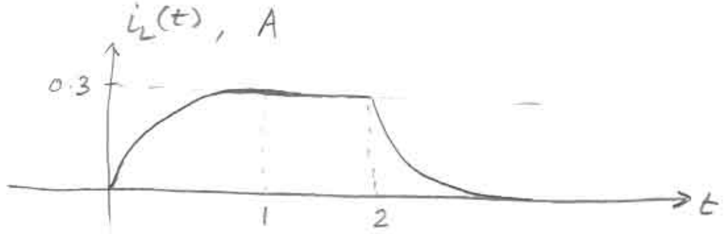
$$= \frac{1.2}{4}\left[e^{-4(t-2)} - e^{-4(0)}\right] + 0.3$$

$$= 0.3\left[e^{-4(t-2)} - 1\right] + 0.3 = 0.3\,e^{-4(t-2)}$$
(for $t>2$)

$$\therefore\ i_L(t) = \begin{cases} 0.3-0.3e^{-4t} & , \ 0\le t\le 2 \\ 0.3\,e^{-4(t-2)} & , \ t>2 \end{cases}$$

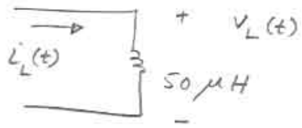
The Sketches of $v_L(t)$ & $i_L(t)$ are shown below:





<u>6.4</u>

$i_L(t) = 18t\,e^{-10t}, \quad t \geq 0$



a) $V_L(t) = L \dfrac{d i_L(t)}{dt} = 50\times10^{-6} \dfrac{d}{dt}\left[18t\,e^{-10t}\right]$

$= 50\times10^{-6}\left[18\,e^{-10t} - 180t\,e^{-10t}\right]$

$= 900\times10^{-6}\,e^{-10t}\left[1 - 10t\right], \quad t > 0$

b) $P_L = i_L v_L = \left(18t\,e^{-10t}\right)\left(0.9\times10^{-3}\,e^{-10t}\right)(1-10t)$

$P_L(200\times10^{-3}) = 18(200\times10^{-3})\,e^{-2}\,(0.9\times10^{-3})(e^{-2})(1-2)$
$= -59.34\ \mu W$

c) Actually delivering power at $t = 200\,ms$.

d) $i_L(200\,ms) = 18(0.2)\,e^{-2} = 0.48721\ A$

$$W(t) = \frac{1}{2} L\, i_L^2(t)$$

$$W(200\,ms) = \frac{1}{2}(50\times10^{-6})(0.48721)^2 = 5.93\ \mu J$$

e) $W(t) = \frac{1}{2} L\, i_L^2(t)$

$$= \frac{1}{2}(50\times10^{-6})(18t\,e^{-10t})^2$$

$$= 25\times10^{-6}(18)^2 t^2 e^{-20t}$$

$$= 8.1\times10^{-3} t^2 e^{-20t} \quad (1)$$

Maximum energy occurs when $W'(t) = 0$

$$\therefore \frac{dW}{dt} = 0 \Rightarrow 8.1\times10^{-3}\,2t\,e^{-20t} + 8.1\times10^{-3} t^2(-20)e^{-20t} = 0$$

$$\therefore 2t - 20t^2 = 0 \Rightarrow t = 0 \text{ or } t = \frac{1}{10}$$

$t = 0$ (rejected because the energy is minimum at $t = 0$).

$\therefore t = \frac{1}{10} = 100\,ms$ is the time instant at which the energy stored in L is maximum.

subtituting in (1) $\Rightarrow W(0.1) = W_{max}$

$$= 8.1\times10^{-3}(0.1)^2 e^{-2}$$

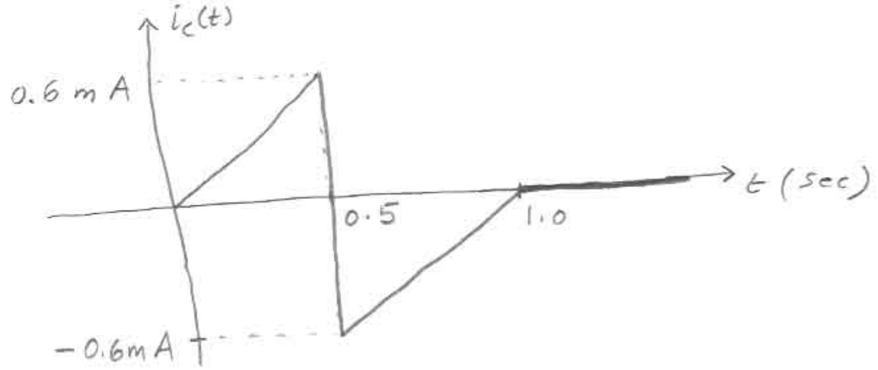
$$= 10.962\ \mu J$$

<u>6.14</u>

$$i_c(t) = c\,\frac{d\,v_c(t)}{dt} = 20\times10^{-6}\,\frac{d}{dt}\begin{cases}30t^2\\30(t-1)^2\\0\end{cases}$$

$$= 20\times10^{-6}\begin{cases}60t\\60(t-1)\\0\end{cases}$$

$$i_c(t) = \begin{cases}1200\times10^{-6}\,t & ,\ 0\le t\le 0.5\\1200\times10^{-6}(t-1) & ,\ 0.5\le t\le 1.0\\0 & ,\ \text{elsewhere}\end{cases}$$



6.18

a) $v_c(t) = \frac{1}{C} \int_{t_0}^{t} i_c(t)\,dt + v_c(t_0)$, for $t \geqslant t_0$

since $v_c(0) = -20 \Rightarrow$

$$v_c(t) = \frac{1}{C} \int_0^t i_c(t)\,dt + v_c(0) = \frac{1}{C}\int_0^t i_c(t)\,dt - 20$$

$$= \frac{1}{0.5\times 10^{-6}} \int_0^t 50\, e^{-2000t} \times 10^{-3}\,dt - 20 , \quad t \geqslant 0$$

$$= 2\times 10^{6} \times 10^{-3} \times 50 \left. \frac{e^{-2000t}}{-2000} \right|_0^t - 20$$

$$= -50\left(e^{-2000t} - 1\right) - 20 = -50\,e^{-2000t} + 30$$

$\therefore\ v_c(t) = 30 - 50\,e^{-2000t}$, $t \geqslant 0$.

$v_c(500\times 10^{-6}) = 30 - 50\,e^{-2000\times 500\times 10^{-6}} = 11.606\,V$

$w_c(t) = \frac{1}{2} C\, V_c^2(t) \Rightarrow$

$W_c(500\times 10^{-6}) = \frac{1}{2}(0.5\times 10^{-6})(11.606)^2$

$= 33.68\,\mu J$

b) $v_c(\infty) = 30 - 50\,e^{-\infty} = 30\,V$

$W_c(\infty) = \frac{1}{2} C\, v_c^2(\infty) = \frac{1}{2}(0.5\times 10^{-6})(30)^2$

$= 225\,\mu J$