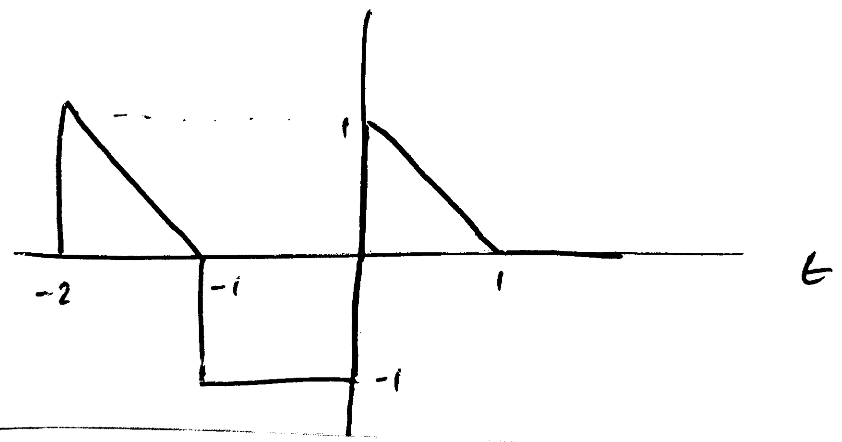
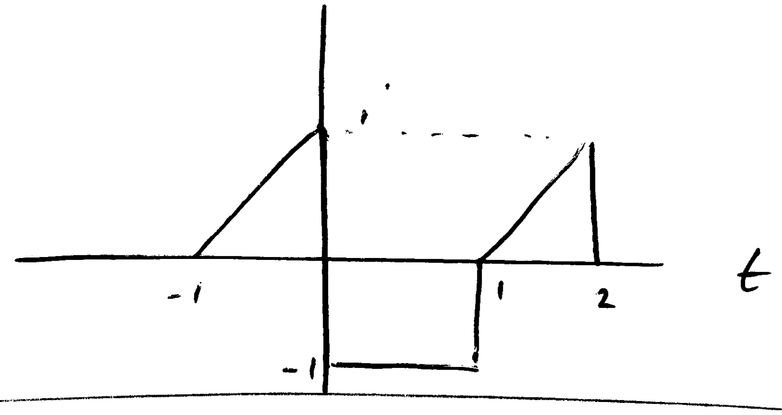
2.2 (a)

