

1.12

HW #1

1/5

a) $P_A = -iV = -15 \times 20 = -300 \text{ W}$

$P_B = iV = 15 \times 20 = +300 \text{ W}$

A generates 300W, which is consumed by B.

∴ Power flow is from A to B.

b) $P_A = -iV = -(-5)(100) = +500 \text{ W}$, $P_B = iV = 5 \times 100 = -500 \text{ W}$

∴ 500W is generated by B and consumed by A.

∴ Power flow is from B to A.

1.15

a) $P_A = iV = (-40)(12) = -480 \text{ W}$

$P_B = -iV = -(-40)(12) = +480 \text{ W}$

Power flows from A to B.

∴ B has the dead battery.

b) $W = PT = 480 \text{ W} \times (1.5 \times 60 \text{ sec}) = 480 \times 1.5 \times 60 \frac{\text{J}}{\text{s}}$
 $= 43,200 \text{ J} = 43.2 \text{ kJ}$

1.17

a) $P = iV = (50e^{-1600t} - 50e^{-400t})(5e^{1600t} - 5e^{-400t})10^{-3}$
 $= 50 \times 5 \times 10^{-3} (e^{-1600t} - e^{-400t})^2$
 $P(625 \mu\text{s}) = P(625 \times 10^{-6}) = 250 \times 10^{-3} \begin{bmatrix} e^{-1600 \times 625 \times 10^{-6}} & -e^{-400 \times 625 \times 10^{-6}} \\ e^{-1600 \times 625 \times 10^{-6}} & -e^{-400 \times 625 \times 10^{-6}} \end{bmatrix}$
 $= 250 \times 10^{-3} \begin{bmatrix} -1 & -0.25 \\ -1 & -0.25 \end{bmatrix}^2 = 250 \times 10^{-3} [0.3679 - 0.7788]^2$
 $= 4.22 \times 10^{-2} \text{ W} = 42.2 \text{ mW}$

$$\begin{aligned}
 a) P(t) &= i(t) v(t) = (20 - 20 e^{-500t}) \times 10^{-3} \times (100 e^{-500t}), \quad t \geq 0 \\
 &= 2000 \times 10^{-3} (e^{-500t} - e^{-1000t}), \quad t \geq 0 \\
 &= 2 (e^{-500t} - e^{-1000t}) \textcircled{1}, \quad t \geq 0
 \end{aligned}$$

To find the maximum power, we first find the time t at which $\frac{dP}{dt} = 0$.

$$\frac{dP}{dt} = 2 (-500 e^{-500t} + 1000 e^{-1000t}) = 0$$

$$\therefore 1000 e^{-1000t} = 500 e^{-500t} \quad (\text{Divide by } e^{-500t})$$

$$\therefore 2 e^{-500t} = 1 \Rightarrow e^{-500t} = \frac{1}{2} \Rightarrow -500t = \ln \frac{1}{2}$$

$$t = \frac{\ln 0.5}{-500} = 1.3863 \times 10^{-3} \text{ s}$$

The power delivered to the circuit is maximum at

$$t = 1.3863 \text{ ms.}$$

To find the maximum power, we substitute

$$t = 1.3863 \text{ ms} \quad \text{in eqn. 1} \Rightarrow$$

$$\begin{aligned}
 P|_{\max} &= P(1.3863 \times 10^{-3}) = 2 \left(e^{-500 \times 1.3863 \times 10^{-3}} - e^{-1000 \times 1.3863 \times 10^{-3}} \right) \\
 &= 2 \left(e^{-0.6932} - e^{-1.3863} \right) = 2(0.5 - 0.25) \\
 &= 0.5 \text{ W}
 \end{aligned}$$

b) Total energy W delivered to the element is 3/5
given by:

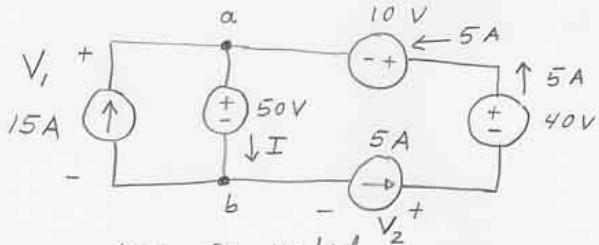
$$W = \int_0^\infty P(t) dt, \text{ because the power is zero for } t < 0.$$

$$= \int_0^\infty 2 (e^{-500t} - e^{-1000t}) dt$$

$$= 2 \left[\frac{e^{-500t}}{-500} - \frac{e^{-1000t}}{-1000} \right]_0^\infty$$

$$= 2 \left[\frac{1}{500} - \frac{1}{1000} \right] = \frac{1}{500} = 2 \text{ mJ}$$

2.8



The interconnection is valid.

