ML BLIND CHANNEL ESTIMATION IN OFDM USING CYCLOSTATIONARITY AND SPECTRAL FACTORIZATION

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ABSTRACT

Channel estimation is vital in OFDM systems for efficient data recovery. In this paper, we propose a blind algorithm for channel estimation that is based on the assumption that the transmitted data in an OFDM system is Gaussian (by central limit arguments). The channel estimate can then be obtained by maximizing the output likelihood function. Unfortunately, the likelihood function turns out to be multi-modal and thus finding the global maxima is challenging. We rely on spectral factorization and the cyclostationarity of the output to obtain the correct channel zeros. The Genetic algorithm is then used to fine tune the obtained solution.

Index Terms— Blind channel estimation, Maximum likelihood estimation, Spectral factorization, and Genetic algorithm.

1. INTRODUCTION

There has been increased interest in Orthogonal Frequency Division Multiplexing (OFDM) due to its high achievable data rates, multipath robustness and simple receiver implementation. This is reflected by the many standards that considered and adopted OFDM, e.g. ADSL, VDSL, power line communication, WiFi (IEEE 802.11a), WiMAX (IEEE 802.16), terrestrial broadcast (DVB-T), and ultrawideband personal area networks (IEEE 802.15.3a) [1].

In OFDM systems, channel must be estimated accurately for high speed communication. The channel estimation techniques present in the literature can be broadly divided into three categories:

1. *Training-based:* These techniques involve sending training data (pilots) along with the data symbols for estimating the channel [2], [3], [4]. Use of pilots decreases the bandwidth efficiency.
2. *Blind:* Blind techniques use the structure of the communication system created by such constraints as finite alphabet constraint [5], cyclic prefix [5], [6], and time and frequency correlation [7], [8]. Blind techniques usually require averaging over many symbols before they converge (this implicitly assumes that the channel remains invariant over these symbols).
3. *Semi-blind:* Semi-blind techniques are hybrid of training-based and blind methods. These methods use pilots to obtain an initial channel estimate and improve the estimate by using a variety of a priori information [7], [9].

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In this paper, we perform blind estimation of block fading channels by utilizing the Gaussian assumption on the time-domain transmitted data, and the cyclostationarity of the transmitted data due to the presence of the cyclic prefix. The advantage of our method is that it provides a blind estimate of the channel from only one output OFDM symbol without the need for training or averaging[1] (contrary to the common practice in blind methods where averaging over several symbols is required [6], [10]). Specifically, the channel estimate is obtained by maximizing the log-likelihood of the channel given the output data. While the frequency domain input in OFDM comes from some finite alphabet (say a QAM constellation), the time domain input can be assumed to be Gaussian (by some central limit theorem arguments [9], [11]). This allows us to write the log-likelihood function of the output given the channel $\boldsymbol{h}$ which would be in terms of the second order statistics of the output. The channel can then be obtained by maximizing the likelihood function.

It is well known that second order methods are phase blind and so it can not uniquely identify non-minimum phase channels [12], [13]. In the OFDM case, however, the input is cyclostationary [6] and thus we can avoid this problem and we can maximize the likelihood function uniquely for the channel estimate. The challenge here is that this maximization is non-convex and thus obtaining the global maxima is a challenge. In this paper, we avoid this problem by first identifying the minimum phase equivalent of the channel. Subsequently, we find all non-minimum phase variations, and find the variation that attains the global maxima of the likelihood function. This gives a good blind estimate that can be further refined using the Genetic algorithm.

2. SYSTEM OVERVIEW

In this paper, an OFDM system is used which involves transmitting data in symbols $\boldsymbol{\mathcal{X}}_i$ of length N each. Each symbol then undergoes an Inverse Discrete Fourier Transform (IDFT) operation to produce the time domain symbol $\boldsymbol{x}_i$, i.e.

$$\boldsymbol{x}_i = \sqrt{N}\boldsymbol{Q}\boldsymbol{\mathcal{X}}_i, \tag{1}$$

where $\boldsymbol{Q}$ is an IDFT matrix of size $N \times N$. A cyclic prefix $\underline{\boldsymbol{x}}_i$ of length L is appended to form the $N+L$ length super-symbol $\overline{\boldsymbol{x}}_i = [\underline{\boldsymbol{x}}_i^T \ \boldsymbol{x}_i^T]^T = [\underline{\boldsymbol{x}}_i^T \ \tilde{\boldsymbol{x}}_i^T \ \underline{\boldsymbol{x}}_i^T]^T$, where $\tilde{\boldsymbol{x}}_i$ is $N-L$ length symbol with cyclic prefix stripped off. The symbol is then transmitted through an FIR channel of maximum length $L+1$, $\boldsymbol{h} = [h_0 \ h_1 \ \dots \ h_L]$. The

