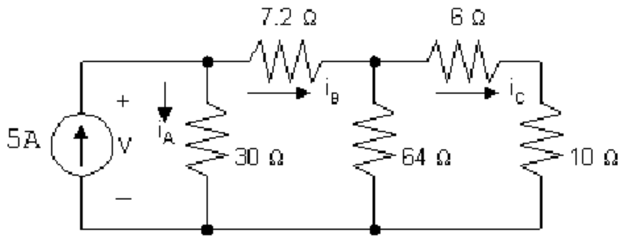
EE 202-132

HW2 -Solution

P1



Start from the right hand side of the circuit and make series and parallel combinations of the resistors until one equivalent resistor remains. Begin by combining the 6 Ω resistor and the 10 Ω resistor in series:

$$6\,\Omega + 10\,\Omega = 16\,\Omega$$

Now combine this 16 Ω resistor in parallel with the 64 Ω resistor:

$$16\,\Omega \| 64\,\Omega = \frac{(16)(64)}{16+64} = \frac{1024}{80} = 12.8\,\Omega$$

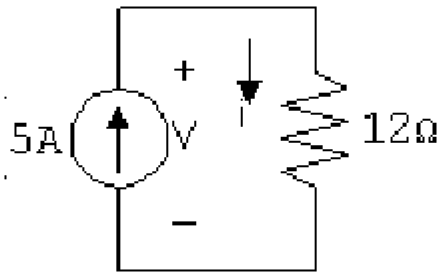
This equivalent 12.8 Ω resistor is in series with the 7.2 Ω resistor:

$$12.8\,\Omega + 7.2\,\Omega = 20\,\Omega$$

Finally, this equivalent 20 Ω resistor is in parallel with the 30 Ω resistor:

$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = \frac{600}{50} = 12\,\Omega$$

Thus, the simplified circuit is as shown:



[a] With the simplified circuit we can use Ohm's law to find the voltage across both the current source and the $12\,\Omega$ equivalent resistor:

$$v = (12\,\Omega)(5\text{ A}) = 60\text{ V}$$

[b] Now that we know the value of the voltage drop across the current source, we can use the formula $p = -vi$ to find the power associated with the source:

$$p = -(60\text{ V})(5\text{ A}) = -300\text{ W}$$

Thus, the source delivers 300 W of power to the circuit.

[c] We now can return to the original circuit, shown in the first figure. In this circuit, $v = 60$ V, as calculated in part (a). This is also the voltage drop across the $30\,\Omega$ resistor, so we can use Ohm's law to calculate the current through this resistor:

$$i_A = \frac{60\text{ V}}{30\,\Omega} = 2\text{ A}$$

Now write a KCL equation at the upper left node to find the current i_B:

$$-5\text{ A} + i_A + i_B = 0 \qquad \text{so} \qquad i_B = 5\text{ A} - i_A = 5\text{ A} - 2\text{ A} = 3\text{ A}$$

Next, write a KVL equation around the outer loop of the circuit, using Ohm's law to express the voltage drop across the resistors in terms of the current through the resistors:

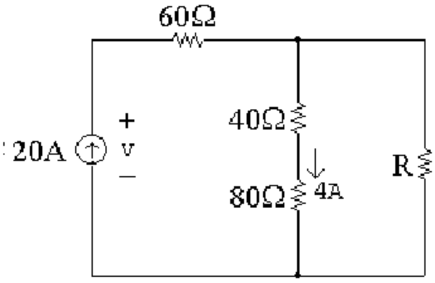
So $\qquad 16i_C = v - 7.2i_B = 60\text{ V} - (7.2)(3) = 38.4\text{ V}$

Thus $\qquad i_C = \dfrac{38.4}{16} = 2.4\text{ A}$

Now that we have the current through the $10\,\Omega$ resistor we can use the formula $p = Ri^2$ to find the power:

$$p_{10\,\Omega} = (10)(2.4)^2 = 57.6\text{ W}$$

P2



[a] We will write a current division equation for the current throught the 80Ω resistor and use this equation to solve for R:

$$i_{80\Omega} = \frac{R}{R + 40\,\Omega + 80\,\Omega}(20\ \text{A}) = 4\ \text{A} \qquad \text{so} \qquad 20R = 4(R + 120)$$

$$\text{Thus} \qquad 16R = 480 \qquad \text{and} \qquad R = \frac{480}{16} = 30\,\Omega$$

[b] With $R = 30\,\Omega$ we can calculate the current through R using current division, and then use this current to find the power dissipated by R, using the formula $p = Ri^2$:

$$i_R = \frac{40 + 80}{40 + 80 + 30}(20\ \text{A}) = 16\ \text{A} \qquad \text{so} \qquad p_R = (30)(16)^2 = 7680\ \text{W}$$

