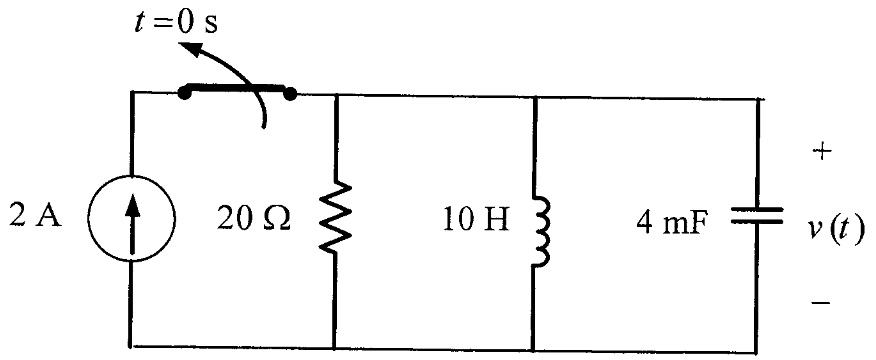
EE 202-01 – Fall 2012(121)

Quiz 6

SER	ID	NAME KEY



For the circuit shown above , the switch was closed for a long time. At t=0 s the switch is opened. Find $v(t)$ for all time ?

$\underline{t<0}$

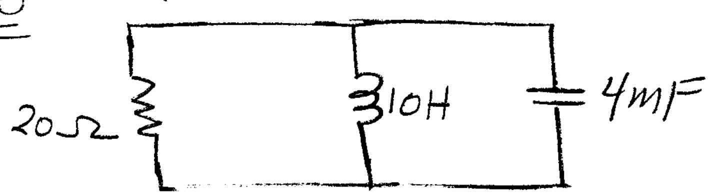
Capacitor open

indictor short $\Rightarrow$ $v(0^-) = 0$

$i_L(0^-) = 2$

$\underline{t>0}$

Parallel RLC



$\alpha = \frac{1}{2RC} = \frac{25}{4}$

$= 6.25$ rad/s

$\omega_0 = \frac{1}{\sqrt{LC}} = 5$ rad/s

$\alpha > \omega_0 \Rightarrow$ overdamped $\quad v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -\frac{25}{4} \pm \frac{15}{4}$

$s_1 = -\frac{25}{4} + \frac{15}{4} = -2.5 \qquad s_2 = -\frac{25}{4} - \frac{15}{4} = -10$

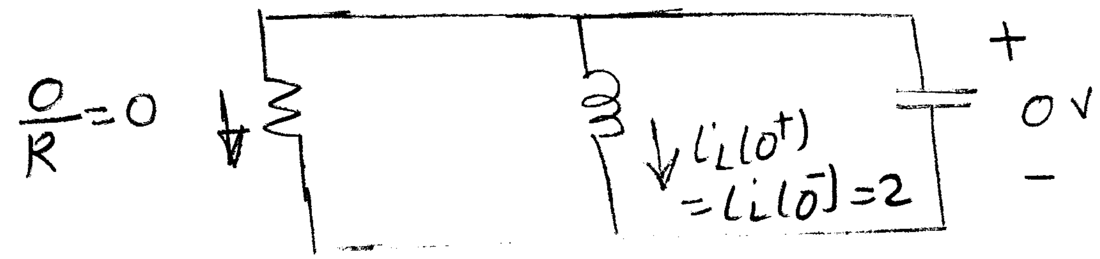
$\Rightarrow v(t) = A_1 e^{-2.5t} + A_2 e^{-10t}$

(continue) $\Rightarrow$

$$v(0^+) = v(0^-) = 0 \Rightarrow A_1 + A_2 = 0 \quad —①$$

$$i_c(t) = c\frac{dv(t)}{dt} \Rightarrow \frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{c}$$