[1]We assume that the channel is block fading which leaves us with one OFDM symbol only to identify the channel.

time-domain input/output relationship is given by[2]

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \tag{2}$$

where $\boldsymbol{y} = [\underline{\boldsymbol{y}}_i^T \ \boldsymbol{y}_i^T]^T$ is the received data of length $N + L$, $\boldsymbol{x} = [\underline{\boldsymbol{x}}_{i-1}^T \ \underline{\boldsymbol{x}}_i^T \ \tilde{\boldsymbol{x}}_i^T \ \underline{\boldsymbol{x}}_i^T]^T$ is the transmitted data of length $N + 2L$,[3] $\boldsymbol{n} = [\underline{\boldsymbol{n}}_i^T \ \boldsymbol{n}_i^T]^T$ is the circular complex Gaussian noise (with vairance σ_n^2) which is independent of $\boldsymbol{x}$, and $\boldsymbol{H}$ is a convolution matrix of size $(N + L) \times (N + 2L)$.

The channel is assumed to be block fading. Moreover, to guard against sign ambiguity inherent in blind techniques, we set the first coefficient of the channel to unity, i.e. $h_0 = 1$.

3. EVALUATING THE LOG LIKELIHOOD FUNCTION

To derive the likelihood function of the output of a linear system, the input is assumed usually to be Gaussian (otherwise writing down the likelihood function can be very difficult). This is usually not true in a data communication system as the input is generated from a finite alphabet. Fortunately in an OFDM system, the time domain input can be assumed to be Gaussian by central limit theorem arguments for large N [11]. This fact can be motivated from the element-by-element version of (1) which reads

$$x_i(j) = \sqrt{N}\boldsymbol{q}_j\boldsymbol{\mathcal{X}}_i$$

where $\boldsymbol{q}_j$ is the jth row of $\boldsymbol{Q}$. This shows that $x_i(j)$ is a large (weighted) sum of iid random variables. From this fact and the fact that noise is also Gaussian, we can conclude that output $\boldsymbol{y}$ will be Gaussian with covariance matrix $\boldsymbol{\Sigma}_{\mathbf{Y}}$ given by (follows from (2))

$$\boldsymbol{\Sigma}_{\mathbf{Y}} = E[\boldsymbol{y}\boldsymbol{y}^H] = \boldsymbol{H}\boldsymbol{\Sigma}_{\mathbf{X}}\boldsymbol{H}^H + \sigma_n^2\boldsymbol{I} \tag{3}$$

where $\boldsymbol{\Sigma}_{\mathbf{X}}$ is the input covariance matrix

$$\begin{aligned}\boldsymbol{\Sigma}_{\mathbf{X}} &= \boldsymbol{E}\begin{bmatrix}\underline{\boldsymbol{x}}_{i-1}\\ \underline{\boldsymbol{x}}_i\\ \tilde{\boldsymbol{x}}_i\\ \underline{\boldsymbol{x}}_i\end{bmatrix}\begin{bmatrix}\underline{\boldsymbol{x}}_{i-1}^H & \underline{\boldsymbol{x}}_i^H & \tilde{\boldsymbol{x}}_i^H & \underline{\boldsymbol{x}}_i^H\end{bmatrix}\\ &= \begin{bmatrix}\boldsymbol{I}_L & 0 & 0 & 0\\ 0 & \boldsymbol{I}_L & 0 & \boldsymbol{I}_L\\ 0 & 0 & \boldsymbol{I}_{N-L} & 0\\ 0 & \boldsymbol{I}_L & 0 & \boldsymbol{I}_L\end{bmatrix}\end{aligned} \tag{4}$$