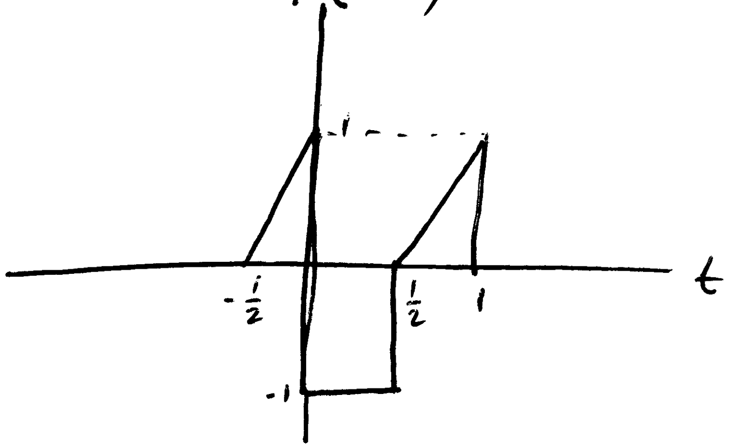
$x(t)$



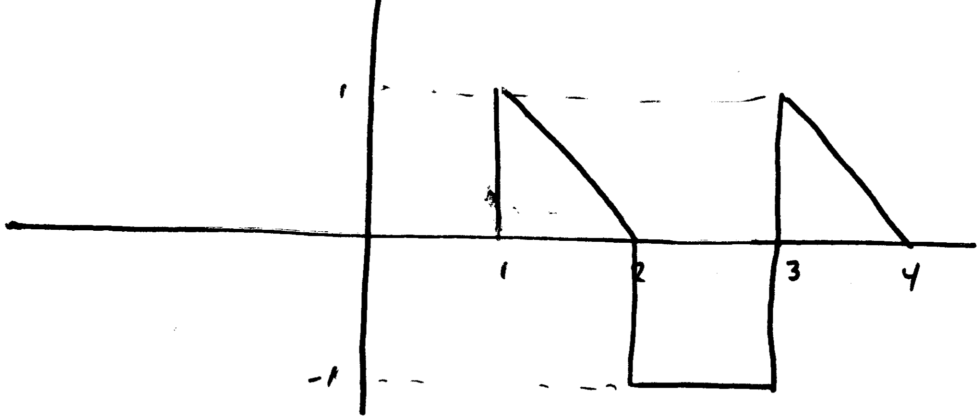
$x(-t)$



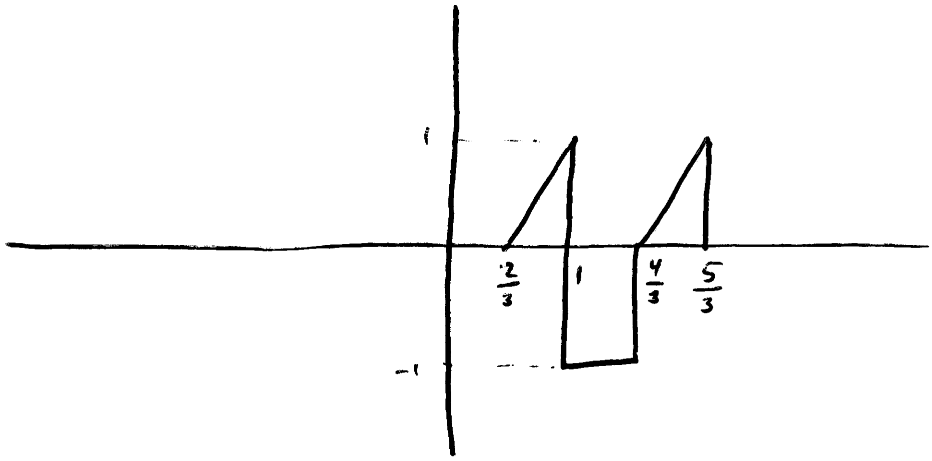
$x(-2t)$



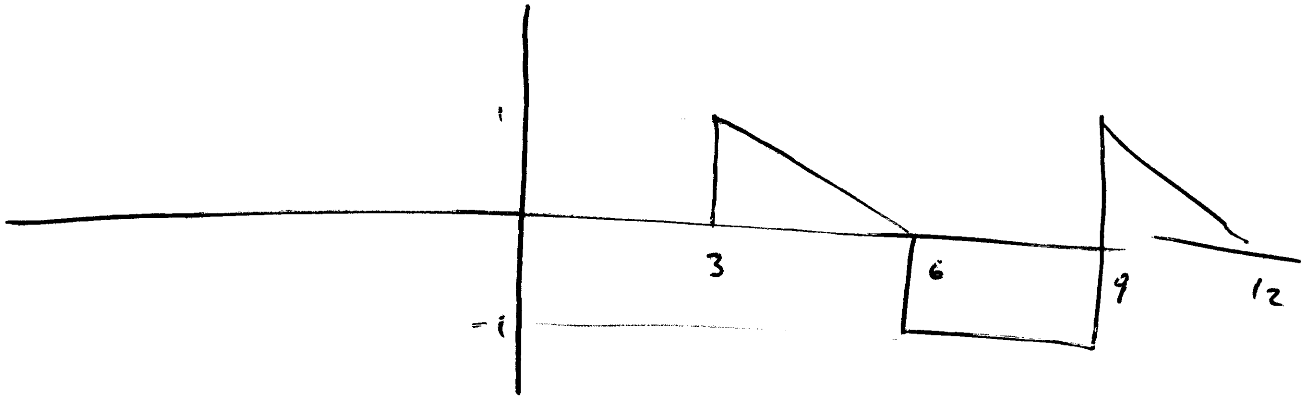
