

**ADAPTIVE BLIND EQUALIZERS ALGORITHMS**

**TERM 001**

**EE 514**

**DETECTION & ESTIMATION**

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## **ABSTRACT**

*Signals when pass through a channel undergo various forms of distortion, most common of which is Inter-symbol-interference, so called ISI. Inter symbol interference induced errors can cause the receiver to misinterpret the received samples. Equalizers are an important part of receivers, which minimizes the linear distortion produced by the channel. If channel characteristics are known a priori, than optimum setting for equalizers can be computed. But in practical systems the channel characteristics are not known a priori, so adaptive equalizers are used. Adaptive equalizers adapt, or change the value of its taps as time progresses. There are two main types of adaptive equalizers, trained equalizers and blind equalizers. In trained equalizers there is a pseudo-random pattern of bits called training sequence known both to receiver and transmitter. But equalizers for which such a initial training period can be avoided are called **BLIND EQUALIZERS**. Blind equalizer as opposed to data trained equalizer, is able to compensate amplitude and delay distortion of a communication channel using only channel output sample and knowledge of basic statistical properties of the data symbol. Among some algorithms of blind equalizers like CMA, Stop and Go, GSA, SGA, SRCA etc., Stop and Go is one of the most important algorithms. One of the major disadvantage of Blind Equalizers is that all blind equalizers converge very slowly. However, one can reduce the convergence time by employing some other techniques. In this term paper, I simulate the various algorithms of Blind Equalization including:*

- ✓ Sato's Algorithm
- ✓ Godard Algorithm (Constant Modulus Algorithm)
- ✓ Stop and Go Algorithm (Picchi and Prati Algorithm)

## **1. INTRODUCTION:**

A Blind Equalizer, as opposed to a data-trained equalizer, is able to compensate amplitude and delay distortions of a communication channel using only the channel output samples and the knowledge of the basic statistical properties of the data symbols. Sato was the one who first introduced the concept of blind equalizers in 1975, since than blind equalization has attracted significant scientific interest due to its potentials in terms of Overhead reduction and Simplification of point to multipoint communication. Training sequences sacrifice bandwidth in order to assist in order to determining the CIR, hence assisting equalization. Blind equalizers do not need any training sequence and therefore conserve bandwidth. Similarly, it offers lot of simplicity in multipoint communication. When a communication link is reset, equalizer adjustment from scratch is necessary. In this case, using a training sequence is inefficient, since the transmitter has to retransmit the training sequence specifically for each receiver which is reset.

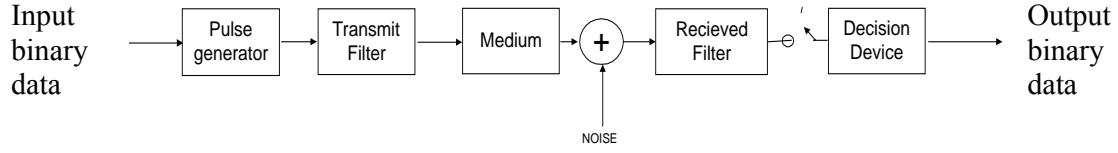
## **2. ADAPTIVE EQUALIZATION:**

In digital communications, a considerable effort has been devoted to the study of data transmission system that utilizes the available channel bandwidth efficiently. The objective here is to design a system that accommodates highest possible rate of data transmission, subject to a specified reliability that is usually measured in terms of the error rate or average probability of symbol error. The transmission of digital data through a linear communication channel is limited by two factors.

- 1- Inter Symbol Interference. (ISI)
- 2- Thermal noise

ISI is caused by dispersion in the transmit filter, the transmission medium, and receive filter. The receiver at its front end generates thermal noise.

For bandwidth-limited channels (e.g., voice-grade telephone channels), we usually find that inter symbol interference is the chief determining factor in the design of high



data rate transmission system.

Block diagram in Figure shows the equivalent base band model of binary pulse-amplitude modulation (PAM) system. The signal applied to the input of the transmitter part of the system consists of binary data sequence, in which each symbol consists of 1 or 0. This sequence is applied to the pulse generator, the output of which is filtered first in the transmitter, then by the medium, and finally in the receiver. Let  $u(k)$  denote the sampled output of the receiver filter in the block diagram; the sampling is performed in synchronism with the pulse generator in the transmitter. The output is compared with a threshold by means of a decision device. If the threshold is exceeded, the receiver makes a decision in favor of symbol 1. Otherwise it decides in favor of symbol 0

Let  $a_k$  be a scaling factor defined by,

$$a_k = \left\{ \begin{array}{l} +1 \text{ if the input consists of symbol 1} \\ -1 \text{ if the input consists of symbol 0} \end{array} \right\}$$

