Enhanced Channel Estimation Using Cyclic Prefix in MIMO STBC OFDM Sytems

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***Abstract*—Channel estimation is an important part of any receiver design. This paper presents an improved iterative joint channel estimation and data detection algorithm for Space Time Block Coded (STBC) MIMO OFDM systems in fast fading environments. The algorithm utilizes both time and frequency correlation information. We show how the Cyclic Prefix (CP) can be used to enhance the joint channel estimation and data detection process. We present two variations of the Expectation Maximization (EM) based Forward Backward (FB) Kalman filter algorithm utilizing the CP information and provide their performance comparison. Simulation results show that the proposed use of CP to aid the EM based FB Kalman algorithm results in improved performance.**

***Index Terms*—Channel estimation, Space Time Block Codes, MIMO OFDM, Kalman filter, expectation maximization.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a technique that has been widely adopted by modern wireless communication standards such as Wireless Local Area Network (WLAN), WIMAX, Digital Audio/Video broadcasting (DAB/DVB), Integrated Services Digital Broadcasting-Terrestrial (ISDB-T) and Digital Terrestrial/Television Multimedia Broadcasting (DTMB). The use of multiple antennas, both at the transmitter and receiver, has gained a lot of interest over the past decade. Various techniques have been proposed to take advantage of the diversity offered by this Multi Input Multi Output (MIMO) setup. Space time block coding is one of such diversity techniques effective in combating channel fading in wireless communication [1].

In OFDM systems, a Cyclic Prefix (CP) is inserted between two successive symbols as guard interval which not only mitigates Inter Symbol Interference (ISI), but also converts the linear convolution between the transmitted OFDM symbol and channel impulse response to a circular one. At the receiver, the CP corrupted by ISI is generally discarded and the ISI free part of the OFDM symbol is used for channel estimation and data detection.

Various methods have been proposed in literature for estimating channel impulse response in OFDM systems. These include pilot based methods [2] - [4], blind methods [5], [6] and semi blind methods [7] - [10]. Semi blind methods offer a compromise between pilot based and blind algorithms. These methods obtain an initial channel estimate using pilots and then use a variety of a priori constraints to improve it further. Various iterative methods have also been proposed. Such methods iterate between data detection and channel estimation, e.g, Expectation Maximization (EM) based methods [11] - [15].

The authors have previously presented a FB Kalman receiver for STBC based MIMO OFDM systems [16] where it was shown how frequency, temporal and spatial correlation can be jointly used to enhance data aided channel estimation process. The present paper is an extension of [16]. In this paper, we show how the CP can be used to further enhance the estimate. The idea is that since the previous OFDM symbol has been received, and is thus known to the receiver, the ISI caused by it in the CP part of the current OFDM symbol can be removed. The ISI free CP part can then be used to aid the channel estimation and data detection algorithm. This is in direct contrast to standard practice of discarding the CP at the receiver. It should be noted that the use of CP observation to aid joint channel estimation and data detection process for Single Input Single Output (SISO) case was proposed in [15]. Nevertheless, its extension to MIMO STBC case is non trivial as not only it involves scaling up the number of transmit and receive antennas, the use of STBC for the MIMO case complicates the matter further. The main contributions of this paper are:

a) We show how the CP information can be used to enhance the joint channel estimation and data detection process of the STBC MIMO OFDM receiver.
b) We provide implementation of two variants of the EM-based FB Kalman filter, namely Cyclic FB Kalman and Helical FB Kalman for STBC MIMO OFDM and provide their performance comparison.
c) Our system model is transparent and tractable facilitating in depth understanding of the problem at hand.

The paper is organized as follows. Following the introduction, the system model is described in Section II. The CP enhanced EM based FB Kalman algorithm is presented in Section III. Section IV gives two different implementations of the algorithm and compares their computational complexity. Simulation results are discussed in Section V while concluding remarks are presented in Section VI.

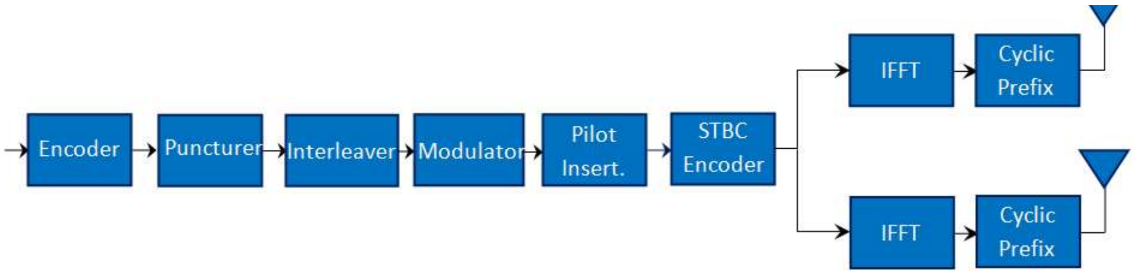

Fig. 1. STBC MIMO OFDM Transmitter.

