**PROBLEM SESSION III**

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
Electrical Engineering Department
EE 340: Introduction to Electromagnetics

3.15 Determine the gradient of the following scalar fields:
(a) \( U = 4xz^2 + 3yz \).
(b) \( V = e^{(2x + 3y)} \cos 5z \).
(c) \( W = 2\rho(z^2 + 1) \cos \phi \).
(d) \( T = 5\rho \ e^{-2z} \sin \phi \).
(e) \( H = r^2 \cos \theta \ \cos \phi \).
(f) \( Q = (\sin \theta \ \sin \phi) / r^3 \).

3.18 Find the divergence and curl of the following vectors:
(a) \( A = e^{xy} ax + \sin xy ay + \cos^2 xz az \).
(b) \( B = \rho z^2 \cos \phi a_\rho + z \sin^2 \phi a_z \).
(c) \( C = r \cos \theta a_\rho - \frac{1}{r} \sin \theta a_\theta + 2r^2 \sin \theta a_\phi \).

3.30 Given that \( E = \frac{1}{r^4} \sin^2 \phi a_\phi \), evaluate
(a) \( \int_S E \cdot dS \)
(b) \( \int_V (\nabla \cdot E) \ dv \)
over the region between the spherical surfaces \( r = 2 \) and \( r = 4 \).

3.33 Calculate the total outward flux of vector \( F = \rho^2 \sin \phi a_\rho + z \cos \phi a_\theta + \rho z a_z \)
through the hollow cylinder defined by \( 2 \leq \rho \leq 3, 0 \leq z \leq 5 \).

3.39 Given the vector field
\[ R = \left( 2x^2 y + yz \right) a_x + \left( xy^2 - xz^2 \right) a_y + \left( cxyz - 2x^2 y^2 \right) a_z \]
determine the value of \( c \) for \( R \) to be solenoidal.

3.40 Given the vector field
\[ T = \left( \alpha xy + \beta z^3 \right) a_x + \left( 3x^2 - \gamma z \right) a_y + \left( 3xz^2 - y \right) a_z \]
is irrotational, determine \( \alpha, \beta, \) and \( \gamma \). Find \( \nabla \cdot T \) at (2, -1, 0).