Then in the absence of thermal noise, we may express

$$\begin{aligned} u(k) &= \sum_n a_n \times p(k-n) \\ &= a_k \times p(0) + \sum_n a_n p(k-n) \quad (n \neq 0) \end{aligned}$$

where  $p(n)$  is the sampled version of the impulse response of the cascaded connection of the transmitter filter, the transmission medium, and the receiver filter. The first term of the right hand side of equation (1) defines the desired symbol, whereas the remaining series represents the inter symbol interference caused by the channel. The inter symbol interference, if left unchecked, can result erroneous

decisions when the sampled signal at the channel output is compared with some pre assigned threshold by means of decision device.

To overcome the ISI, control of the time-sampled function  $p(n)$  is required. In principle, if the characteristics of the transmission medium are known precisely, then it is virtually always possible to design a pair of transmit and receive filters that will make the effect of intersymbol interference (at sampling times) arbitrary small. But the use of a fixed pair of transmits and receives filters, designed on the basis of average channel characteristics, may not adequately reduce the ISI.. This suggests the need for an “Adaptive Equalizer” that provides precise control over the time response of the channel.

Among the basic philosophies for equalization of data transmission systems are pre equalization at the transmitter and pre equalization at the receiver. Since the former technique require the use of feed back path, we will only consider equalization at the receiver, where the adaptive equalizer is placed after the receive filter. In theory, the effect of ISI may be made arbitrary small by making the number of adjustable coefficients (tap weights) in the adaptive equalizer infinitely large.

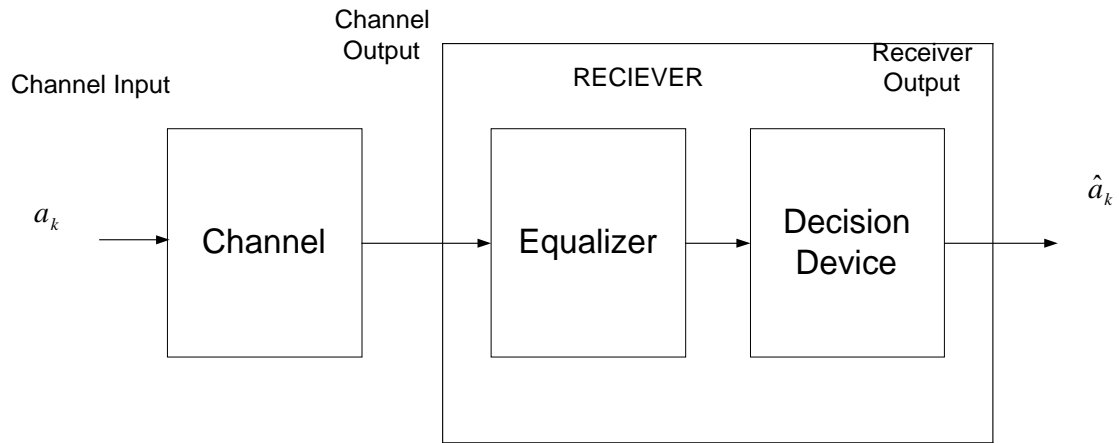
An adaptive filtering algorithm requires the knowledge of the “desired” response so as to form the error signal needed for adaptive process to function. In theory, the transmitted sequence (originating at the transmitter output) is the “desired” response for the adaptive equalization. In practice, however, with the adaptive equalizer located at the receiver, the equalizer is physically separated from the origin of its ideal desired response.

One method to generate the desired response locally in the receiver is the use of training sequence, in which a replica of the desired response is stored in the receiver.

Naturally the generator of this stored reference has to be electronically synchronized with the known transmitted sequence. With a known training sequence, the adaptive filtering algorithm used to adjust the equalizer coefficients corresponds

mathematically to searching for the unique minimum of quadratic error-performance surface.

Second method is decision directed method, in which, a good facsimile of the transmitted sequence is being produced at the output of the decision device in the receiver. Accordingly, if this output were the correct transmitted sequence, it may be used as the “desired” response for the purpose of Adaptive Equalization.