A. Notation

Scalars are denoted with small-case letters (e.g., x), vectors with small-case boldface letters (e.g., $\mathbf{x}$), and matrices with uppercase boldface letters (e.g., $\mathbf{X}$). Frequency domain vectors are represented as $\check{\mathbf{x}}$ while a hat over a variable indicates an estimate of the variable (e.g., $\hat{\mathbf{h}}$ is an estimate of $\mathbf{h}$). We use $(.)^{\mathrm{H}}$ to denote conjugate transpose, $\otimes$ to denote Kronecker product, $*$ represents linear convolution, $\mathbf{I}_N$ to denote the size $N \times N$ identity matrix and $\mathbf{0}_{M\times N}$ to denote the all zero $M \times N$ matrix. The operation $\operatorname{diag}(\check{\mathbf{x}})$ transforms the vector $\check{\mathbf{x}}$ into a matrix with diagonal $\check{\mathbf{x}}$. Given a sequence of vectors $\mathbf{h}_{r_x}^{t_x}$ for $r_x = 1 \cdots R_x$ and $t_x = 1 \cdots T_x$, we define the following stack variables

$$\mathbf{h}_{r_x} = \begin{bmatrix} \mathbf{h}_{r_x}^1 \\ \vdots \\ \mathbf{h}_{r_x}^{T_x} \end{bmatrix} \quad \text{and} \quad \mathbf{h} = \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_{R_x} \end{bmatrix} \tag{1}$$

II. System Model

This section provides an overview of the STBC MIMO OFDM system under study.

A. Overview of the STBC MIMO OFDM System

A block diagram of the STBC MIMO OFDM transmitter is shown in Figure 1. After encoding, puncturing, interleaving, mapping and pilot insertion, the OFDM symbols are sent to the space time encoder. For a set of N_u uncoded OFDM symbols $\{\check{\mathbf{s}}(1), \ldots, \check{\mathbf{s}}(N_u)\}$ that are to be transmitted over T_x antennas and N_c time slots, the ST coding is performed using the set of $T_x \times N_c$ matrices $\{\mathbf{A}(1), \mathbf{B}(1), \ldots, \mathbf{A}(N_u), \mathbf{B}(N_u)\}$ [17]. The OFDM symbol transmitted from antenna t_x at time n_c is given by [17]

$$\check{\mathbf{x}}_{t_x}(n_c) = \sum_{n_u=1}^{N_u} a_{t_x,n_c}(n_u) \operatorname{Re} \check{\mathbf{s}}(n_u) + j b_{t_x,n_c}(n_u) \operatorname{Im} \check{\mathbf{s}}(n_u) \tag{2}$$

where $a_{t_x,n_c}(n_u)$ is the (t_x, n_c) element of $\mathbf{A}(n_u)$ and $b_{t_x,n_c}(n_u)$ is the (t_x, n_c) element of $\mathbf{B}(n_u)$. At each antenna t_x, the frequency domain OFDM symbol $\check{\mathbf{x}}_{t_x}$ is converted to time domain symbol $\mathbf{x}_{t_x} = N\mathbf{Q}\check{\mathbf{x}}_{t_x}$ where $\mathbf{Q}$ is an $N \times N$ Inverse Discrete Fourier Transform (IDFT) matrix. Each antenna then appends a CP $\underline{\mathbf{x}}_{t_x}$, of length P, to $\mathbf{x}_{t_x}$ and then transmits the $N + P$ length symbol $\bar{\mathbf{x}}_{t_x}$.

B. Channel Model

The channel is assumed to be frequency selective and time variant. It is assumed that the channel response $\mathbf{h}_{r_x}^{t_x}$ between the transmit antenna t_x and receive antenna r_x, with P distinct paths, remains constant over one ST block and changes from the current block to the next according to the dynamical equation

$$h_{r_x,\,t+1}^{t_x}(p) = \alpha(p) h_{r_x,\,t}^{t_x}(p) + \sqrt{(1-\alpha^2(p))e^{-\beta p}}\, u_{r_x,\,t}^{t_x}(p) \tag{3}$$

Here, $u_{r_x,\,t}^{t_x}(p)$ is an iid matrix with entries that are $\mathcal{N}(0, 1)$ and $\alpha(p)$ is related to the Doppler frequency $f_D(p)$ by $\alpha(p) = J_0(2\pi f_D(p) T_s)$ for the p^{th} path. The variable T_s is the time duration of one ST block while β corresponds to the exponent of the channel decay profile while the factor $\sqrt{(1-\alpha^2(p))e^{-\beta p}}$ ensures that each link maintains the exponential decay profile ($e^{-\beta p}$) for all time. By stacking (3) over all taps, transmit and receive antennas, we get

$$\mathbf{h}_{t+1} = (\mathbf{I}_{T_x R_x} \otimes \mathbf{F})\, \mathbf{h}_t + (\mathbf{I}_{T_x R_x} \otimes \mathbf{G})\, \mathbf{u}_t \tag{4}$$

where $\mathbf{u}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_u)$. The matrices $\mathbf{F}$ and $\mathbf{G}$ are a function doppler delay and power delay profile obtained by stacking alpha and the exponential factor that appear in (3). Thus the model (4) captures both time and frequency correlation information (see [16] for further detail on the construction of these matrices).