$x(t-3)$



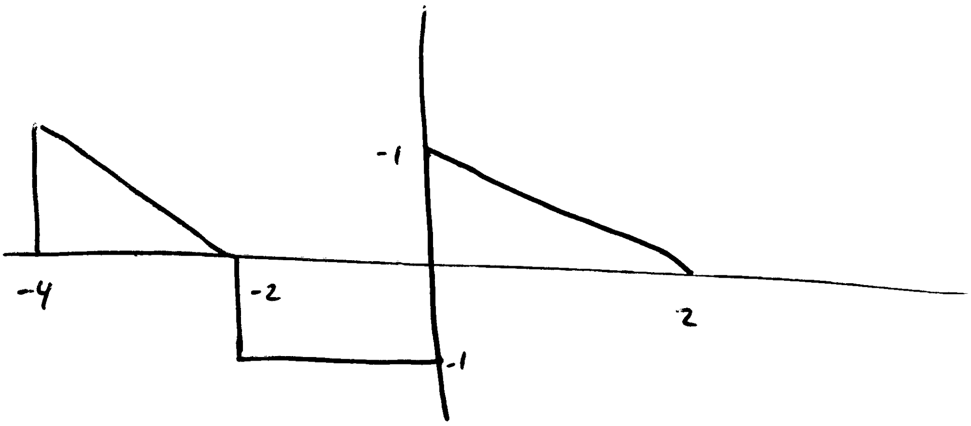
$$x(3-3t) = x(-3(t-1))$$



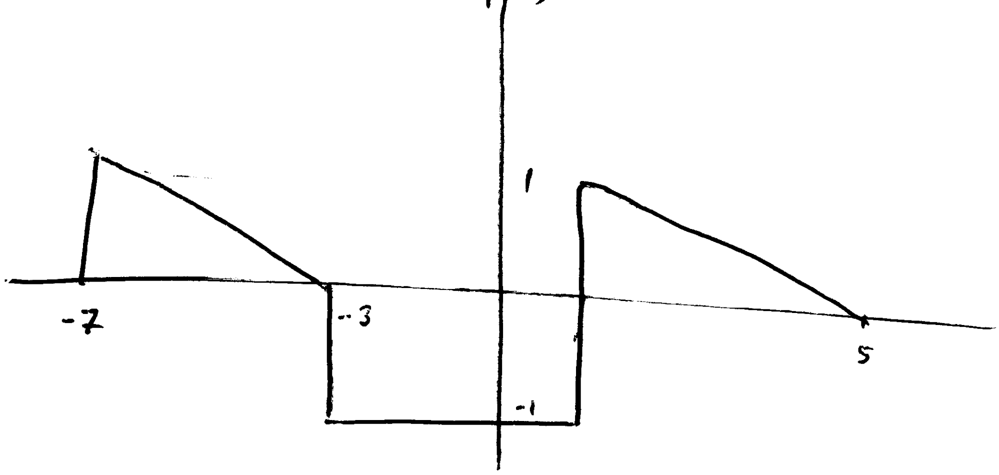
$$x\left(-3+\frac{t}{3}\right) = x\left(\frac{1}{3}(t-9)\right)$$