The probability density function (pdf) of the output $\boldsymbol{y}$ can thus be written as

$$P(\boldsymbol{y}|\boldsymbol{h}) = \frac{1}{\det(\boldsymbol{\Sigma}_{\mathbf{Y}})} \ \exp(-\boldsymbol{y}^H{\boldsymbol{\Sigma}_{\mathbf{Y}}}^{-1}\boldsymbol{y}) \tag{5}$$

which yields the following log-likelihood function

$$\mathcal{L}(\boldsymbol{y}|\boldsymbol{h}) \ = \ -\ln \det(\boldsymbol{\Sigma}_{\mathbf{Y}}) \ - \ \boldsymbol{y}^H{\boldsymbol{\Sigma}_{\mathbf{Y}}}^{-1}\boldsymbol{y} \tag{6}$$

3.1. Maximum Likelihood Estimation of the Channel IR

We obtain the maximum-likelihood (ML) channel estimate by maximizing the log-likelihood function, i.e.

$$\begin{aligned}\hat{\boldsymbol{h}}_{\text{ML}} &= \max_{\boldsymbol{h}} \ \mathcal{L}\\ &= \max_{\boldsymbol{h}} \ -\ln \det(\boldsymbol{\Sigma}_{\mathbf{Y}}) \ - \ \boldsymbol{y}^H{\boldsymbol{\Sigma}_{\mathbf{Y}}}^{-1}\boldsymbol{y}\end{aligned} \tag{7}$$

[2]The symbol index has been omitted for simplification.

[3]Note that we are assuming the channel to be constant over the vector $\boldsymbol{x}$ consisting of the current OFDM symbol and the preceding cyclic prefix.

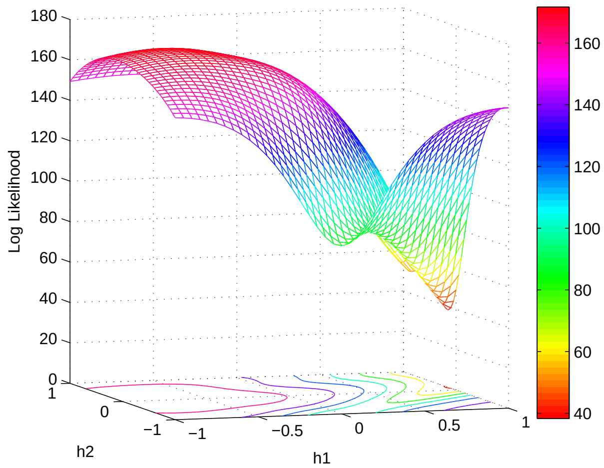

Fig. 1. 3D plot of likelihood function against channel taps

This maximization depends only on the output data $\boldsymbol{y}$ and the channel $\boldsymbol{h}$ (through the dependance of $\boldsymbol{\Sigma}_{\mathbf{Y}}$ on $\boldsymbol{h}$).

Unfortunately, as is evident from Figure 1, the likelihood function is not uni-modal. In this figure, the input is BPSK modulated data of length $N = 64$ and a cyclic prefix of length $L = 2$ is used. Channel is considered to be an FIR of length $L + 1 = 3$ with first tap fixed at 1 and the log-likelihood function is plotted against the remaining two channel taps. It can be seen that the likelihood function is non-convex with several local maxima and thus finding the global maxima can be challenging.

4. BLIND CHANNEL ESTIMATION ALGORITHM

The proposed algorithm performs blind channel estimation in three steps:

1. The output data stripped off the cyclic prefix is used to estimate the minimum phase equivalent of the channel IR. This is done using spectral factorization.
2. The system phase is identified by maximizing the likelihood function over all possible phase combinations.
3. The estimate obtained in step 2 is then used to kick start the Genetic algorithm which further refines the channel estimate.

In what follows, we discuss these steps in detail.

4.1. Estimation of the minimum phase equivalent channel

As we mentioned in the introduction, the output of an OFDM system is cyclostationary due to the presence of cyclic prefix. However, if we discard the cyclic prefix from the input and output, then the system appears as a linear system with stationary input and stationary output. We can thus use the second order statistics of the output to obtain the minimum phase equivalent of the channel. This is a standard spectral factorization problem [14].

Specifically, the power spectrum of the received data $\boldsymbol{y}$ (with cyclic prefix stripped off) is given by

$$\boldsymbol{P}_y(e^{jw}) = \boldsymbol{P}_x(e^{jw})|\boldsymbol{H}(e^{jw})|^2 \tag{8}$$

Note that since $\boldsymbol{y}$ no more contains the cyclic prefix, it is stationary and so its power spectrum is well defined which can be factored into a product of the form [14]

$$\boldsymbol{P}_y(z) = \sigma_x^2 \boldsymbol{H}(z)\boldsymbol{H}^*(1/z^*) \tag{9}$$

which implies that if the system function $\boldsymbol{H}(z)$ has a zero at $z = z_0$, the power spectrum of the received data will have a zero at $z = z_0$ and another at the conjugate reciprocal position $z = 1/z_0^*$.