**Channel Equalization in a Data Communication System**

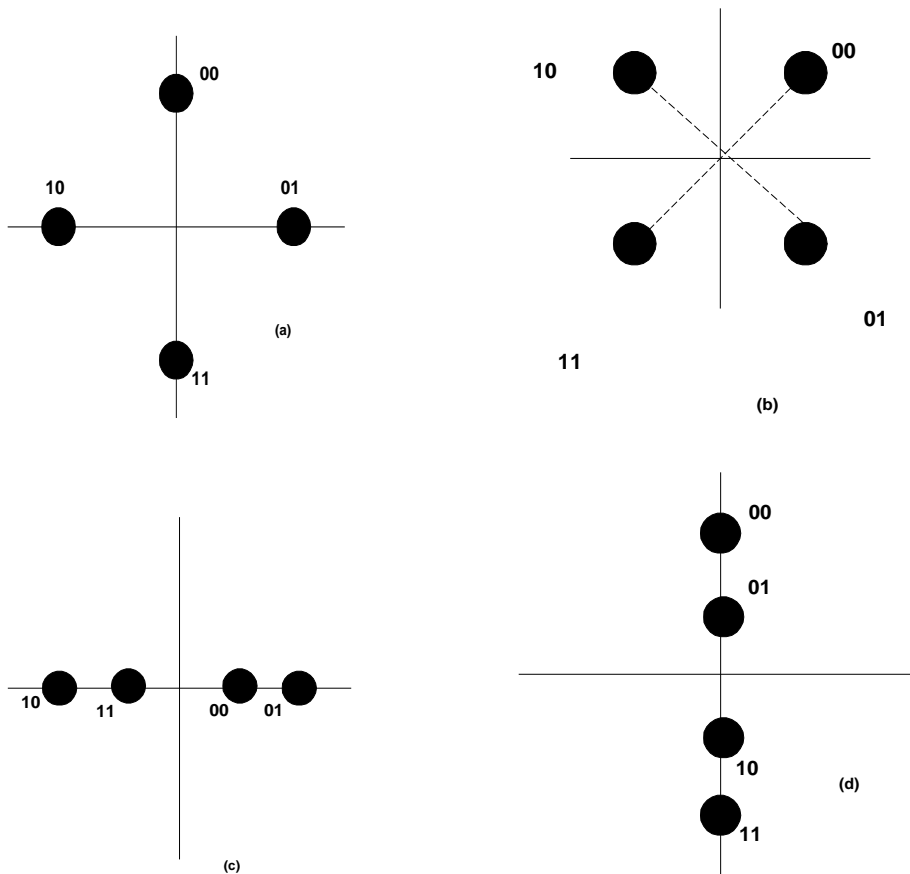
### **3. BLIND EQUALIZATION:**

Blind Equalization perform channel equalization without the aid of a training sequence. The term blind is used because the equalizer performs the equalization blindly on the data without a reference signal. Instead the blind equalizer relies on the knowledge of signal’s structure and its statistics to perform the equalization. The major advantage of blind equalizers is that there is no training sequence at the start up, hence no bandwidth is wasted by its transmission. The major drawback is that the equalizer will typically take a longer to converge as compared to a trained equalizer. The need for blind equalizers in the field of data communications is greatly discussed by Godard[2], in the context of multipoint networks. Blind joint equalization and carrier recovery may also find application in digital radio link over multipath fading

channels. Moreover, in highly non stationary environments like digital mobile communications, it is impractical to use training sequences. Furthermore, application of blind equalization is also important, in other areas, such as geographical signal processing.

### 3.1 QAM SIGNALS

Suppose in a typical communication system, if the data rate is 32kb/s, and the signaling rate is 16kb/s, than every bit transmitted must carry two bits of information. This mean that we must have four points on the constellation, and clearly this can be done in many ways. Figure shows some four points constellation.



The two bits of information associated with each constellation point are marked on the figure. In part (a) and (b) so called quadrature modulation has been used as the points can only be uniquely described using two orthogonal coordinate axes, each passing

through the origin. The orthogonal coordinate axes have a phase rotation of 90 degrees with respect to each other, and hence they have so called quadrature relationship. The pair of coordinate axes can be associated with a pair of quadrature carriers, normally have a sine and cosine waves, which can be independently modulated and then transmitted within the same frequency band. Due to their orthogonality they can be separated by the receiver. This implies that whatever symbol is chosen on one axis it will have no effect on the demodulated on the y axis. Data can therefore be independently transmitted via these two quadrature or orthogonal channels without any increase in error rate. For the constellation of part (a) and (b) we have a constant amplitude signal, but the carrier phase values at the beginning of each symbol period in figure (b) would be either 45 deg, 135 deg, 225 deg or 315 deg. These are two magnitude values and two phase values for the constellations in figure (c) and (d). In general, grouping n bits into one signaling symbol yields  $2^n$  constellation points, which are often referred to as phasors, or complex vectors.