$t = 0^+$



KCL

$$i_R(0^+) + i_L(0^+) + i_c(0^+) = 0$$

$$0 + 2 + i_c(0^+) = 0$$

$$\Rightarrow i_c(0^+) = -2\ A$$

$$\Rightarrow \frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{c} = \frac{-2}{4\times10^{-3}} = -500$$

$$\frac{dv(t)}{dt} = -2.5A_1e^{-2.5t} - 10A_2e^{-10t}$$

$$\frac{dv(0^+)}{dt} = -500 = -2.5A_1 - 10A_2 \quad —②$$

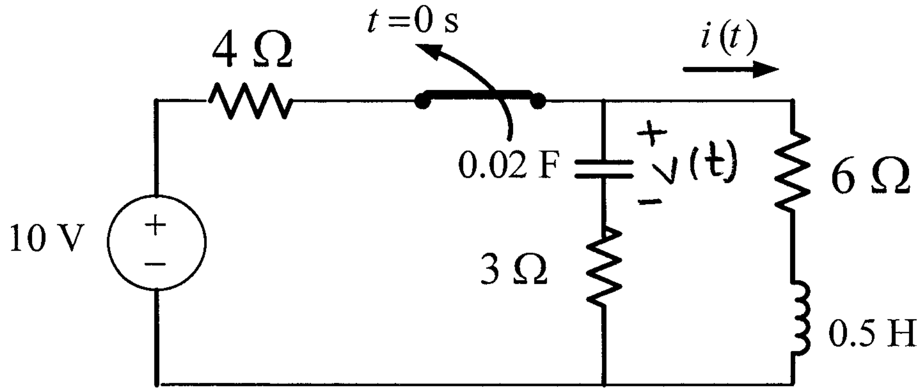
solving ① and ② $\Rightarrow$ $A_1 = -66.67$ $\quad A_2 = 66.67$

$$\Rightarrow v(t) = 66.67\left(e^{-10t} - e^{-2.5t}\right)\ V$$

EE 202-02 – Fall 2012(121)

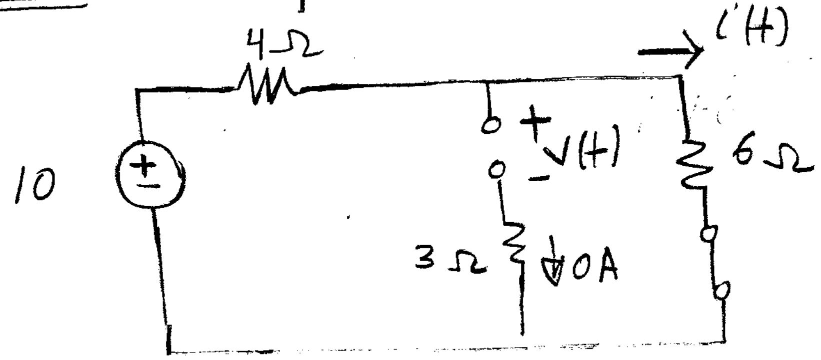
Quiz 6

SER	ID	NAME KEY



For the circuit shown above , the switch was closed for a long time. At t=0 s the switch is opened. Find $i(t)$ for all time ?

