

Enhanced Equalization in OFDM Systems Using Cyclic Prefix

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Abstract—Orthogonal frequency division multiplexing (OFDM) is a modulation technique that provides high achievable data rates in wireless environment by combating multipath effects and has relatively simple implementation. In this paper, it is demonstrated that how the cyclic prefix can be used to enhance the operation of an OFDM equalizer when the channel is perfectly known at the receiver or is obtained through training. This method relies on transforming the OFDM channel into two subchannels (circular and linear) that share the same input and are characterized by the same channel parameters. Conventionally, data is recovered by utilizing only the circular subchannel while in this method, both the circular and linear subchannels are used to recover the data. The improved performance of the proposed method is demonstrated with the help of simulations for constant modulus as well as non-constant modulus data.

Keywords—OFDM, Pilot based channel estimation, Equalization, Least squares.

I. INTRODUCTION

OFDM has emerged as a modulation scheme that can achieve high data rates by efficiently combating multipath effects. This is achieved by dividing the frequency selective fading channel into parallel frequency-flat channels. The additional advantages of simple receiver implementation and high spectral efficiency due to orthogonality of the carriers, contribute towards the increasing interest in OFDM. Due to the above mentioned advantages, many standards considered and employed OFDM, including those for digital audio and video broadcasting (DAB and DVB), high speed modems over digital subscriber lines, local area wireless broadband standards such as the HIPER-LAN/2 and WiFi (IEEE 802.11a) [1], WIMAX (IEEE 802.16), and, ultrawideband personal area networks (IEEE 802.15.3a) [2].

In order to achieve high data rate in OFDM, receivers must estimate the channel efficiently and subsequently the data. Many techniques have been presented by the researchers for channel estimation in OFDM systems. They can be broadly divided into three categories:

- 1) The training-based techniques involve sending pilots (symbols which are known to the receiver) with the data symbols so that the channel can be estimated and hence the data at the receiver [3], [4], [5].
- 2) Blind techniques only utilize some inherent structure of the communication system which is produced by constraints including (but not limited to) finite alphabet constraint [6], [7], cyclic prefix [7] - [10], linear precoding [11], [12], and virtual carriers [13].

- 3) Semi-blind techniques use pilots as well as the constraints of the communication system for channel estimation [14], [15], [16].

A. Approach and Contribution of the Paper

The method presented in this paper is based on the fact that the presence of cyclic prefix at the input introduces redundancy in the data and enables us to divide the OFDM channel into two parallel subchannels: circular and linear. The circular subchannel relates the input and output OFDM symbols in frequency domain and is free of intersymbol interference (ISI) while the linear subchannel carries all the burden of ISI as it relates the input and output cyclic prefixes. This decomposition of OFDM channel into linear and circular subchannels was exploited by the authors in [7] for blind and semiblind channel estimation and data detection in OFDM. On the contrary, in this paper it is demonstrated that how the cyclic prefix can be used to enhance the operation of the receiver when operating in training or perfectly known channel modes. Specifically, the cyclic prefix observation enhances the BER performance especially when the channel exhibits zeros on the FFT grid.

B. Paper Organization and Notation

This paper is organized as follows. After introducing the notation for variables and operations used in the paper in Table I and II respectively, a careful study is performed in Section II of the elements of an OFDM transmission decomposing it into a circular subchannel described in the frequency domain and a linear subchannel described in the time domain. In Section III, the conventional equalization used in OFDM systems and its disadvantages are described. The enhanced equalization technique using the cyclic prefix which is the main focus of this paper is presented in Section IV. Section V presents the simulation results followed by the conclusion in Section VI.