### **3.2 BLIND EQUALIZATION IN "QAM" DATA TRANSMISSION SYSTEMS**

A typical QAM (Quadrature amplitude modulated) data transmission system consists of transmitter, a channel, and a receiver., where the unknown channel represents all the interconnections between the transmitter and the receiver. The transmitter generates a zero mean, independent input data sequence  $\{a_k\}$ , each element of which comes from a finite alphabet A of the QAM symbols (or constellation). The data sequence  $\{a_k\}$  is sent through the channel whose output  $\{x_k\}$  is the receiver input. The function of the receiver is to restore the original data  $\{a_k\}$  from the observation  $\{x_k\}$  by outputting a sequence of estimate  $\{\hat{a}_k\}$ . Assume the channel is linear, causal, and (BIBO) stable with transfer function  $H(q^{-1})$ . Its input-output relation can be written as

$$H(q^{-1}) \times a_k = \sum_{i=0}^{\infty} h_i \times a_{k-i} , \quad a_{k-i} \in A \subset \Delta , \quad h_i \in \Delta$$



such that more than one non-zero element exists in Where  $\{h_i\}$  is the impulse response of the channel. When the channel is ideal with no ISI such that only one non-zero element exists in the sequence  $\{h_i\}$ , then the channel output becomes

$$x_k = h_v \times a_{k-v} \quad , \quad h_v \neq 0, \quad v \in Z_+$$

Which is simply the scaled version of the input with finite delay  $v$ . In this ideal case, the original sequence can be recovered by removing the constant scaling factor through an appropriate memory less nonlinear decision device. The difficulty arises when the channel is imperfect such that more than one non-zero element in the channel impulse response. In this case an undesirable ISI is introduced at the channel output from which a simple memory less decision device might not be able to recover the original data sequence.

A linear channel equalizer is a linear filter  $G(q^{-1})$  applied to the channel output to eliminate its ISI by essentially canceling the channel dynamics. All the ISI is removed if

$$G(q^{-1}) H(q^{-1}) = cq^{-v} \quad , \quad 0 \neq c \in \Delta \quad , \quad v \in Z_+$$

such that the equalizer output becomes  $z_k = ca_{k-v}$ . The desired response can be rewritten as

$$G(q^{-1}) = cq^{-v} H(q^{-1}) \quad ,$$

Which means that the equalizer attempts to achieve the inverse of the channel transfer function with a possible gain difference and/or a constant time delay.

In blind equalization, the original sequence is unknown to the receiver except for its probabilistic or statistical properties over the known alphabet  $A$ . Based on the second-order statistics of the input and output signal of the channel, the spectral magnitude of the channel can be determined and so that of the channel inverse. Thus if the channel is known to be of minimum or maximum phase, only second order statistics need to be used to identify  $H(q^{-1})$ , with which one can construct, respectively, a truncated causal or an anticausal inverse of the channel using an FIR Equalizer. Unfortunately, communication channels usually have mixed phase. Hence, the blind equalization problem for general channels cannot be solved through schemes

that only use second order statistics. If the input sequence is gaussian, so is the output. Therefore, only second order statistics are useful if the input is gaussian which means that blind equalization is unattainable for gaussian input with mixed-phase channels.

In summary, the following facts about the blind equalization are generally noted.

- A mixed-phase linear dynamical channel is identifiable from its output (and knowledge of the distribution of its input) only when the input is not gaussian.
- Second order statistics alone are generally insufficient for blind equalization.
- No channel transfer function zeros are allowed on the unit circle, since spectral nulls require a linear equalizer with infinite gain to compensate.
- An exact causal inverse of a nonminimum-phase channel is unstable. However an anticausal representation can be truncated and delayed with a finite delay  $v$  to enable a causal equalizer to approximate the desired response.
- Most practical QAM systems employ a constellation, which has symmetrical properties. If the input data is independent and uniformly distributed over constellation, a  $k\pi/2$  phase rotation does not cause any statistical changes in the channel output. Thus the data recovered from blind equalization will be subjected to a phase ambiguity of  $k\pi/2$ , and the best possible result would be

$$G(q^{-1})H(q^{-1}) = e^{k\pi/2}q^{-v}, k \in \{0,1,2,3\}$$

Thus remaining phase ambiguity can be resolved through differential encoding.

### **3.3 BLIND EQUALIZATION ALGORITHMS**

Sato was the first who introduced the idea of blind equalization for multilevel pulse amplitude modulation, and after it Godard combined Sato idea with a DD algorithm and obtain a new blind equalization scheme for QAM data transmission. Sato proposed algorithm which was designed only for real valued signal and PAM. However, its complex valued extension is straightforward which was derived by Godard.