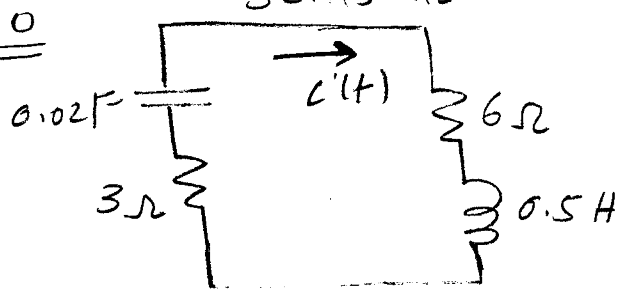
t<0 Capacitor open inductor short



$i(0^-) = \frac{10}{4+6} = 1A$

$v(0^-) = 6(1) = 6V$

t>0 series RLC



$R_{eq} = 3 + 6 = 9\Omega$

$\alpha = \frac{R_{eq}}{2L} = 9$

$\omega_0 = \frac{1}{\sqrt{LC}} = 10$

$\alpha < \omega_0 \Rightarrow$ underdamped

$i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$

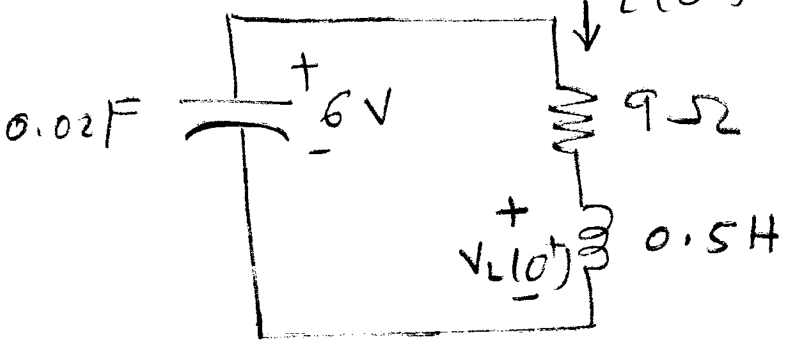
$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 4.359$

$$\Rightarrow \quad i(t) = e^{-9t}\left(B_1 \cos 4.359t + B_2 \sin 4.359t\right)$$

$$i(0^+) = i(0^-) = 1 = B_1 \qquad — \quad (1)$$

$$V_L(t) = L\frac{di(t)}{dt} \Rightarrow \frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L}$$

<u>$t = 0^+$</u>



<u>KVL</u> $\quad -6 + 9i(0^+) + V_L(0^+) = 0$

$$\Rightarrow V_L(0^+) = 6 - 9(1) = -3$$

$$\Rightarrow \frac{di(0^+)}{dt} = \frac{-3}{0.5} = -6 \quad A/s$$

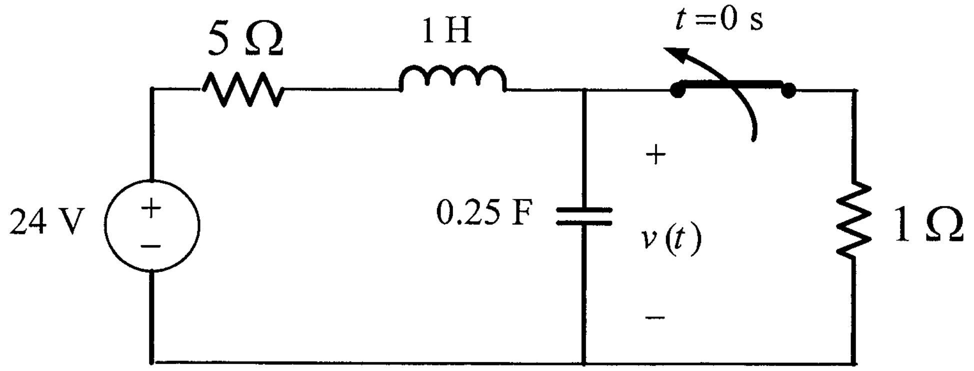
$$\frac{di(t)}{dt} = -9e^{-9t}\left(B_1 \cos 4.359t + B_2 \sin 4.359t\right) + e^{-9t}(4.359)\left(-B_1 \sin 4.359 + B_2 \cos 4.359\right)$$

$$\frac{di(0^+)}{dt} = -6 = -9(B_1 + 0) + 4.359(-0 + B_2)$$

$$\Rightarrow B_2 = 0.6882 \quad \Rightarrow \boxed{i(t) = e^{-9t}\left(\cos 4.359t + 0.6882 \sin 4.359t\right)}$$

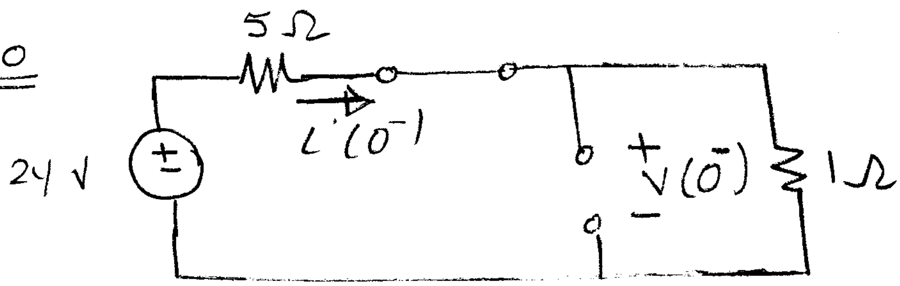
EE 202-07 – Fall 2012(121)
Quiz 6

SER	ID	NAME KEY



For the circuit shown above , the switch was closed for a long time. At t=0 s the switch is opened. Find $v(t)$ for all time ?