II. SYSTEM OVERVIEW

In an OFDM system, data is transmitted in symbols \mathcal{X}_i of length N each. The symbol undergoes an Inverse Discrete Fourier Transform (IDFT) operation to produce the time domain symbol \mathbf{x}_i , i.e.

$$\mathbf{x}_i = \sqrt{N}Q\mathcal{X}_i \quad (1)$$

where Q is the $N \times N$ IDFT matrix. When juxtaposed, these symbols result in the sequence $\{\mathbf{x}_k\}$. We assume a channel \underline{h} of maximum length $L + 1$. To avoid ISI caused by passing through the channel, a cyclic prefix \underline{x}_i (of length L) is appended to \mathbf{x}_i , resulting in super-symbol $\bar{\mathbf{x}}_i$ as defined

TABLE I
NOTATION USED IN THE PAPER FOR VARIABLES

Variable	Notation employed
Scalars	Small-case letters (e.g. x)
Vectors	Small-case boldface letters (e.g. \mathbf{x})
Matrices	Upper-case boldface letters (e.g. \mathbf{Q})
Vectors in frequency domain	Calligraphic notation (e.g. \mathcal{H})
Individual entries of a vector	$h(l)$
Estimate of a variable	Hat over the variable (e.g. $\hat{\mathbf{x}}$)
Variables as function of time	Time index appears as a subscript (e.g. \mathbf{x}_i)
Cyclic prefix	Underlined vector (e.g. $\underline{\mathbf{x}}_i$)
Super symbol	Overlined vector (e.g. $\overline{\mathbf{x}}_i = [\underline{\mathbf{x}}_i^T \mathbf{x}_i^T]^T$)

TABLE II
NOTATION USED IN THE PAPER FOR OPERATIONS

Operation	Notation employed
Transpose	T
Conjugate transpose / Hermitian	H
Convolution	$*$
Circular convolution	\circledast
Element by element multiplication	\odot

in Table I. The concatenation of these symbols produces the underlying sequence $\{\overline{\mathbf{x}}_k\}$. When passed through the channel $\underline{\mathbf{h}}$, the sequence $\{\overline{\mathbf{x}}_k\}$ produces the output sequence $\{\overline{\mathbf{y}}_k\}$ i.e.

$$\overline{\mathbf{y}}_k = \underline{\mathbf{h}}_k * \overline{\mathbf{x}}_k + \overline{\mathbf{n}}_k \quad (2)$$

where $\overline{\mathbf{n}}_k$ is the additive white Gaussian noise and $*$ stands for linear convolution. Motivated by the symbol structure of the input, it is convenient to partition the output into length $N + L$ symbol as

$$\overline{\mathbf{y}}_i = [\underline{\mathbf{y}}_i^T \quad \mathbf{y}_i^T]^T \quad (3)$$

This is a natural way to partition the output because the prefix $\underline{\mathbf{y}}_i$ actually absorbs all ISI that takes place between the adjacent symbols $\overline{\mathbf{x}}_{i-1}$ and $\overline{\mathbf{x}}_i$. Moreover, the remaining part \mathbf{y}_i of the symbol depends on the i th input OFDM symbol \mathbf{x}_i only. These facts allow us to partition the total OFDM channel described by (2) into two subchannels that is described next.

A. Circular Convolution (Subchannel)

Due to the presence of the cyclic prefix, the input and output OFDM symbols \mathbf{x}_i and \mathbf{y}_i are related by circular convolution, i.e.

$$\mathbf{y}_i = \mathbf{h}_i \circledast \mathbf{x}_i + \mathbf{n}_i \quad (4)$$

where \mathbf{h}_i is a length- N zero-padded version of $\underline{\mathbf{h}}_i$. In the frequency domain, the cyclic convolution (4) reduces to the element-by-element operation

$$\mathcal{Y}_i = \mathcal{H}_i \odot \mathcal{X}_i + \mathcal{N}_i \quad (5)$$

where \mathcal{H}_i , \mathcal{X}_i , \mathcal{N}_i , and \mathcal{Y}_i , are the DFT's of \mathbf{h}_i , \mathbf{x}_i , \mathbf{n}_i , and \mathbf{y}_i respectively