C. Input/Output Relationship

The input/output relationship is given by

$$\bar{\mathbf{y}}_{r_x}^{t_x} = \mathbf{h}_{r_x}^{t_x} * \bar{\mathbf{x}}_{t_x} + \bar{\mathbf{n}}_{r_x} \tag{5}$$

where $\bar{\mathbf{n}}_{r_x}$ is the noise at receiver r_x. The receiver removes the CP from the received time domain symbol $\bar{\mathbf{y}}_{r_x}^{t_x}$ to obtain the ISI free symbol $\mathbf{y}_{r_x}^{t_x}$. After CP removal, the input/output relation of OFDM system at receive antenna r_x for OFDM symbol n_c is best described in frequency domain as

$$\begin{aligned} \check{\mathbf{y}}_{r_x}(n_c) = {} & [\operatorname{diag}(\check{\mathbf{x}}_1(n_c)) \cdots \operatorname{diag}(\check{\mathbf{x}}_{T_x}(n_c))] \\ & (\mathbf{I}_{T_x} \otimes \mathbf{Q}_{P+1}^{\mathrm{H}}) \mathbf{h}_{r_x} + \check{\mathbf{n}}_{r_x}(n_c) \end{aligned} \tag{6}$$

where $\check{\mathbf{y}}_{r_x}^{t_x}$, $\check{\mathbf{x}}_{r_x}^{t_x}$ and $\check{\mathbf{n}}_{r_x}^{t_x}$ are DFTs of $\mathbf{y}_{r_x}^{t_x}$, $\mathbf{x}_{r_x}^{t_x}$ and $\mathbf{n}_{r_x}^{t_x}$ respectively and $\mathbf{Q}_{P+1}$ is a matrix with only the first $P + 1$ rows of $\mathbf{Q}$. From (5), we deduce that the CP at the input and output is related by linear convolution. Specifically, if we concatenate all CPs at the input into a sequence $\underline{\mathbf{x}}_{t_x}$ and the CPs at the output into the corresponding sequence $\underline{\mathbf{y}}_{r_x}^{t_x}$, then it can be shown that the two sequences are related by linear convolution [18]

$$\underline{\mathbf{y}}_{r_x}^{t_x} = \mathbf{h}_{r_x}^{t_x} * \underline{\mathbf{x}}_{t_x} + \underline{\mathbf{n}}_{r_x} \tag{7}$$

From this, we deduce that the CP of OFDM symbol, $\mathbf{y}_{r_x}^{t_x}$, is related to the input CPs $\underline{\mathbf{x}}_{t_x,t-1}$ and $\underline{\mathbf{x}}_{t_x,t}$ by

$$\underline{\mathbf{y}}_{r_x}(n_c) = \underline{\mathbf{X}}(n_c)\mathbf{h}_{r_x} + \underline{\mathbf{n}}_{r_x}(n_c) \tag{8}$$

where $\underline{\mathbf{X}}$ is constructed from $\underline{\mathbf{x}}_{t_x,t-1}$ and $\underline{\mathbf{x}}_{t_x,t}$ according to equation (9) given on top of the next page. Combining (6) and (8) and collecting them for all N_c symbols, we obtain the total input/output relationship

$$\check{\underline{\mathbf{y}}}_{r_x} = \overline{\mathbf{X}}\mathbf{h}_{r_x} + \check{\underline{\mathbf{n}}}_{r_x} \tag{10}$$

where

$$\check{\underline{\mathbf{y}}}_{r_x} = \begin{bmatrix} \check{\mathbf{y}}_{r_x}(1) \\ \underline{\mathbf{y}}_{r_x}(1) \\ \vdots \\ \check{\mathbf{y}}_{r_x}(N_c) \\ \underline{\mathbf{y}}_{r_x}(N_c) \end{bmatrix}, \quad \check{\underline{\mathbf{n}}}_{r_x} = \begin{bmatrix} \check{\mathbf{n}}_{r_x}(1) \\ \underline{\mathbf{n}}_{r_x}(1) \\ \vdots \\ \check{\mathbf{n}}_{r_x}(N_c) \\ \underline{\mathbf{n}}_{r_x}(N_c) \end{bmatrix}, \text{ and}$$