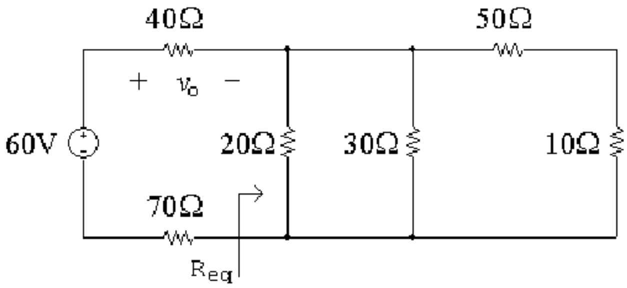
[c] Write a KVL equation around the outer loop to solve for the voltage v, and then use the formula $p = -vi$ to calculate the power delivered by the current source:

$$-v + (60\,\Omega)(20\ \text{A}) + (30\,\Omega)(16\ \text{A}) = 0 \qquad \text{so} \qquad v = 1200 + 480 = 1680\ \text{V}$$

$$\text{Thus,} \qquad p_{\text{source}} = -(1680\ \text{V})(20\ \text{A}) = -33{,}600\ \text{W}$$

Thus, the current source generates 33,600 W of power.

P3



[a] First we need to determine the equivalent resistance to the right of the 40 Ω and 70 Ω resistors:

$$R_{eq} = 20\,\Omega \| 30\,\Omega \| (50\,\Omega + 10\,\Omega) \qquad \text{so} \qquad \frac{1}{R_{eq}} = \frac{1}{20\,\Omega} + \frac{1}{30\,\Omega} + \frac{1}{60\,\Omega} = \frac{1}{10\,\Omega}$$

Thus, $\quad R_{eq} = 10\,\Omega$

Now we can use voltage division to find the voltage v_o:

$$v_o = \frac{40}{40 + 10 + 70}(60\text{ V}) = 20\text{ V}$$

[b] The current through the 40 Ω resistor can be found using Ohm's law:

$$i_{40\Omega} = \frac{v_o}{40} = \frac{20\text{ V}}{40\,\Omega} = 0.5\text{ A}$$

This current flows from left to right through the 40 Ω resistor. To use current division, we need to find the equivalent resistance of the two parallel branches containing the 20 Ω resistor and the 50 Ω and 10 Ω resistors:

$$20\,\Omega \| (50\,\Omega + 10\,\Omega) = \frac{(20)(60)}{20 + 60} = 15\,\Omega$$

Now we use current division to find the current in the 30 Ω branch:

$$i_{30\Omega} = \frac{15}{15 + 30}(0.5\text{ A}) = 0.16667\text{ A} = 166.67\text{ mA}$$

[c] We can find the power dissipated by the $50\,\Omega$ resistor if we can find the current in this resistor. We can use current division to find this current

from the current in the $40\,\Omega$ resistor, but first we need to calculate the equivalent resistance of the $20\,\Omega$ branch and the $30\,\Omega$ branch:

$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = 12\,\Omega$$

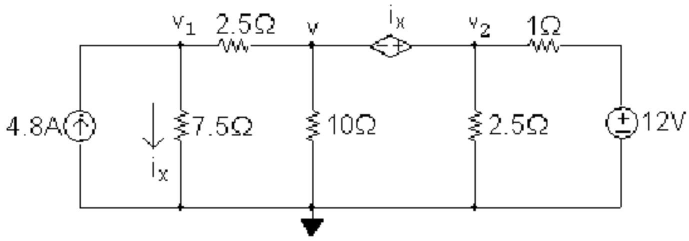
Current division gives:

$$i_{50\Omega} = \frac{12}{12+50+10}(0.5\text{ A}) = 0.08333\text{ A}$$

Thus, $\quad p_{50\Omega} = (50)(0.08333)^2 = 0.34722\text{ W} = 347.22\text{ mW}$

P4

Redraw the circuit identifying the three node voltages and the reference node:



Note that the dependent voltage source and the node voltages v and v_2 form a supernode. The v_1 node voltage equation is

$$\frac{v_1}{7.5} + \frac{v_1 - v}{2.5} - 4.8 = 0$$

The supernode equation is

$$\frac{v - v_1}{2.5} + \frac{v}{10} + \frac{v_2}{2.5} + \frac{v_2 - 12}{1} = 0$$

The constraint equation due to the dependent source is

$$i_x = \frac{v_1}{7.5}$$

The constraint equation due to the supernode is

$$v + i_x = v_2$$

Place this set of equations in standard form:

$$\begin{array}{lllllllll}
v_1\left(\frac{1}{7.5}+\frac{1}{2.5}\right) & + & v\left(-\frac{1}{2.5}\right) & + & v_2(0) & + & i_x(0) & = & 4.8 \\
v_1\left(-\frac{1}{2.5}\right) & + & v\left(\frac{1}{2.5}+\frac{1}{10}\right) & + & v_2\left(\frac{1}{2.5}+1\right) & + & i_x(0) & = & 12 \\
v_1\left(-\frac{1}{7.5}\right) & + & v(0) & + & v_2(0) & + & i_x(1) & = & 0 \\
v_1(0) & + & v(1) & + & v_2(-1) & + & i_x(1) & = & 0
\end{array}$$

Solving this set of equations gives $v_1 = 15$ V, $v_2 = 10$ V, $i_x = 2$ A, and $v = 8$ V.