$$\begin{aligned} \mathcal{H}_i &= \mathbf{Q}^H \mathbf{h}_i, \quad \mathcal{X}_i = \frac{1}{\sqrt{N}} \mathbf{Q}^H \mathbf{x}_i, \\ \mathcal{N}_i &= \frac{1}{\sqrt{N}} \mathbf{Q}^H \mathbf{n}_i, \quad \text{and} \quad \mathcal{Y}_i = \frac{1}{\sqrt{N}} \mathbf{Q}^H \mathbf{y}_i \end{aligned} \quad (6)$$

B. Linear Convolution (Subchannel)

From (2), it can also be deduced that the cyclic prefixes at the input and output are related by linear convolution. Specifically, if all the cyclic prefixes at the input are concatenated into a sequence $\{\underline{\mathbf{x}}_k\}$ and the cyclic prefixes at the output into the corresponding sequence $\{\underline{\mathbf{y}}_k\}$, then it can be shown that the two sequences are related by linear convolution [17]

$$\underline{\mathbf{y}}_k = \underline{\mathbf{h}}_k * \underline{\mathbf{x}}_k + \underline{\mathbf{n}}_k \quad (7)$$

From this, it can be deduced that the cyclic prefix of OFDM symbol \mathbf{y}_i is related to the input cyclic prefixes $\underline{\mathbf{x}}_{i-1}$ and $\underline{\mathbf{x}}_i$ by

$$\underline{\mathbf{y}}_i = \underline{\mathbf{X}}_i \underline{\mathbf{h}}_i + \underline{\mathbf{n}}_i \quad (8)$$

where $\underline{\mathbf{X}}_i$ is constructed from $\underline{\mathbf{x}}_{i-1}$ and $\underline{\mathbf{x}}_i$ according to

$$\underline{\mathbf{X}}_i = \underline{\mathbf{X}}_{U_{i-1}} + \underline{\mathbf{X}}_{L_i} \quad (9)$$

and where

$$\underline{\mathbf{X}}_{U_{i-1}} = \begin{bmatrix} 0 & \underline{x}_{i-1}(L-1) & \cdots & \underline{x}_{i-1}(0) \\ 0 & 0 & \cdots & \underline{x}_{i-1}(1) \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \underline{x}_{i-1}(L-1) \end{bmatrix},$$

$$\text{and } \underline{\mathbf{X}}_{L_i} = \begin{bmatrix} \underline{x}_i(0) & 0 & \cdots & 0 \\ \underline{x}_i(1) & \underline{x}_i(0) & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \underline{x}_i(L-1) & \cdots & \underline{x}_i(0) & 0 \end{bmatrix}$$

In the next section, a brief description of the equalization performed in conventional OFDM systems is presented.

III. CONVENTIONAL EQUALIZATION

Conventionally in OFDM systems, the cyclic prefix is discarded at the receiver and only the circular subchannel is used for data recovery. Consider the input/output equation of circular subchannel in equation (5), the data detection is performed on a bin by bin basis according to

$$\hat{\mathbf{x}}(l) = \frac{\mathcal{Y}(l)}{\mathcal{H}(l)} \quad l = 1, 2, \dots, N \quad (10)$$

The disadvantage of this method is that it is very sensitive to channel nulls. It is not possible to recover the data completely if there is even a single zero on the FFT grid of the channel impulse response (IR). In what follows, a new equalization technique to overcome this problem is presented.

IV. ENHANCED EQUALIZATION USING CYCLIC PREFIX

The use of cyclic prefix for blind and semiblind data detection was exploited in [7]. Here, the importance of cyclic prefix observation is demonstrated when the channel is known perfectly or an estimate of it is available at the receiver through training symbols.