$$\overline{\mathbf{X}} = \begin{bmatrix} \{\mathrm{diag}(\check{\mathbf{x}}_1(1)) & \cdots & \mathrm{diag}(\check{\mathbf{x}}_{T_x}(1))\}\,(\mathbf{I}_{T_x} \otimes \mathbf{Q}_{P+1}^{\mathrm{H}}) \\ \underline{\mathbf{X}}_1(1) & \cdots & \underline{\mathbf{X}}_{T_x}(1) \\ \{\mathrm{diag}(\check{\mathbf{x}}_1(2)) & \cdots & \mathrm{diag}(\check{\mathbf{x}}_{T_x}(2))\}\,(\mathbf{I}_{T_x} \otimes \mathbf{Q}_{P+1}^{\mathrm{H}}) \\ \underline{\mathbf{X}}_1(2) & \cdots & \underline{\mathbf{X}}_{T_x}(2) \\ \vdots & \cdots & \vdots \\ \{\mathrm{diag}(\check{\mathbf{x}}_1(N_c)) & \cdots & \mathrm{diag}(\check{\mathbf{x}}_{T_x}(N_c))\}(\mathbf{I}_{T_x} \otimes \mathbf{Q}_{P+1}^{\mathrm{H}}) \\ \underline{\mathbf{X}}_1(N_c) & \cdots & \underline{\mathbf{X}}_{T_x}(N_c) \end{bmatrix}$$

By further collecting this relationship over all receive antennas, we obtain

$$\check{\underline{\mathbf{y}}}_t = (\mathbf{I}_{R_x} \otimes \overline{\mathbf{X}}_t)\mathbf{h}_t + \check{\underline{\mathbf{n}}}_t \tag{11}$$

This equation includes the input/output relationship at all frequency bins, for all transmit and receive antennas and for all OFDM symbols including the CPs of the t^{th} ST block.

III. CP Enhanced Joint Estimation Detection Algorithm

In order to facilitate the reader to better understand the working of the CP enhanced EM based joint channel estimation and data detection algorithm, we first consider two extreme scenarios 1) when the input is completely known at the receiver 2) when the input is completely unknown at the receiver. The final algorithm will be a compromise between these two extreme cases owing to its semi blind nature.

A. Known Input Case

Let us first consider the case when the input is completely known at the receiver. For a sequence of $T+1$ input and output ST symbols $\overline{\mathbf{X}}_0^T$ and $\check{\underline{\mathbf{y}}}_0^T$ (where $\overline{\mathbf{X}}_0^T$ denotes the sequence $\overline{\mathbf{X}}_0, \overline{\mathbf{X}}_1, \cdots, \overline{\mathbf{X}}_T$), the maximum a posteriori estimate of $\mathbf{h}_0^T$ is obtained by maximizing the log likelihood function

$$\mathcal{L} = \ln p(\check{\underline{\mathbf{y}}}_0^T | \overline{\mathbf{X}}_0^T, \mathbf{h}_0^T) + \ln p(\mathbf{h}_0^T) \tag{12}$$

which simplifies to

$$\begin{aligned} \mathcal{L} = & -\sum_{t=1}^{T} \|\check{\underline{\mathbf{y}}}_t - (\mathbf{I}_{R_x} \otimes \overline{\mathbf{X}}_t)\mathbf{h}_t\|^2_{\frac{1}{\sigma_n^2}} - \|\mathbf{h}_0\|^2_{\Pi_0^{-1}} \\ & - \sum_{t=1}^{T} \|\mathbf{h}_t - (\mathbf{I}_{T_x R_x} \otimes \mathbf{F})\mathbf{h}_{t-1}\|^2_{(G R_u G^{\mathrm{H}})^{-1}} \end{aligned} \tag{13}$$

The estimate of $\mathbf{h}_t$ is obtained by applying the FB Kalman to the state space model defined by (4) and (10). Starting from initial conditions $\mathbf{h}_{0|-1} = \mathbf{0}$ and $\mathbf{P}_{0|-1} = \mathbf{\Pi}_0$, the FB Kalman filter is given as

Forward run: For $i = 1, \ldots, T$, calculate

$$\mathbf{R}_{e,t} = \sigma_n^2 \mathbf{I}_{T_x R_x N} + (\mathbf{I}_{R_x} \otimes \overline{\mathbf{X}}_t)\mathbf{P}_{t|t-1}(\mathbf{I}_{R_x} \otimes \overline{\mathbf{X}}_t^{\mathrm{H}}) \tag{14}$$

$$\mathbf{K}_t = \mathbf{P}_{t|t-1}(\mathbf{I}_{R_x} \otimes \overline{\mathbf{X}}_t^{\mathrm{H}})\mathbf{R}_{e,t}^{-1} \tag{15}$$

$$\begin{aligned} \hat{\mathbf{h}}_{t|t} = & \left(\mathbf{I}_{T_x R_x (P+1)} - \mathbf{K}_t(\mathbf{I}_{R_x} \otimes \overline{\mathbf{X}}_t)\right)\hat{\mathbf{h}}_{t|t-1} \\ & + \mathbf{K}_t \check{\underline{\mathbf{y}}}_t, \end{aligned} \tag{16}$$