Thus, if we are able to approximately model $\boldsymbol{P}_y(z)$ from the received data only, we can estimate the zeros of the system function $\boldsymbol{H}(z)$. Depending on the system function, the power spectrum of the received data can be modeled by either auto-regressive (AR), moving-average (MA) or auto-regressive moving-average (ARMA) models. As we have assumed an FIR channel of length $L + 1$, its system function will have the form

$$\boldsymbol{H}(z) = \sum_{k=0}^{L} h_k z^{-k},$$

where h_k are the coefficients of the $L + 1$ length channel (described in Section 2). Thus MA model of order L can be used for spectrum estimation of the received data, i.e.

$$\hat{\boldsymbol{P}}_y(e^{jw}) = \left| \sum_{k=0}^{L} \hat{\boldsymbol{h}}_k e^{-jkw} \right|^2 \tag{10}$$

where the MA parameters $\boldsymbol{h}_k$ (channel coefficients) can be estimated using the Durbin's algorithm [14]. These estimated parameters represent the minimum phase equivalent of the channel IR.

4.2. Identification of the system phase

Let $\{z_0, z_1, ..., z_{L-1}\}$ denote the zeros of the minimum phase equivalent channel. The power spectral density (10) will not change if we replace any zeros z_i with its corresponding conjugate reciprocal $1/z_i^*$. However, the log-likelihood function (6) which is not phase blind will change depending on the mixed phase possibility we choose. There are 2^L such possibilities that we can use to construct the mixed phase IR and evaluate the corresponding log-likelihood function. We can then choose the channel with largest value of the log-likelihood.

The estimate obtained using spectral factorization can be used to initialize the Genetic algorithm which we describe next.

4.3. Genetic Algorithm

Genetic algorithm (GA) is an iterative stochastic search algorithm which was introduced by Holland [15] in 1975. GA finds the best solution in a population of candidate solutions (called chromosomes) based on natural selection (survival of the fittest) and evolution. Each chromosome has a fitness value associated with it which in our algorithm is found by evaluating the likelihood function in Equation (6). The next generation is produced by using genetic operators like crossover and mutation.

A. Crossover: Crossover is a technique of combining the features of two parent chromosomes to form two offspring. We selected the *BLX*-α algorithm (with α = 0.5) for implementing crossover due to its superior performance in real coded genetic algorithms [16].

B. Mutation: Mutation is a method in which an arbitrary element of a selected chromosome is altered to prevent the premature convergence of GA to suboptimal solutions. GA is able to avoid local minima/maxima due to this operator. We have used the *Non-uniform mutation* algorithm as it is very appropriate for real coded genetic algorithms [16].

5. COMPUTATIONAL COMPLEXITY

Both spectral factorization and GA require calculation of the log-likelihood function (Equation (6)) that involves the computation of inverse and determinant of output autocorrelation matrix $\boldsymbol{\Sigma}_\mathbf{Y}$ (of size $(N + L) \times (N + L)$). We will rely on block matrix calculations to simplify these matrix operations. The output autocorrelation matrix $\boldsymbol{\Sigma}_\mathbf{Y}$ can be decomposed as

$$\boldsymbol{\Sigma}_\mathbf{Y} = \boldsymbol{G}\boldsymbol{G}^H + \sigma_n^2 \boldsymbol{I} \tag{11}$$

where $\boldsymbol{G}$ is a square matrix of size $N + L$ given by[4]

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{B} & \boldsymbol{O} & \cdots & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{C} & \boldsymbol{B} & \cdots & \boldsymbol{O} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \cdots & \boldsymbol{B} \\ \boldsymbol{O} & \boldsymbol{B} & \boldsymbol{O} & \cdots & \boldsymbol{C} \end{bmatrix} \tag{12}$$

with

$$\boldsymbol{B} = \begin{bmatrix} h_0 & 0 & \cdots & 0 \\ h_1 & h_0 & \cdots & 0 \\ \vdots & & \ddots & \\ h_{L-1} & h_{L-2} & \cdots & h_0 \end{bmatrix} \tag{13}$$

$$\text{and } \boldsymbol{C} = \begin{bmatrix} h_L & h_{L-1} & \cdots & h_1 \\ 0 & h_L & \cdots & h_2 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & h_L \end{bmatrix} \tag{14}$$