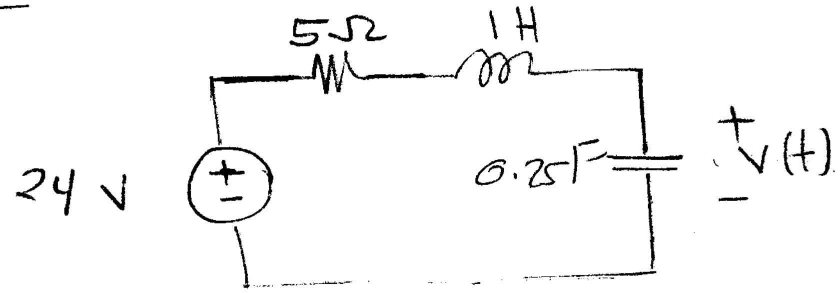
t<0



$v(0^-) = \frac{1}{5+1} 24 = 4 V$

$i(0^-) = \frac{24}{6} = 4 A$

t>0 step response series RLC



$\alpha = \frac{R}{2L} = 2.5$

$w_0 = \frac{1}{\sqrt{LC}} = 2$

$\alpha > w_0 \Rightarrow$ overdamped $v(t) = V_f + A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$V_f = v(\infty) = 24 V$

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - w_0^2} = -2.5 \pm 1.5$

$s_1 = -1, s_2 = -4$

$v(t) = 24 + A_1 e^{-t} + A_2 e^{-4t}$

$\Rightarrow$

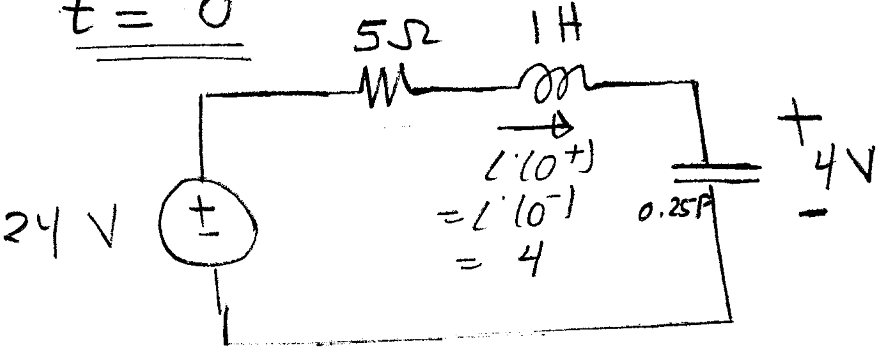
$$v(0^+) = v(0^-) = 4 = 24 + A_1 + A_2$$

$$\Rightarrow A_1 + A_2 = -20 \quad — ①$$

$$i_c(t) = c\frac{dv(t)}{dt} \Rightarrow \frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{c}$$

$t = 0^+$

$i_c(0^+) = 4$ A



$$\Rightarrow \frac{dv(0^+)}{dt} = \frac{4}{0.25} = 16 \text{ V/s}$$

$$\frac{dv(t)}{dt} = -A_1 e^{-t} - 4A_2 e^{-4t}$$

$$\Rightarrow \frac{dv(0^+)}{dt} = 16 = -A_1 - 4A_2 \quad — ②$$

solving ① and ②

$$\Rightarrow A_1 = \frac{-64}{3} \qquad A_2 = \frac{4}{3}$$

$$\Rightarrow v(t) = 24 + \frac{4}{3}(-16e^{-t} + e^{-4t}) \text{ V} \quad t > 0$$