### **3.3.1) Sato Algorithm**

Sato's pioneering contribution in 1975, describe first blind equalization algorithm proposed, which was designed for real-valued signals only. However, its extension to complex-valued signals and QAM is straightforward. Sato algorithm dedicated to real valued signal  $z(n)$ , which uses the following cost function:

$$J(n) = E [ ( z(n) - \gamma )^2 ]$$

Where  $\gamma$  is the sato's scaling coefficient and  $E[\ ]$  represents the expectation over all possible transmitted data sequence, and  $z(n)$  is the equalizer output. It is clear that this cost function is forcing the absolute value of the equalized signal to a fixed value  $\gamma$ . For these multilevel constellations the minimization of the Sato cost function of above equation, may not seem to lead to correct update of the equalizer taps at each iteration. Sato's coefficient update equation is given as:

$$C(n+1) = C(n) - \mu y(n) \varepsilon^{\text{sato}}(n)$$

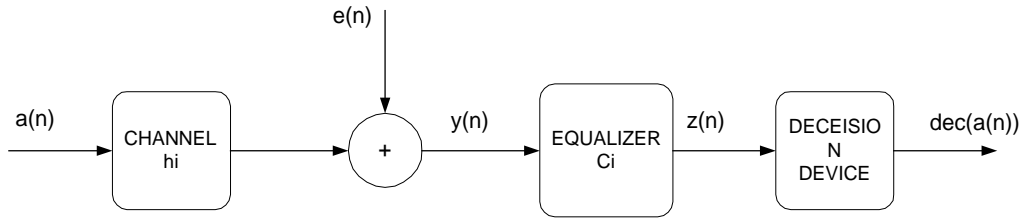
Where  $\varepsilon^{\text{sato}}(n)$  is the Sato's error defined as:

$$\varepsilon^{\text{sato}}(n) = z(n) - \gamma \text{csgn}(z(n))$$

so taps of equalizer are updated according to the equation defined above. As it can be seen from above equation, this algorithm uses only the sign of equalized output values  $z(n)$  in order to update equalizer coefficients  $C(n)$ . Setting the value of  $\gamma$  in above equation is very important, since it actually directs the signal  $z(n)$  to the point of its convergence that is to the original constellation points. A way of achieving this is by constraining the mean value of the error update term of above equation is zero. Therefore, the optimum value is set for  $\gamma$  in the minimum mean squared error sense. The optimum value of  $\gamma$  set by Sato in order to achieve minimum MSE is given by:

$$\gamma = E [ a(n)^2 ] / E [ \text{abs}( a(n) ) ]$$

which is only for real valued signals, where  $a(n)$  is the signal to be transmitted. A typical equalization scheme looks like:



**EQUALIZED COMMUNICATION SYSTEM**

### **3.3.2) Constant Modulus Algorithm (CMA)**

Godard algorithm which he developed for complex valued signal is the most popular scheme for blind equalization of QAM signals. The CMA attempts to minimize the constant modulus cost function  $J_{CM}$ . CMA adjust the taps of equalizer in an attempt to minimize the difference between samples squared magnitude and Godard dispersion constant  $R_2$ , which depends only on input data symbols  $\{a_k\}$ . So value of  $J_{CM}$  depends on difference between squared magnitude of received samples  $z(n)$  and the Godard dispersion constant  $\gamma$ .

$$J_{CM} = E [ ( \text{abs}( z(n) )^2 - R_2 )^2 ]$$

Where  $z(n)$  is the equalizer output at time  $n$ . The equalizer coefficient update equation in CMA uses a gradient descent to minimize  $J_{CM}$ . The equation is given by Godard as,

$$C(n+1) = C(n) - \mu y(n) z(n) [ ( \text{abs}( z(n) )^2 - R_2 ]$$

Where in order to find out the value of  $R_2$ , Godard uses the exactly same method as used by Sato, to obtain the value of  $R_2$ ,

$$R_2 = E [ a(n)^4 ] / E [ \text{abs}( a(n) )^2 ]$$

### **3.3.3) Modified Constant Modulus Algorithm (CMA)**

Another modified version of Godard's CMA was proposed by Wesolowsky, employing a cost function, which relies on both the real and imaginary parts of the equalized signal  $z(n)$ .

$$J(n) = E [ ( \text{abs}( \text{Re}\{z(n)\} )^2 - R_{2,R} )^2 + ( \text{abs}( \text{Im}\{z(n)\} )^2 - R_{2,I} )^2 ]$$

The idea behind this cost function, as compared to the CMA cost function, is that both the real and imaginary parts of the signal are forced to a constant value and, therefore, the random phase ambiguity of CMA now becomes only 90 degrees. This is meaningful in a pure phase modulation, in which case the CMA may converge to an arbitrary phase shifted solution. For QAM though, the 90 degree symmetry of the constellation make it possible for both algorithm to converge to a 90 degree phase shifted solution. Now here the coefficient update equation is:

$$C(n+1) = C(n) - \lambda y(n) \cdot [\text{Re}(z(n)) \cdot ((\text{Re}(z(n))^2 - R_{2,R}) + j \text{Im}(z(n)) \cdot ((\text{Im}(z(n))^2 - R_{2,I}))].$$