Alternatively, we can write $\boldsymbol{G}$ in the following block form

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{D} \\ \boldsymbol{O} & \boldsymbol{A} \end{bmatrix} \tag{15}$$

where

$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{O} & \cdots & \boldsymbol{O} \end{bmatrix}$$

and

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{B} & \boldsymbol{O} & \cdots & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{C} & \boldsymbol{B} & \cdots & \boldsymbol{O} \\ \vdots & \ddots & \ddots & \cdots & \vdots \\ \boldsymbol{B} & \boldsymbol{O} & \boldsymbol{O} & \cdots & \boldsymbol{C} \end{bmatrix} \tag{16}$$

It is easy to see that the matrix $\boldsymbol{A}$ is circulant so it is diagnolizable by an FFT matrix. So, $\boldsymbol{\Sigma}_\mathbf{Y}$ can now be written as

$$\begin{aligned} \boldsymbol{\Sigma}_\mathbf{Y} &= \boldsymbol{G}\boldsymbol{G}^H + \sigma_n^2 \boldsymbol{I} \\ &= \begin{bmatrix} \boldsymbol{C}\boldsymbol{C}^H + \boldsymbol{D}\boldsymbol{D}^H + \sigma_n^2 \boldsymbol{I} & \boldsymbol{D}\boldsymbol{A}^H \\ \boldsymbol{A}\boldsymbol{D}^H & \boldsymbol{A}\boldsymbol{A}^H + \sigma_n^2 \boldsymbol{I} \end{bmatrix} \end{aligned} \tag{17}$$

In the following sections, we use the above structure of $\boldsymbol{\Sigma}_\mathbf{Y}$ to calculate its inverse and determinant.

[4] In writing (12), we assume that N is a multiple of L to simplify the subsequent exposition. Computational complexity can still be reduced in a similar manner even if this condition is not valid

5.1. Calculating $\mathbf{\Sigma_Y}^{-1}$

To calculate $\mathbf{\Sigma_Y}^{-1}$, we use the block inversion formula [17] (page 30, formula (2)) shown in equation (18), which is valid provided the inverses involved exist. There are two inverses that we need to calculate here. 1) $\mathbf{\Omega}^{-1}$, and 2) $(\mathbf{\Gamma} - \mathbf{\Phi}\mathbf{\Omega}^{-1}\mathbf{\Psi})^{-1}$. Given the structure of $\mathbf{\Sigma_Y}$ in Equation (17), this boils down to calculating

$$(\boldsymbol{A}\boldsymbol{A}^H + \sigma_n^2\boldsymbol{I})^{-1} \tag{19}$$

and

$$\left(\boldsymbol{C}\boldsymbol{C}^H + \boldsymbol{D}\boldsymbol{D}^H + \sigma_n^2\boldsymbol{I} - \boldsymbol{D}\boldsymbol{A}^H(\boldsymbol{A}\boldsymbol{A}^H + \sigma_n^2\boldsymbol{I})^{-1}\boldsymbol{A}\boldsymbol{D}^H\right)^{-1} \tag{20}$$

As $\boldsymbol{A}$ is circulant (from (16)), we can write it as

$$\boldsymbol{A} = \boldsymbol{Q}^H\Lambda\boldsymbol{Q} \tag{21}$$

where $\boldsymbol{Q}$ is the IDFT matrix and Λ is the DFT of the first row of $\boldsymbol{A}$ (which is the DFT of $[h_L \; h_{L-1} \; \dots \; h_0 \; \dots \; 0]$). It is easy then to see that the inverse in (19) can be computed as follows

$$\begin{aligned}(\boldsymbol{A}\boldsymbol{A}^H + \sigma_n^2\boldsymbol{I})^{-1} &= \boldsymbol{Q}^H\left(|\Lambda|^2 + \sigma_n^2\boldsymbol{I}\right)^{-1}\boldsymbol{Q} \\ &= \boldsymbol{Q}^H\begin{bmatrix}\frac{1}{|\lambda_1|^2+\sigma_n^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{|\lambda_N|^2+\sigma_n^2}\end{bmatrix}\boldsymbol{Q}\end{aligned}$$