Apply KCL at node $b \Rightarrow I = 15 + 5 = 20 \text{ A}$

By KVL around the outer circuit \Rightarrow

$$-V_1 - 10 + 40 + V_2 = 0, \text{ but } V_1 = 50 \text{ V}$$

$$\therefore -50 - 10 + 40 + V_2 = 0 \Rightarrow V_2 = 20 \text{ V}$$

$$P_{15A} = -iV = -15 \times 50 = -750 \text{ W (generated)}$$

$$P_{50V} = IV = 20 \times 50 = 1000 W \text{ (dissipated)}$$

4/5

$$P_{10V} = IV = 5 \times 10 = 50 W \text{ ()}$$

$$P_{40V} = -IV = -5 \times 40 = -200 W \text{ (generated)}$$

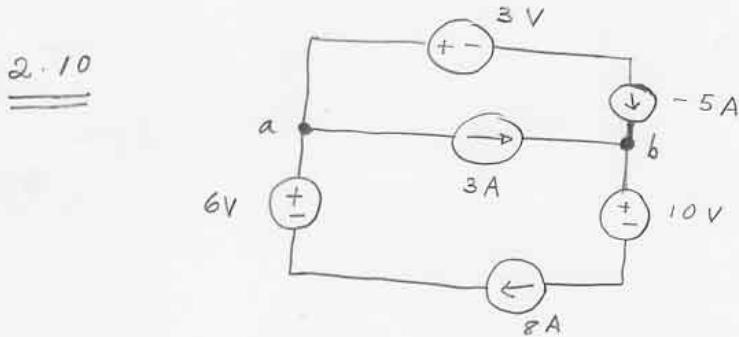
$$P_{5A} = -IV = -5 \times 20 = -100 W \text{ (generated)}$$

$$\text{Total power generated} = 750 + 200 + 100 = 1050 W.$$

$$\text{" " dissipated} = 1000 + 50 = 1050 W.$$

$$\therefore \text{Total power generated} = \text{Total power dissipated}$$

$$= \text{total power developed in the circuit} = 1050 W.$$



KCL is not satisfied at terminal a,

because:

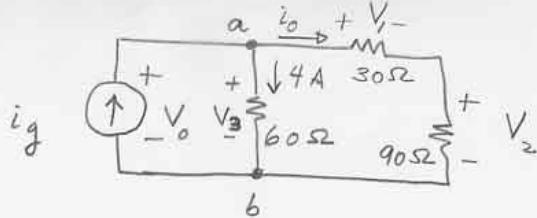
$$\left. \begin{aligned} \sum i_{\text{entering}} &= 8 \\ \sum i_{\text{leaving}} &= 3 + (-5) = -2 \end{aligned} \right\} \begin{aligned} \sum i_{\text{entering}} &\neq \\ \sum i_{\text{leaving}} & \end{aligned}$$

\therefore Interconnection is invalid.

2.16

5/5

a)



$$\text{Ohm's law} \Rightarrow V_3 = +IV = 4 \times 60 = 240V$$

$$KVL \Rightarrow -V_3 + V_1 + V_2 = 0 \Rightarrow -240 + V_1 + V_2 = 0$$

$$\text{Ohm's law} \Rightarrow -240 + 30i_o + 90i_o = 0$$

$$\therefore 120i_o = 240 \Rightarrow i_o = \frac{240}{120} = 2A$$

$$KCL \text{ at node } a \Rightarrow i_g = 4 + i_o = 4 + 2 = 6A.$$

b) $V_o = V_3 = 240V$

c) $P_{i_g} = -i_g V_o = -6 \times 240 = -1440W$ (generated).

$$P_{60\Omega} = i^2 R = (4)^2 \times 60 = 960W$$

$$P_{30\Omega} = i^2 R = (2)^2 \times 30 = 120W$$

$$P_{90\Omega} = i^2 R = (2)^2 \times 90 = 360W$$

$$\therefore \sum P_{\text{gen}} = 1440W$$

$$\sum P_{\text{dis}} = 960 + 120 + 360 = 1440W$$

$$\therefore \sum P_{\text{gen}} = \sum P_{\text{dis}} = 1440W$$