Consider the input/output equation of the circular subchannel given by equation (5). It can be rewritten in the following form

$$\mathcal{Y}_i = \mathcal{H}_i \odot \mathcal{X}_i + \mathcal{N}_i = \text{diag}(\mathcal{H}_i) \mathcal{X}_i + \mathcal{N}_i \quad (11)$$

Now, consider the input/output equation of the linear subchannel (equation (8)), reproduced here for convenience

$$\underline{\mathbf{y}}_i = \underline{\mathbf{X}}_i \underline{\mathbf{h}}_i + \underline{\mathbf{n}}_i \quad (12)$$

Substituting the value of $\underline{\mathbf{X}}_i$ from equation (10) in the above equation results in

$$\underline{\mathbf{y}}_i = (\underline{\mathbf{X}}_{U_{i-1}} + \underline{\mathbf{X}}_{L_i}) \underline{\mathbf{h}}_i + \underline{\mathbf{n}}_i \quad (13)$$

Moving the known part $\underline{\mathbf{X}}_{U_{i-1}} \underline{\mathbf{h}}_i$ to the left hand side,

$$\underline{\mathbf{y}}_i - \underline{\mathbf{X}}_{U_{i-1}} \underline{\mathbf{h}}_i = \underline{\mathbf{X}}_{L_i} \underline{\mathbf{h}}_i + \underline{\mathbf{n}}_i \quad (14)$$

and exchanging the roles of $\underline{\mathbf{X}}_{L_i} \underline{\mathbf{h}}_i$ as

$$\underline{\mathbf{X}}_{L_i} \underline{\mathbf{h}}_i = \underline{\mathbf{H}}_L \underline{\mathbf{x}}_i \quad (15)$$

$$= \underline{\mathbf{H}}_L \mathbf{Q}_{N-L+1} \mathcal{X}_i \quad (16)$$

where the second line follows from the fact that

$$\underline{\mathbf{x}}_i = \mathbf{Q} \mathcal{X}_i$$

and that $\underline{\mathbf{x}}_i$ consists of the last $L + 1$ elements of \mathbf{x}_i . Thus (14) can be rewritten as

$$\underline{\mathbf{y}}_i - \underline{\mathbf{X}}_{U_{i-1}} \underline{\mathbf{h}}_i = \underline{\mathbf{H}}_L \mathbf{Q}_{N-L+1} \mathcal{X}_i + \underline{\mathbf{n}}_i \quad (17)$$

Combining (11) and (17) yields an $N + L$ system of equations in the unknown OFDM symbol \mathcal{X}_i

$$\begin{bmatrix} \mathcal{Y}_i \\ \underline{\mathbf{y}}_i - \underline{\mathbf{X}}_{U_{i-1}} \underline{\mathbf{h}}_i \end{bmatrix} = \begin{bmatrix} \text{diag}(\mathcal{H}_i) \\ \underline{\mathbf{H}}_L \mathbf{Q}_{N-L+1} \end{bmatrix} \mathcal{X}_i + \begin{bmatrix} \mathcal{N}_i \\ \underline{\mathbf{n}}_i \end{bmatrix} \quad (18)$$

This system can be solved for \mathcal{X}_i using *least squares*¹

$$\hat{\mathcal{X}}_i = (\mathbf{B}^H \mathbf{B})^{-1} \mathbf{B}^H \mathbf{C} \quad (19)$$

where

$$\mathbf{B} = \begin{bmatrix} \text{diag}(\mathcal{H}_i) \\ \underline{\mathbf{H}}_L \mathbf{Q}_{N-L+1} \end{bmatrix} \text{ and } \mathbf{C} = \begin{bmatrix} \mathcal{Y}_i \\ \underline{\mathbf{y}}_i - \underline{\mathbf{X}}_{U_{i-1}} \underline{\mathbf{h}}_i \end{bmatrix}$$

Thus, the advantage of incorporating this enhanced equalization in the receiver design over conventional equalization (discussed in Section III) is complete data recovery even in the presence of channel nulls. To compare the performance of these two methods, simulation results are presented in the next section.