The values of  $R_{2,R}$  and  $R_{2,I}$  can be calculated by exactly the same method as used by Sato and Godard in their algorithms.

$$R_{2,R} = E [\text{Re}(a(n))^4] / E [\text{Re}(a(n))^2]$$

$$R_{2,I} = E [\text{Im}(a(n))^4] / E [\text{Im}(a(n))^2]$$

Wesolowsky also showed that the MCMA exhibits slightly faster convergence than the classical CMA, in particular for medium distortion channels. He also showed that his algorithm offers slightly better steady state performance.

### **3.3.4 STOP AND GO" DECISION DIRECTED ALGORITHM FOR BLIND EQUALIZATION**

In my term paper, I am using "STOP AND GO" decision directed algorithm, proposed by **GIORGIO PICCHI** and **GIANCARLO PRATI**, which is not based on the knowledge of complex output error  $e_n$ .

The term "STOP AND GO" indicates that every time, tap coefficients are not adapted, instead the reliability of error is checked and then the algorithm decides either the tap coefficients should be adapted or not. It is worth exploring the possibility of retaining the advantages of simplicity and smoothness of the decision directed algorithm while attempting to substantially improve its blind convergence capabilities. The basic idea in the paper of **picchi prati** was that this possibility could be achieved by stopping adaptation when the reliability (in a probabilistic sense) of the self-decided output error is not high enough. More precisely an easy to generate "binary-valued flag" tells the equalizer whether the output error on the current decision may be reliably

used in the standard Decision Directed algorithm. If not, adaptation is stopped for that iteration.

The stop and go operation made of the standard DD algorithm is an attractive solution of the problem of blind joint equalization and carrier recovery. As a matter of fact, this algorithm has been successfully implemented and tested in a new commercial 64-QAM digital radio system.

In this algorithm, two SATO-like errors

$$\tilde{e}_{n,R} = y_{n,R} - (\text{sgn } y_{n,R})\beta_n$$

$$\tilde{e}_{n,I} = y_{n,I} - (\text{sgn } y_{n,I})\beta_n$$

are generated.

where  $y_{n,R}$  and  $y_{n,I}$  are the real and imaginary components of the output of the equalizer.  $\beta_n$  being a suitable real value possibly changing with  $n$ , and used to determine on which intervals of  $y_R$  and  $y_I$  axes, the error on the decided symbol may be used for adaptation. More specifically, the DD Algorithm now uses the following **binary-valued flags**.

$$f_{n,R} = \begin{cases} 1 & \text{if } \text{sgn } \hat{e}_{n,R} = \text{sgn } \tilde{e}_{n,R} \\ 0 & \text{if } \text{sgn } \hat{e}_{n,R} \neq \text{sgn } \tilde{e}_{n,R} \end{cases}$$

$$f_{n,I} = \begin{cases} 1 & \text{if } \text{sgn } \hat{e}_{n,I} = \text{sgn } \tilde{e}_{n,I} \\ 0 & \text{if } \text{sgn } \hat{e}_{n,I} \neq \text{sgn } \tilde{e}_{n,I} \end{cases}$$

where  $\hat{e}_n$  is the estimated error, determined by the relation.

$$\hat{e}_n = (y_{n,R} - \hat{d}_{n,R}) + j(y_{n,I} - \hat{d}_{n,I})$$

Note that  $\hat{d}_n$  is the decision made in each interval that our output of equalizer  $y_n$  is nearest to which one signal in the given 64-QAM-signal constellation.

When the event  $\{\text{sgn } \hat{e}_{n,R} = \text{sgn } \tilde{e}_{n,R}\}$  occurs and the choice of  $\beta_n$  is proper, the conditional error probability,

$$\begin{aligned}
P_{n/go} &= P\{no\_error \mid go\} = P\{\text{sgn } \hat{e}_n, R \\
&= \text{sgn } e_n, R \mid \text{sgn } \hat{e}_n, R = \text{sgn } \tilde{e}_n, R\} \\
&= P\{\text{sgn } \hat{e}_n, I = \text{sgn } e_n, I \mid \text{sgn } \hat{e}_n, I = \text{sgn } \tilde{e}_n, I\}
\end{aligned}$$

is sufficiently high, and  $\text{sgn } \hat{e}_n, R$  may be confidently used in the DD Algorithm. The two flags  $f_n, R$  and  $f_n, I$  therefore restrict operation of algorithm to a region of higher reliability. It can be shown that by a direct measure of some performance parameters, a proper choice of  $\beta_n$  can be made, favorable on the average. Therefore  $\beta_n$  is used as a reference point to identify regions on each axis where  $\hat{e}_n, R$  and  $\hat{e}_n, I$  may be used more confidently. The conditional error probability  $p_{n/go}$  and its complement  $q_{n/go} = P\{error \mid go\} = 1 - p_{n/go}$  corresponds in a sense to  $p_n$  and  $q_n$ . The performance related to the probabilities is better than all other algorithms.