It is easy to calculate the $L \times L$ inverse in (20). However, it involves an $N \times N$ matrix inversion, namely $\boldsymbol{A}(\boldsymbol{A}\boldsymbol{A}^H + \sigma_n^2\boldsymbol{I})^{-1}\boldsymbol{A}^H$ which can be evaluated as

$$\boldsymbol{A}^H(\boldsymbol{A}\boldsymbol{A}^H + \sigma_n^2\boldsymbol{I})^{-1}\boldsymbol{A} = \boldsymbol{Q}^H\begin{bmatrix}\frac{|\lambda_1|^2}{|\lambda_1|^2+\sigma_n^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{|\lambda_N|^2}{|\lambda_N|^2+\sigma_n^2}\end{bmatrix}\boldsymbol{Q}$$

So, evaluating inverse of $(N+L)\times(N+L)$ matrix $\mathbf{\Sigma_Y}$ reduces to calculating an FFT (to find Λ) and to calculating the $L \times L$ inverse of (20).

5.2. Calculating $\det(\mathbf{\Sigma_Y})$

In order to calculate determinant of $\mathbf{\Sigma_Y}$, we use [17] (page 50, formula (6))

$$\det\begin{bmatrix}\mathbf{\Gamma} & \mathbf{\Phi} \\ \mathbf{\Psi} & \mathbf{\Omega}\end{bmatrix} = \det(\mathbf{\Omega})\det(\mathbf{\Gamma} - \mathbf{\Phi}\mathbf{\Omega}^{-1}\mathbf{\Psi}) \tag{22}$$

Now, from (17),

$$\begin{aligned}\mathbf{\Omega} &= \boldsymbol{A}\boldsymbol{A}^H + \sigma_n^2\boldsymbol{I} &(23)\\ \Rightarrow \det(\mathbf{\Omega}) &= \det\left(\boldsymbol{Q}^H(|\Lambda|^2 + \sigma_n^2\boldsymbol{I})\boldsymbol{Q}\right) &(24)\\ &= \det(|\Lambda|^2 + \sigma_n^2\boldsymbol{I}) &(25)\end{aligned}$$

We already know how to calculate $\mathbf{\Omega}^{-1}$ from (22), thus $\det(\mathbf{\Gamma} - \mathbf{\Phi}\mathbf{\Omega}^{-1}\mathbf{\Psi})$ involves calculating the determinant of an $L \times L$ matrix.

To summarize, the inverse and the determinant of the $(N+L)\times(N+L)$ matrix $\mathbf{\Sigma_Y}$ essentially reduces to calculating

1. N-point FFT (to find Λ)
2. Inverse and determinant of an $L \times L$ matrix.

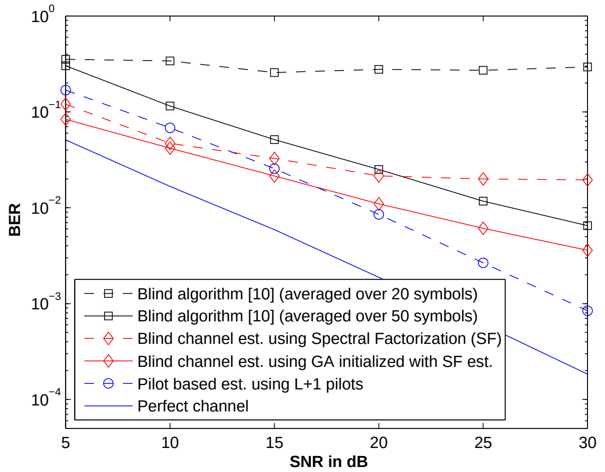

Fig. 2. Comparison with Blind algorithm [10] for BPSK modulated data with $N = 16$ and $L = 4$

6. SIMULATION RESULTS

An OFDM system with $N = 16$ BPSK modulated symbols and cyclic prefix of length $L = 4$ is considered. The channel IR consists of 5 iid Rayleigh fading taps. Moving average (MA) model of order L is used to model the power spectrum of the received data in spectral factorization (SF) method. The estimate obtained by SF is used to initialize the GA. The parameters used in implementing the GA are listed in Table 1.