¹We use *least squares* to solve for \mathcal{X}_i although better estimates can be obtained using e.g. *ML sequence detection*

V. SIMULATIONS AND RESULTS

We consider a realistic OFDM system with $N = 128$ subcarriers and a cyclic prefix of length $L = 32$. The channel IR consists of 33 iid Rayleigh fading taps and the OFDM symbol consists of BPSK or 16QAM modulated data. In this section, it is assumed that the receiver either knows the channel perfectly or estimates it using $L + 1$ pilots. We then compare the performance of the receiver in these two scenarios when (i) data is detected using only the circular subchannel, (ii) data is detected using both the circular and linear subchannels with errors propagated (i.e. the error corrupted symbol detected in the current iteration is used as it is in the next iteration), and (iii) data is detected using both the channels with no errors propagated (i.e. it is assumed that the previous symbol has been detected perfectly in the next iterations).

A. BER vs SNR Comparison for BPSK-OFDM over a Rayleigh channel

In Figure 1, the performance of the receiver is compared for the above scenarios for BPSK-OFDM over a Rayleigh channel. It can be seen that the performance of the receiver improves when both subchannels are used for data recovery. The improvement is significant at high SNR. It can also be noticed that the case when errors are propagated does not perform well at low SNR but as the SNR increases, its performance is improved and becomes equal to the case when no errors are propagated.

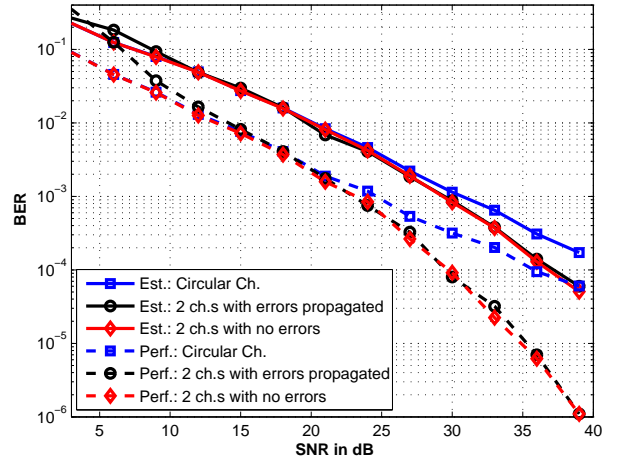


Fig. 1. Comparison of perfect and pilot based estimation with enhanced equalization using cyclic prefix for BPSK-OFDM over a Rayleigh channel

B. BER vs SNR Comparison for BPSK-OFDM over channel with persistent nulls

In Figure 2, the performance of the receiver with enhanced equalization using cyclic prefix is compared with the conventional one using only circular subchannel when channel IR has zeros on FFT grid. It can be observed that the case when data is estimated using only circular subchannel reaches an error

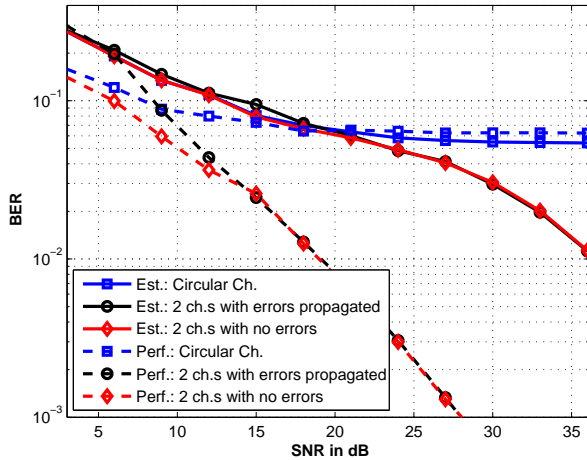


Fig. 2. Comparison of perfect and pilot based estimation with enhanced equalization using cyclic prefix for BPSK-OFDM over channel with zeros on FFT grid