$$\begin{aligned}
p_{n/go} &= P\{no\_error \mid go\} = P\{\text{sgn } \hat{e}_n, R \\
&= \text{sgn } e_n, R, f_n, R = 1\} = P\{\text{sgn } \hat{e}_n, R = \text{sgn } e_n, R, \\
&\text{sgn } \hat{e}_n, R = \text{sgn } \tilde{e}_n, R\} = (1 - p_{sc})p_n
\end{aligned}$$

$$\begin{aligned}
q_{n/go} &= P\{error \mid go\} = P\{\text{sgn } \hat{e}_n, R \\
&\neq \text{sgn } e_n, R, f_n, R = 1\} = P\{\text{sgn } \hat{e}_n, R \neq \text{sgn } e_n, R, \\
&\text{sgn } \hat{e}_n, R = \text{sgn } \tilde{e}_n, R\} = (1 - p_{se})q_n
\end{aligned}$$

A proper choice of  $\beta_n$  causes  $q_{n/go}$  to be sufficiently smaller than  $q_{n/DD}$ , that is, adaptation stops with probability  $p_{se}$  when a sign error occurs for a component of  $\hat{e}_n$ . This makes the Decision Directed Algorithm converge.

For the baud-rate feed forward linear equalizer adopting the stop-and-go algorithm, the filter coefficient update equation is given as

$$c(n+1) = c(n) + e^{SG}(n)^*$$

where  $\alpha$  is the adaptation step size,  $c^T(n) = [c_0(n), c_1(n), c_2(n), \dots, c_{N-1}(n)]$  is the coefficient vector,  $x^T(n) = [x(n), x(n-1), \dots, x(n-N+1)]$  is the input vector,  $e^{SG}(n)$ , produced by processing the equalizer output  $y(n)$ , is the error signal at time  $n$ , and  $N$  is the number of equalizer tap coefficients.

The stop and go Algorithm uses the Decision Directed algorithm (DD) error signal and allows coefficient updates only when the DD error signal is considered to be reliable.

The DD error signal is given by

$$\hat{e}(n) = y(n) - \hat{d}(n)$$

where  $\hat{d}(n)$  is the decision made on the data symbol  $a(n-D)$  from  $y(n)$  and  $D$  is the overall propagation delay of the channel and the equalizer. A method of judging the reliability of the DD error signal is to see whether the sign of the DD error coincides with a sato-like error

$$\tilde{e}(n) = y(n) - \beta^{SG} [\text{sgn}(yR(n)) + j \text{sgn}(yI(n))]$$

where  $R$  and  $I$  denotes the Real and Imaginary components of the Equalizer output  $y(n)$  and  $\beta^{SG}$  is a suitable real value chosen depending upon the data constellation. Hence the real and imaginary parts of the error signal for the stop and go algorithm are given as follows

$$e_{\gamma}^{SG}(n) = f_{\gamma}(n) \hat{e}_{\gamma}(n), \quad \text{for } \gamma = I, R \text{ (i.e. indicating Real and Imaginary parts)}$$

$$\text{where } f_{\gamma}(n) = \begin{cases} 1, & \text{if } \text{sgn}(\tilde{e}_{\gamma}(n)) = \text{sgn}(\hat{e}_{\gamma}(n)) \\ 0, & \text{otherwise} \end{cases} \quad \gamma = I, R$$



## **8. SIMULATION RESULTS**

The operation of a blind transversal adaptive equalizer can be verified using simulation experiments. In these simulations I use the channel reported by G. Picchi and G. Prati in [1].