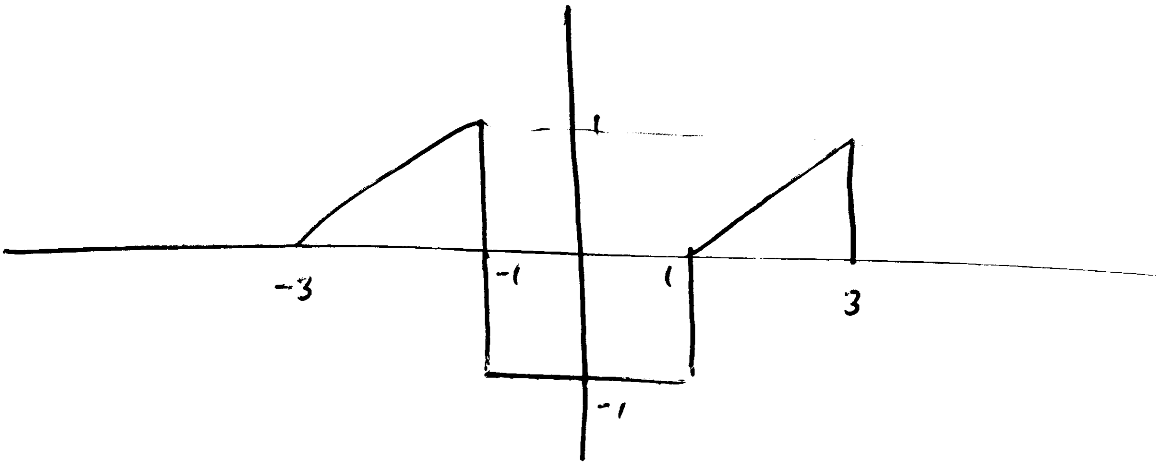
$$x\left(\frac{t}{2}\right)$$



$x\left(\frac{t-1}{4}\right)$

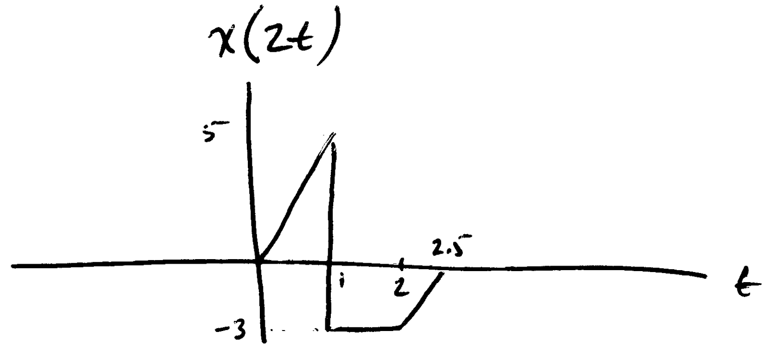


$x\left[(-t-1)/2\right] = x\left(-\frac{1}{2}(t+1)\right)$

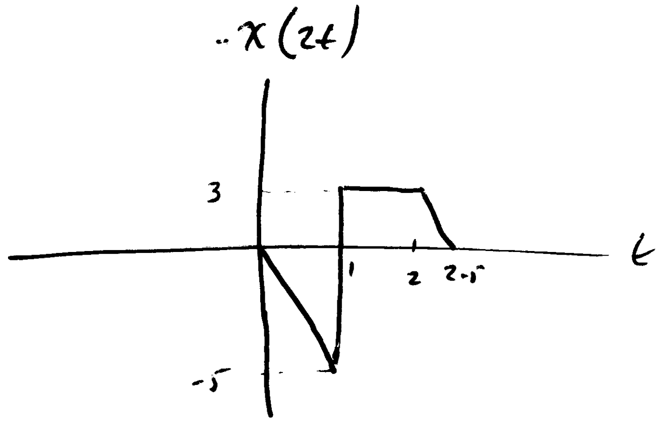


2.4 We need the following:

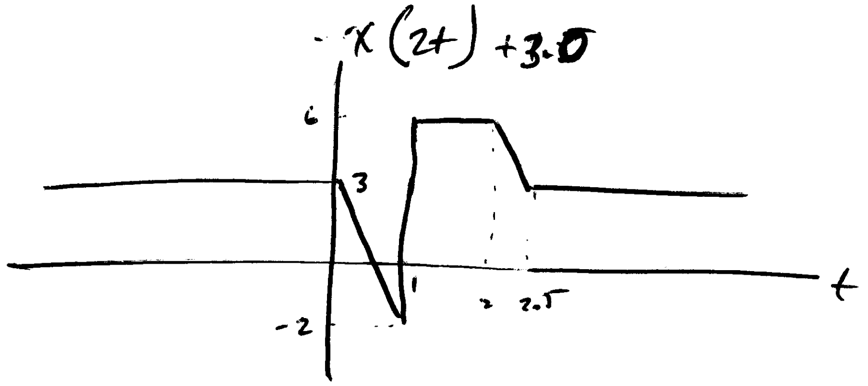
1. Scale in time by 2 (compress in time by 2)