Table 1. Simulation Parameters used to implement GA

Population Size	50
Number of Generations	100
Cross-over Scheme	BLX-α ($\alpha = 0.5$)
Cross-over Probability	0.8
Mutation Scheme	Non-Uniform Mutation
Mutation Probability	0.08
Number of Elite Chromosomes	5

To bench mark our algorithm, we compare it with 1) the subspace blind algorithm proposed by Muquet et al. in [10], 2) a training based method in which the channel is estimated using $L+1$ pilots, and 3) the perfectly known channel state information case. As opposed to our algorithm in which the estimation is done using only a single OFDM block, the subspace method in [10] requires the channel to remain fixed over several OFDM blocks. The subspace algorithm was implemented in two ways. One with 50 blocks of data to make the covariance matrix full rank and the other with 20 blocks. Only a single block was used for channel estimation in our algorithm. In Figure 2, as expected, the best performance is achieved for the perfectly known channel case, followed by the pilot based case.

$$\begin{bmatrix}\mathbf{\Gamma} & \mathbf{\Phi} \\ \mathbf{\Psi} & \mathbf{\Omega}\end{bmatrix}^{-1} = \begin{bmatrix}(\mathbf{\Gamma} - \mathbf{\Phi}\mathbf{\Omega}^{-1}\mathbf{\Psi})^{-1} & -(\mathbf{\Gamma} - \mathbf{\Phi}\mathbf{\Omega}^{-1}\mathbf{\Psi})^{-1}\mathbf{\Phi}\mathbf{\Omega}^{-1} \\ -\mathbf{\Omega}^{-1}\mathbf{\Psi}(\mathbf{\Gamma} - \mathbf{\Phi}\mathbf{\Omega}^{-1}\mathbf{\Psi})^{-1} & \mathbf{\Omega}^{-1} + \mathbf{\Omega}^{-1}\mathbf{\Psi}(\mathbf{\Gamma} - \mathbf{\Phi}\mathbf{\Omega}^{-1}\mathbf{\Psi})^{-1}\mathbf{\Phi}\mathbf{\Omega}^{-1}\end{bmatrix} \tag{18}$$

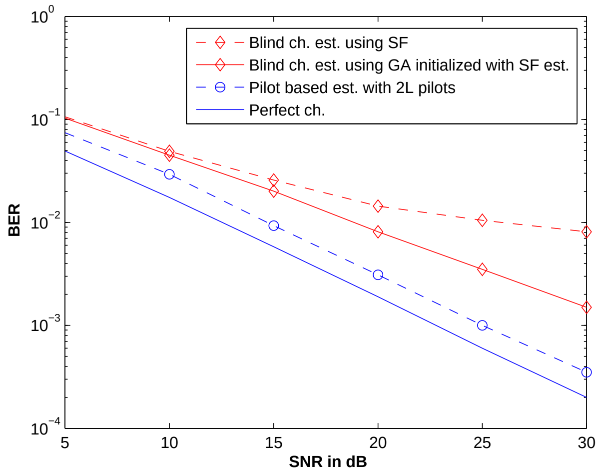

Fig. 3. BER vs SNR comparison for BPSK modulated data with $N = 64$ and $L = 4$

It can be observed that our blind algorithm (using SF and GA) easily outperforms the subspace algorithm presented in [10] even when it is uses 50 blocks of data. The BER curve of our algorithm using only SF becomes flat at high SNR. When this estimate is used to initialize the Genetic algorithm, it refines the estimate and thus results in better performance even at high SNR but this estimate still appears to flatten at high SNR which is due to the small number of carriers N.[5] It can also be seen that the performance of the subspace algorithm [10] becomes worse when only 20 data blocks are used as the covariance matrix is not full rank.

In Figure 3, the performance of our algorithm with large number of carriers ($N = 64$) is demonstrated. It can be seen that the BER curve of our algorithm (using SF and GA) does not flattens at high SNR in this case.

7. CONCLUSION

In this paper, a blind channel estimation method for OFDM system with block fading channel was presented. The method avoids any latency or storage by estimating the channel from current symbol only. It was argued in this paper that the time-domain transmitted data in OFDM is Gaussian. Using this assumption and the cyclic prefix, an algorithm was devised to perform channel estimation in three steps. First, a minimum phase equivalent of the channel IR is estimated using spectral factorization. This is followed by identifying the channel phase by maximizing the likelihood function over all possible phase combinations. Finally, the Genetic algorithm is used to refine the channel estimate. The Genetic algorithm can also be replaced by the steepest descent algorithm which is the subject of a future paper. Methods to reduce the computational complexity involved in calculating the likelihood function have also been discussed. Simulation results show the favorable performance of the proposed algorithm.

[5] The Gaussian assumption on the transmitted data is valid for large N.

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