Transmissions using 64-QAM modulations

$$\text{Re}(d_n), \text{Im}(d_n) \in \{\pm 1, \pm 3, \pm 5, \pm 7\}$$

was simulated. The additive gaussian noise samples were added on the output of the channel.

$$\text{SNR} = 30 \text{ dB}$$

$$E[|x_n|^2] = 42,$$

### **Figure 1** (Sato Algorithm)

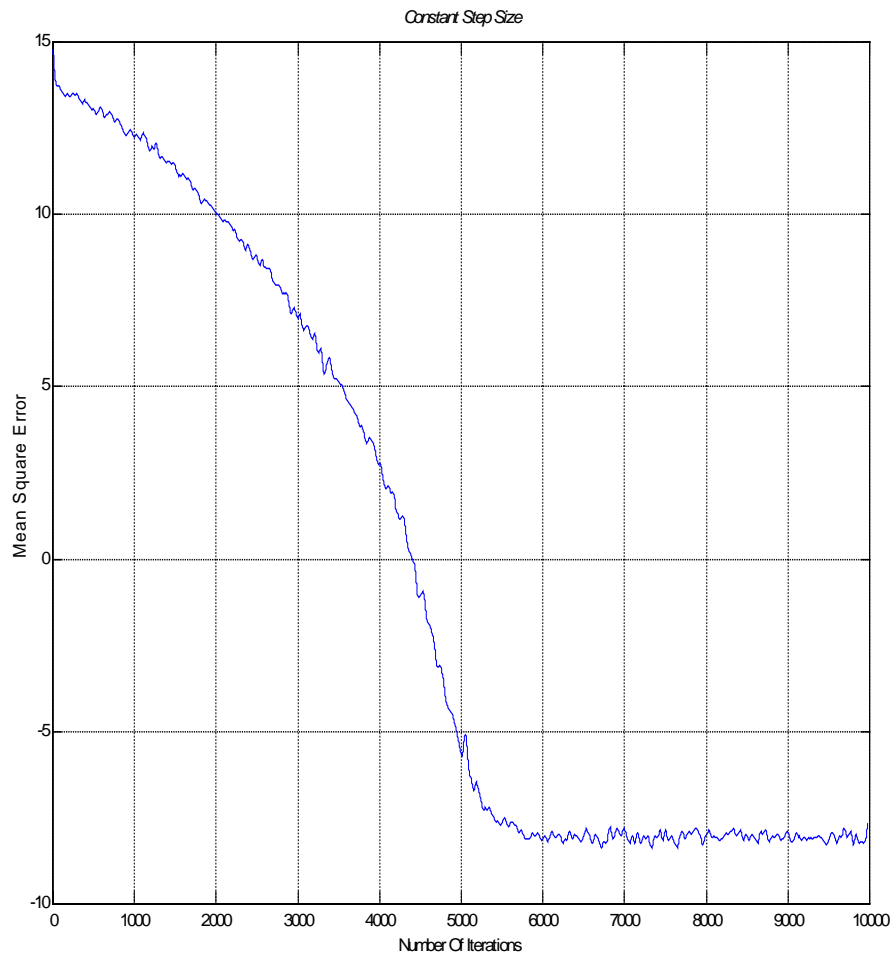
Shows the Equalized 64-Qam Constellation and curve for Mean Square Error versus iteration number for the constant length and step size of  $\gamma_n = 5 \times 10^{-4}$  by using Sato Algorithm. Constant length means that all the tap coefficients are adapted in each iteration. Length of the equalizer is taken to be 15.

**Figure 2** (Godard Algorithm)

Shows the Equalized 64-Qam Constellation and curve for Mean Square Error versus iteration number for the constant length and step size of  $\gamma_n = 5 \times 10^{-4}$  by using Godard Algorithm. Constant length means that all the tap coefficients are adapted in each iteration. Length of the equalizer is taken to be 15.

**Figure 3** (Stop and Go Algorithm)

Shows the Equalized 64-Qam Constellation and curve for Mean Square Error versus iteration number for the constant length and step size of  $\gamma_n = 5 \times 10^{-4}$  by using Stop & Go Algorithm. Constant length means that all the tap coefficients are adapted in each iteration. Length of the equalizer is taken to be 15.



## **8. CONCLUSIONS**

From the above curve 1 and curve 2 of Mean Square Error, we conclude that the automatic step size control results in the saving of about 800 iterations as compared with the algorithm with a constant step size. Hence constant step size is lower than that which ensures the fastest convergence, however the steady-state mean square error is then only slightly larger than its minimum irreducible value. From the curves 1 and curve 3 of Mean Square Error, we conclude that that the filter control length results in substantial shortening of the initial convergence time. One can also observe that thanks to smoothing based on condition (2), the convergence is faster and more stable. (Let us note that control of the filter length associated with changing the step size outperforms the operation of the constant length adaptive equalizer with the variable step size.)

The proposed method of automatic control of the step size and filter length for the blind equalizer results in substantial acceleration of the equalizer's convergence. One has to admit that the propose procedure increases the computational complexity of the adaptive blind equalizer very slightly and thus can be easily implemented.

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