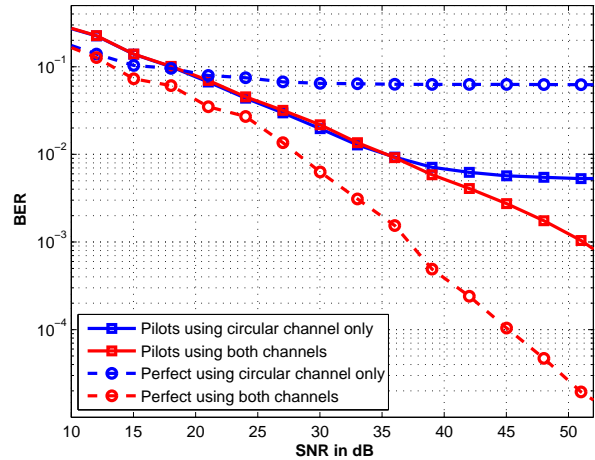


Fig. 4. Comparison of perfect and pilot based estimation with enhanced equalization using cyclic prefix for 16QAM-OFDM over channel with zeros on FFT grid

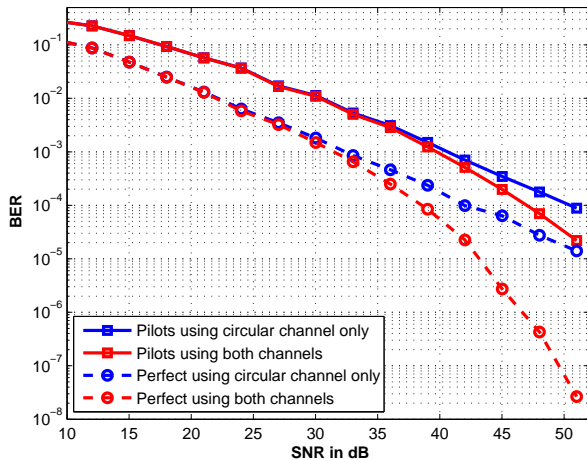


Fig. 3. Comparison of perfect and pilot based estimation with enhanced equalization using cyclic prefix for 16QAM-OFDM over a Rayleigh channel

floor as expected. No such error floor is observed if both the linear and circular subchannels are used for data detection. As noticed in Figure 1, the performance of the case when errors are propagated improves with increasing SNR.

C. BER vs SNR Comparison for 16QAM-OFDM over a Rayleigh channel

Figure 3 shows the performance of the receiver with enhanced equalization using cyclic prefix for 16QAM modulated (non-constant modulus) data over a Rayleigh channel. Similar to the BPSK modulated data case, the performance of the receiver when both subchannels are considered is better as compared to its performance when only circular subchannel is used for data recovery. The improvement is quite significant at high SNR.

D. BER vs SNR Comparison for 16QAM-OFDM over channel with persistent nulls

The performance of receiver with enhanced equalization using cyclic prefix for 16QAM modulated data is shown in Figure 4. The case when only circular subchannel is used for data recovery flattens at high SNR while the case when both subchannel are used does not show any error floor.

VI. CONCLUSION

In this paper, the advantage of using cyclic prefix observation in OFDM systems is demonstrated when the channel is known either perfectly or an estimate of it is available (by using pilots) at the receiver. This is in contrast to conventional OFDM systems where the cyclic prefix is discarded at the receiver. The performance of the receiver using both the linear and circular subchannels is considerably better than the conventional receiver that uses only the circular subchannel for data detection. Its superior performance can be seen especially in the case when the channel impulse response has zeros on the FFT grid.

ACKNOWLEDGEMENT

The author would like to pay special gratitude to Dr. T. Y. Al-Naffouri for the support and guidance provided in this work. The author would also like to thank King Fahd University of Petroleum & Minerals for supporting this work.

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