$$\hat{\mathbf{h}}_{t+1|t} = (\mathbf{I}_{T_x R_x} \otimes \mathbf{F})\hat{\mathbf{h}}_{t|t}, \tag{17}$$

$$\begin{aligned} \mathbf{P}_{t+1|t} = & (\mathbf{I}_{T_x R_x} \otimes \mathbf{F})\left(\mathbf{P}_{t|t-1} - \mathbf{K}_t \mathbf{R}_{e,t} \mathbf{K}_t^{\mathrm{H}}\right)(\mathbf{I}_{T_x R_x} \otimes \mathbf{F}^{\mathrm{H}}) \\ & + \mathbf{G}\mathbf{R}_u \mathbf{G}^{\mathrm{H}} \end{aligned} \tag{18}$$

where $N = 2(N_{\text{CP}} + N_{\text{Data}})$ with N_{CP} and N_{data} being the length of CP and data portion of the OFDM symbol.

Backward run: Starting from $\boldsymbol{\lambda}_{T+1|T} = \mathbf{0}$ and for $t = T, T-1, \ldots, 0$, calculate

$$\begin{aligned} \boldsymbol{\lambda}_{t|T} = & \left(\mathbf{I}_{P+N} - (\mathbf{I}_{R_x} \otimes \overline{\mathbf{X}}_t^{\mathrm{H}})\mathbf{K}_t^{\mathrm{H}}\right)(\mathbf{I} \otimes \mathbf{F}^{\mathrm{H}})\boldsymbol{\lambda}_{t+1|T} \\ & + (\mathbf{I} \otimes \overline{\mathbf{X}}_t)\mathbf{R}_{e,t}^{-1}\left(\check{\underline{\mathbf{y}}}_t - (\mathbf{I} \otimes \overline{\mathbf{X}}_t)\hat{\mathbf{h}}_{t|t-1}\right) \end{aligned} \tag{19}$$

$$\hat{\mathbf{h}}_{t|T} = \hat{\mathbf{h}}_{t|t-1} + \mathbf{P}_{t|t-1}\boldsymbol{\lambda}_{t|T} \tag{20}$$

The desired estimate is $\hat{\mathbf{h}}_{t|T}$.

B. Unknown Input Case (The Expectation Maximization Approach)

For the unknown input case, we maximize the log likelihood function *averaged* over the entire sequence $\overline{\mathbf{X}}_0^T$ instead of maximizing over $\overline{\mathbf{X}}_t$. The channel estimate obtained thus, will then be used for data detection. This process is iterated J times to improve the joint estimate and is known as the Expectation Maximization (EM) approach. For the EM case, the log likelihood function is related to the log likelihood in (12) by

$$\overline{\mathcal{L}} = E_{\overline{\mathbf{X}}_0^T | \left(\mathbf{h}_0^T, \check{\underline{\mathbf{y}}}_0^T\right)^{(j-1)}}[\mathcal{L}] \tag{21}$$

which simplifies to

$$\begin{aligned} \overline{\mathcal{L}} = & -\sum_{t=1}^{T} \|\mathbf{h}_t - \mathbf{F}\mathbf{h}_{t-1}\|^2_{(\mathbf{G}\mathbf{R}_u\mathbf{G}^{\mathrm{H}})^{-1}} - \|\mathbf{h}_0\|^2_{\Pi_0^{-1}} \\ & - \sum_{t=0}^{T} \left\|\check{\underline{\mathbf{y}}}_t - \left(\mathbf{I}_{R_x} \otimes E[\overline{\mathbf{X}}_t]\right)\mathbf{h}_t\right\|^2_{\frac{1}{2\sigma_n^2}} \end{aligned} \tag{22}$$

$$\underline{\mathbf{X}}(n_c) = \begin{bmatrix} \underline{x}_{t_x,t}(0) & \underline{x}_{t_x,t-1}(L-1) & \cdots & \underline{x}_{t_x,t-1}(0) \\ \underline{x}_{t_x,t}(1) & \underline{x}_{t_x,t}(0) & \cdots & \underline{x}_{t_x,t-1}(1) \\ \vdots & \ddots & \ddots & \vdots \\ \underline{x}_{t_x,t}(L-1) & \cdots & \underline{x}_{t_x,t}(0) & \underline{x}_{t_x,t-1}(L-1) \end{bmatrix} \quad (9)$$