2. Flip its amplitude (multiply by -1)

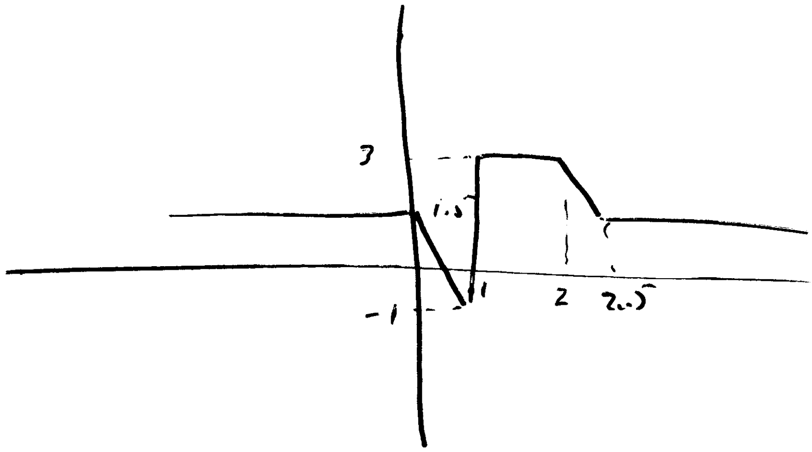


3. Add 3.0 to signal



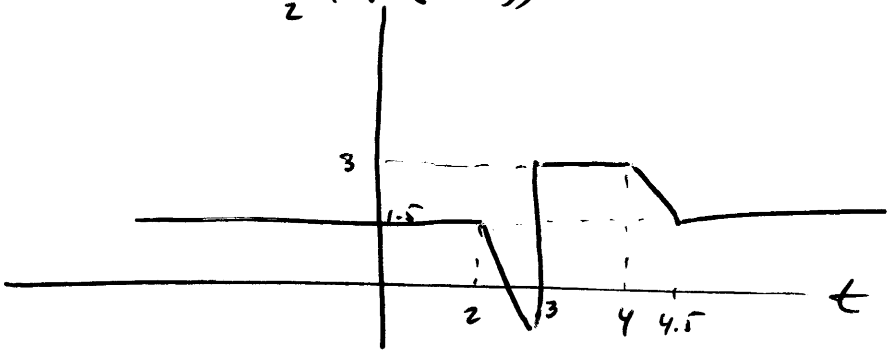
4. Multiply the whole signal by $\frac{1}{2}$ (to scale it down)

$$-\frac{1}{2}x(2t)+1.5$$



5. Shift in time by 2 to the right

$$-\frac{1}{2}x(2(t-2))+1.5$$



$\Longrightarrow$ $\boxed{y(t) = -\frac{1}{2}x(2(t-2))+1.5}$

Also,

$$y(t)-1.5 = -\frac{1}{2}x(2(t-2)) \quad \text{[Add -1.5 to both sides]}$$

$$x(2(t-2)) = 3-2y(t) \quad \text{[multiply by } -\tfrac{1}{2}\text{]}$$

$$x(2(t)) = 3-2y(t+2) \quad \text{[substitute } t+2 \text{ for } t\text{]}$$

$$\boxed{x(t) = 3-2y\left(\frac{t}{2}+2\right)} \quad \text{[substitute } \tfrac{t}{2} \text{ for } t\text{]}$$

Do parts (b) and (d) yourself.

2.10

a) $x(t) = 3t$ (Odd)

b) $x(t) = 5u(t)$ (Neither)

c) $x(t) = 5\cos(3t)$ (Even)

d) $x(t) = 5 + e^{-t} + e^{t}$ (Even)

e) $x(t) = 5 + e^{-t}$ (Neither)

f) $x(t) = \sin(3t - \frac{\pi}{2})$ (Even)

$= -\cos(3t)$

2.21 c)

i) $= \cancel{\sin(0)}^{0} \int_{-\infty}^{\infty} \cancel{\delta(t)\,dt}^{1} = 0$

ii) $= \sin(-6) \cdot \int_{-\infty}^{\infty} \cancel{\delta(t+2)\,dt}^{1} = \sin(-6)$

$= +0.2794$

iii) $= \sin(3(0+2)) \int_{-\infty}^{\infty} \cancel{\delta(t)\,dt}^{1} = \sin(6) =$

$= -0.2794$

iv) $= \sin(\cancel{3(-2+2)}^{0}) \cdot \int_{-\infty}^{\infty} \cancel{\delta(t+2)\,dt}^{1} = 0$

v) $= \cos(3(\cancel{-2+2})^{1}) \cdot \underbrace{\int_{-\infty}^{\infty} \delta(2t+4)\,dt}_{= \frac{1}{2}} = \frac{1}{2}$