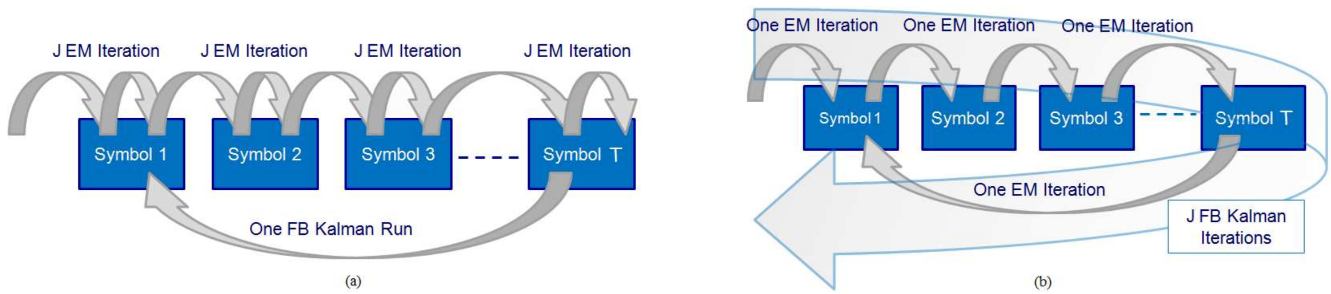

Fig. 2. Graphical view of the two implementations of EM based Kalman filter (a) Helical FB Kalman (b) Cyclic FB Kalman.

where $E[.]$ represents the mean of the unknown input sequence which depends on $\operatorname{Re}\check{\mathbf{s}}(n_u)$ and $\operatorname{Im}\check{\mathbf{s}}(n_u)$ as can be seen from equations (2) and (11) and is given by[1]

$$E[\operatorname{Re}\check{\mathbf{s}}(n_u)|\tilde{\mathcal{Y}}(n_u), \check{\mathbf{h}}] = \frac{\sum_{i=1}^{|\mathcal{R}|} r_i e^{-\frac{|\tilde{\mathcal{Y}}(n_u)-\|\check{\mathbf{h}}\|^2 r_i|^2}{2\sigma_n^2}}}{\sum_{i=1}^{|\mathcal{R}|} e^{-\frac{|\tilde{\mathcal{Y}}(n_u)-\|\check{\mathbf{h}}\|^2 r_i|^2}{2\sigma_n^2}}} \quad (23)$$

where $\mathcal{R}$ is the set of possible values of $\check{\mathbf{s}}$ and $\tilde{\mathcal{Y}}$ is the transformed version of $\check{\mathbf{y}}$ used for data detection (see [16] for details). The channel estimate at the j^{th} iteration of the CP enhanced EM algorithm is obtained by applying the FB Kalman (14)–(20) to the following state-space model

$$\mathbf{h}_{t+1} = (\mathbf{I}_{T_x R_x} \otimes \mathbf{F})\mathbf{h}_t + (\mathbf{I}_{T_x R_x} \otimes \mathbf{G})\mathbf{u}_t \quad (24)$$

$$\check{\check{\mathbf{y}}}_t = \left(\mathbf{I}_{R_x} \otimes E[\overline{\mathbf{X}}_t]\right)\mathbf{h}_t + \check{\check{\mathbf{n}}}_t \quad (25)$$

C. Initial Channel Estimation

In practice the OFDM symbol will consist of unknown data sequence along with known pilot sequence. The initial channel estimate is thus obtained using the dynamical channel model (4) together with a version of (10) pruned of all data symbols reproduced here for convenience

$$\mathbf{h}_{t+1} = (\mathbf{I}_{T_x R_x} \otimes \mathbf{F})\,\mathbf{h}_t + (\mathbf{I}_{T_x R_x} \otimes \mathbf{G})\,\mathbf{u}_t \quad (26)$$

$$\check{\check{\mathbf{y}}}_{t_{I_p}} = (\mathbf{I}_{R_x} \otimes \overline{\mathbf{X}}_{t_{I_p}})\mathbf{h}_t + \check{\check{\mathbf{n}}}_{t_{I_p}} \quad (27)$$

where I_p denotes the pilot position index set.

D. Data Aided Semi blind Channel Estimate

The initial channel estimate obtained using the pilots is used to initialize the iterative FB Kalman algorithm. The subsequent iterations make use of the soft data EM approach outlined in subsection III-B above.

[1]The interested reader is requested to go through Appendix B of [16] to understand how to calculate this moment.

IV. Two Implementations of the EM based FB Kalman Filter

When the EM algorithm is used in conjunction with the FB Kalman filter, we are presented with two possible approaches to implement the iterative procedure. We can 1) iterate over channel estimation and data detection for each symbol 2) or iterate over FB Kalman runs for a sequence of T symbols. The two implementations are named Helical and Cyclic FB Kalman based on the similarity to their iterative process to these shapes.

A. Helical FB Kalman

In Helical FB Kalman filter, J iterations are performed between channel estimation and data detection for each OFDM symbol, where the channel estimate obtained is used to improve the data estimate and vice versa, until the T^{th} OFDM symbol is processed and then the backward run of the Kalman filter is performed once. Here the EM approach is used to improve the estimation/detection process at *each* OFDM symbol. The name is derived from the fact that this implementation seems to move in a Helical manner with respect to the time axes as shown in Figure 2(a).

B. Cyclic FB Kalman

In Cyclic FB Kalman filter, a sequence of T OFDM symbols is first processed by the forward and backward runs of the Kalman filter. The channel estimate thus obtained is used for data detection which is then used to start the subsequent iteration of the FB Kalman. This process is repeated J times before the next batch of T OFDM symbols is processed. The implementation is named so as it draws circles of iterations between channel estimation and data detection as shown in Figure 2(b).

C. Computational Complexity

Let C_{M} be the complexity of calculating the moment $E[\overline{\mathbf{X}}]$ while C_{F} and C_{B} be the complexity of the forward and backward runs of the Kalman filter respectively. Both the forward and backward runs require a matrix inversion, consequently C_{F} and C_{B} are $\mathcal{O}\left([T_x R_x N]^3\right)$. Moment calculation requires matrix multiplication hence $C_{\mathrm{M}} \sim \mathcal{O}(N_{\mathrm{Data}}^2)$. For a frame of T OFDM symbols, the complexity of the Helical FB Kalman calculates to $T \times [J \times (C_{\mathrm{F}} + C_{\mathrm{M}})] + C_{\mathrm{B}}$ while that of Cyclic FB Kalman calculates to $J \times (T \times C_{\mathrm{F}} + C_{\mathrm{B}} + C_{\mathrm{M}})$.

V. Simulation Results

We implement the MIMO OFDM system using the transmitter and receiver presented in Figures 1 and 5 respectively. We use a 1/2 rate convolutional encoder as an outer encoder and 16-QAM as the modulation scheme. The commonly used rate-1 Alamouti code is implemented with two transmitters ($T_x = 2$) and two time slots ($N_c = 2$) [17]. Moreover, the number of receivers is also set to 2 ($R_x = 2$). A MIMO channel model similar to the one presented in [16] is used with parameters, $\alpha = 0.85$, $\beta = 0.2$, $P = 16$, and transmit and receive correlation matrices

$$\boldsymbol{T}(p) = \begin{bmatrix} 1 & \zeta \\ \zeta & 1 \end{bmatrix} \quad \text{and} \quad \boldsymbol{R}(p) = I$$

where $\zeta = 0.2$. Each data packet consists of 12 OFDM symbols transmitted over 6 Alamouti blocks. Each OFDM symbol consists of 64 frequency tones and a CP of length 16. The number of EM iterations is fixed at 2. The first ST block comprises of 16 pilots while the number of pilots in subsequent blocks remains fixed at either 6 or 12. In the following subsections, these two cases will be referred as "12 pilots" and "6 pilots" cases.

A. Cyclic vs Helical EM-based FB Kalman

Figure 3 compares the performance of the two implementations of EM-based FB Kalman discussed in Section IV when 12 pilots are used. It demonstrates the performance of both implementations when no outer code is used and we also compare it with the coded data case. It can be seen that the performance of both the Cyclic and Helical implementations is very close in case of uncoded data while the Helical implementation easily outperforms the Cyclic one in the coded data case. It can be observed that in the coded data case, the gain in SNR for Helical implementation at a BER of 10^{-6} is around 1.5 dB. We also study how the performance of the two different implementations of EM-based FB Kalman is affected by different number of pilots. Figure 4 presents the performance of both implementations for the cases when "6 pilots" and "12 pilots" are used. It is observed that the performance of both the implementations improves when more pilots are used and the performance of Helical implementation is better than Cyclic one in both the cases.