Because of this

$$\boxed{\int \delta(\alpha t) = \frac{1}{|\alpha|} \int \delta(t) \quad \text{see table 2-3}}$$

2.23 a)

$$y_3(t) = T_3\left[T_1(x(t))\right]$$

$$y_5(t) = T_5\left[x(t)\right]$$

$$y(t) = T_2\left[T_1(x(t))\right] + T_4\left[y_3(t) + y_5(t)\right]$$

$$= T_2\left[T_1(x(t))\right] + T_4\left[T_3(T_1(x(t))) + T_5(x(t))\right]$$

2.24

$$m(t) = T_2\left[x(t) + T_4(y(t))\right]$$

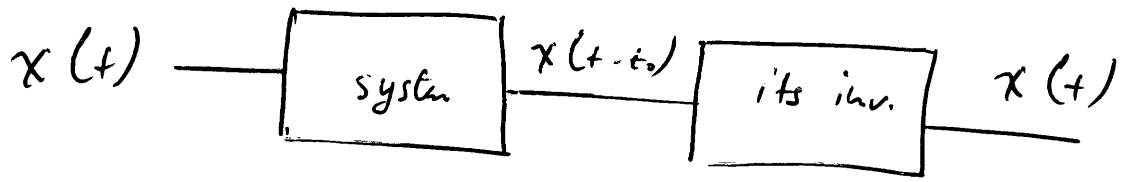
$$y(t) = T_3\left[T_1(x(t)) + m(t)\right]$$

$$= T_3\left[T_1(x(t)) + T_2\left\{x(t) + T_4(y(t))\right\}\right]$$

2.29 $y(t) = x(t - t_0)$

i) Memoryless if $t_0 = 0$ otherwise Not memoryless.

ii) Invertible (inverse is the system
$y(t) = x(t + t_0)$



iii) Causal if $t_0 \geq 0$ (otherwise not causal)

iv) Stable (If input is bounded, output will be bounded.)

v) Time invariant (assuming t_0 is constant and does not change)

vi) Linear (put $x_1(t)$ and then $x_2(t)$ then put $a_1 x_1(t) + a_2 x_2(t)$, you will get $y(t) = a_1 y_1(t) + a_2 y_2(t)$)

2.30	a	b	c	e
	$y(t)=x(t)+2$	$y(t)=t\,x(t)$	$y(t)=\int_{-\infty}^{t} x(2\tau)\,d\tau$	$y(t)=x(2t)$
Memoryless	Yes Depends only on x at t.	Yes	No output depends on $x(t_0)$ for $t_0<t$	No $y(3)=x(6)$ $y(-2)=x(-4)$
Invertible	Yes inv $y(t)=x(t)-2$	No Assume $x(t)=t$ $\Rightarrow y(t)=t^2$ see(*) below	Yes inv. is $y(t)=\frac{d}{dt}x(\frac{t}{2})$	Yes inv. $y(t)=x(\frac{t}{2})$
Time inv.	Yes	No (Put same input at diff. times, output will be diff.	No (because of the 2.)	No (same)
Causal	Yes	Yes	No (Because at $t=5$ for example $y(5)$ is function of $x(10)$	No (because at $t=5$, $y(5)$ is function of $x(10)$.
Stable	Yes	No (put bounded input $\Rightarrow$ output is unbounded as $t\to\pm\infty$)	No (If $x(t)=1$ for all t $y(t)=\infty$)	Yes
Linear	No $y_1(t)=x_1(t)+2$ $y_2(t)=x_2(t)+2$ $y_3(t)=(a_1x_1(t)+a_2x_2(t))+2$ $\neq a_1y_1(t)+a_2y_2(t)$	Yes $y_1(t)=t\,x_1(t)$ $y_2(t)=t\,x_2(t)$ $y_3(t)=t(a_1x_1+a_2x_2)$ $=a_1y_1(t)+a_2y_2(t)$	Yes same	Yes same

* for $x(t)=-3$ and 3
$y(t)=9$.