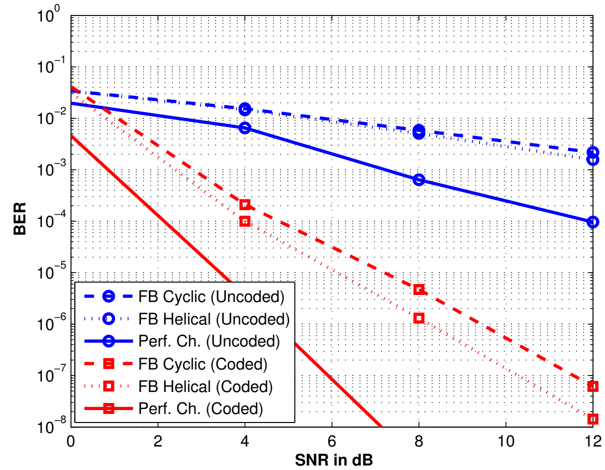

Fig. 3. Performance comparison for coded and uncoded data

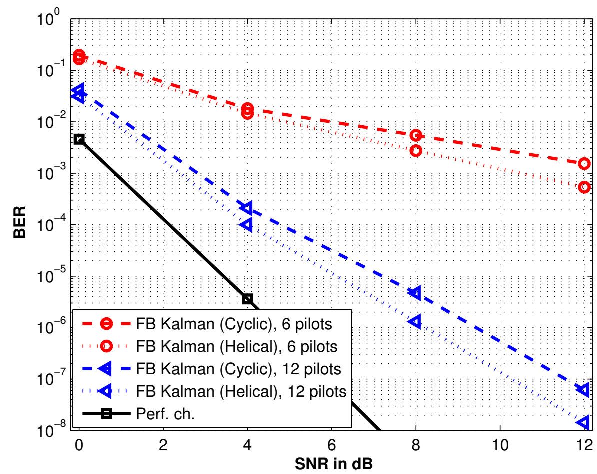

Fig. 4. Performance using different number of pilots

B. Enhanced Performance Using CP

In the previous subsection it was observed that Helical EM-based FB Kalman implementation performs better than the Cyclic implementation. In this subsection, we study the gain in performance by utilizing the CP information available instead of discarding it (as is done generally). Figure 6 compares the performance of both Cyclic and Helical EM-based FB Kalman implementations when CP information is used. We note that the performances of both algorithms improve when CP is used.

VI. Conclusion

This paper proposes an iterative CP enhanced data aided channel estimation algorithm for STBC MIMO OFDM systems. In contrast to the standard approach of discarding the CP at the receiver, the paper shows how it can be used to aid the joint channel estimation and data detection process. The algorithm utilizes both the frequency and time correlation

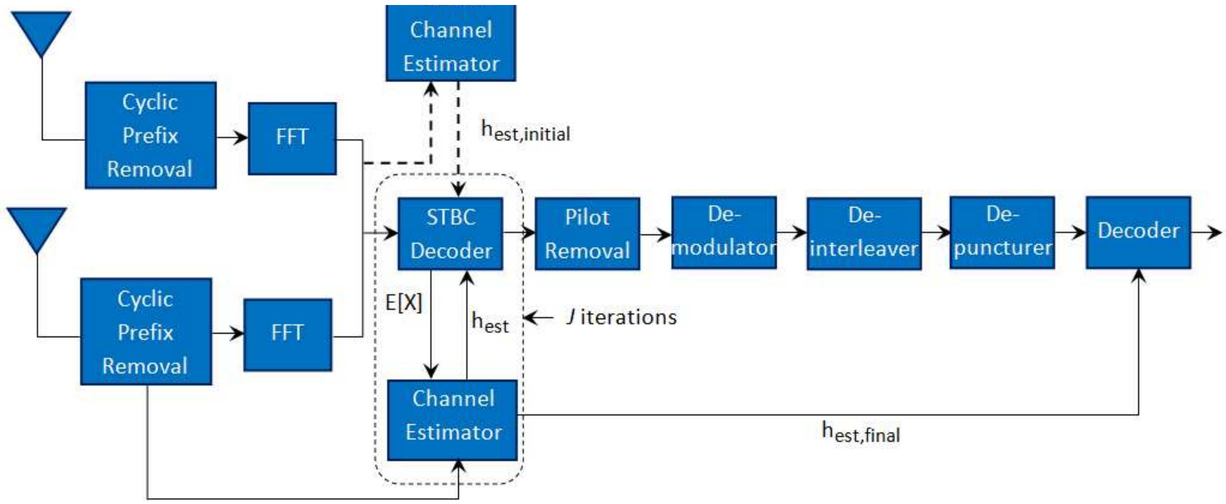

Fig. 5. CP enhanced STBC MIMO OFDM receiver.

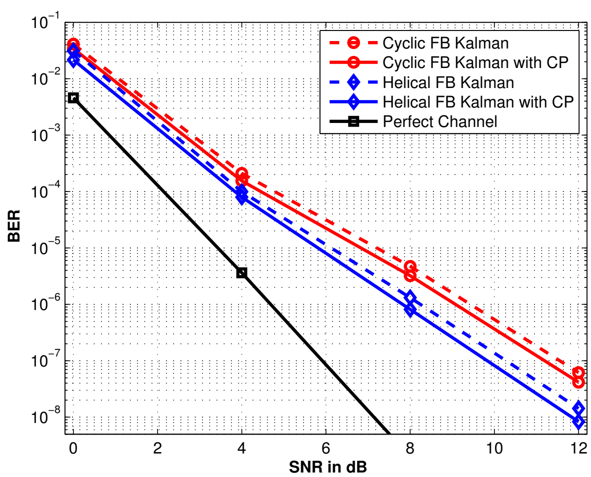

Fig. 6. Enhancement in performance using the CP information

information at the receiver leading to a CP enhanced EM based FB Kalman filter. Taking advantage of the iterative nature of the algorithm, two different implementations of the FB Kalman filter (Helical and Cyclic) are presented. The simulation results show that the low complexity Helical implementation of EM-based FB Kalman performs better than the Cyclic implementation and the use of CP improves the performance of both the implementations.

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