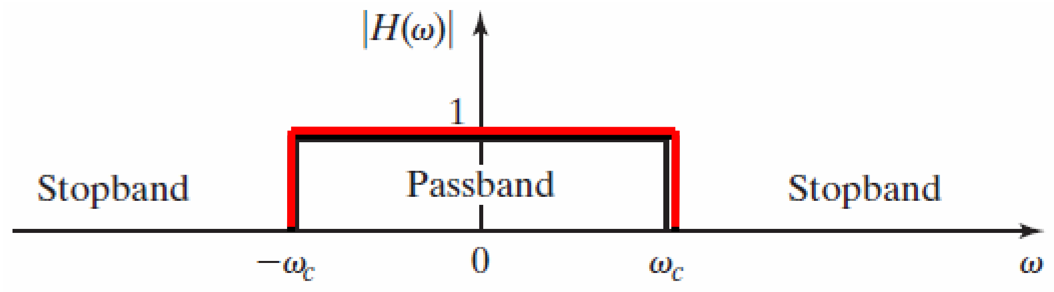
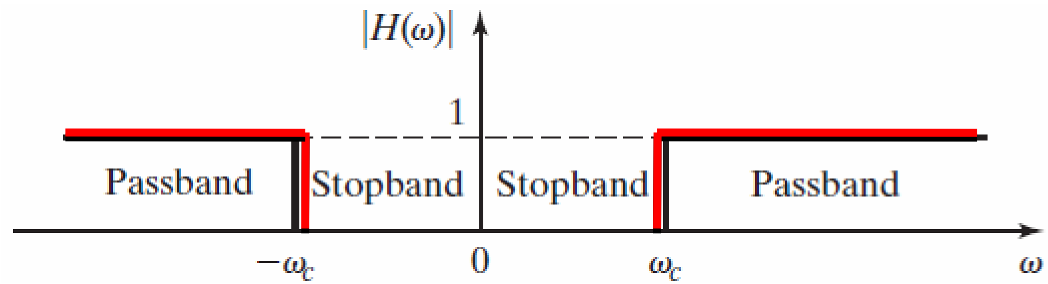
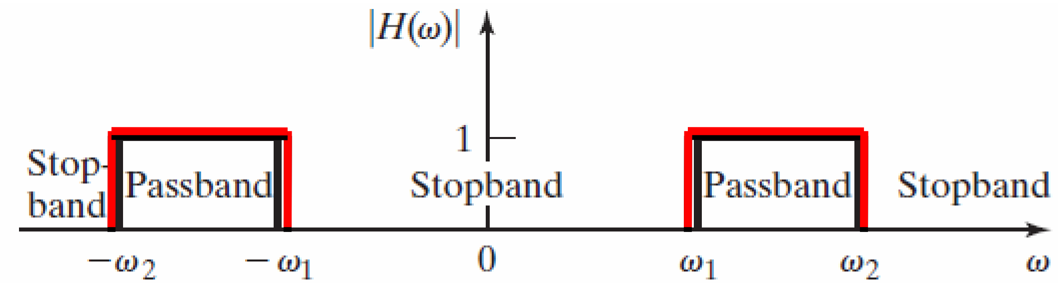
6.1 IDEAL FILTERS



ideal low-pass filter

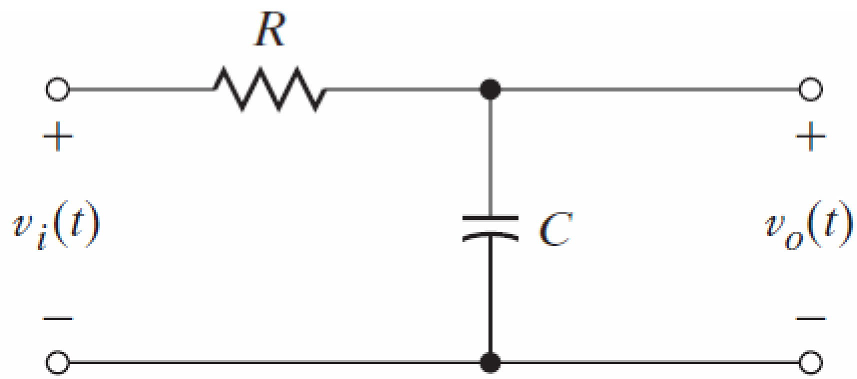


high-pass filter



bandpass filter

RC Low-Pass Filter



$$v_o(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

$$v_i(t) = Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

$$V_i(\omega) = RI(\omega) + \frac{1}{j\omega C}I(\omega)$$

$$V_o(\omega) = \frac{1}{j\omega C}I(\omega)$$

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} \qquad = \frac{1}{1 + j\omega RC}$$

$$\omega_c = \frac{1}{RC} \qquad H(\omega) = \frac{1}{1 + j\omega/\omega_c} \qquad = |H(\omega)|e^{j\Phi(\omega)}$$

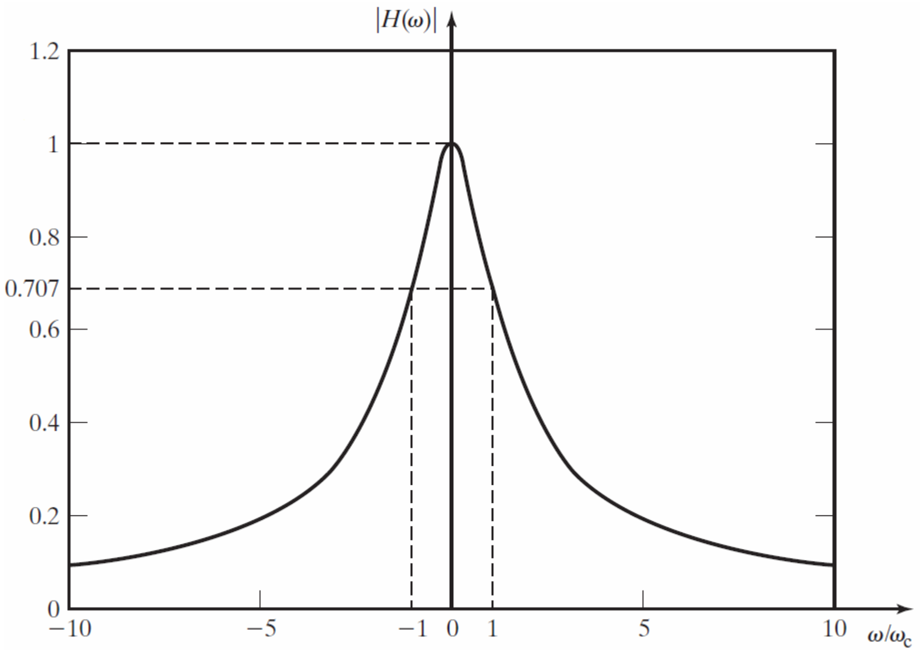
The magnitude and phase frequency spectra of the filter are described by the equations

$$H(\omega) = \frac{1}{1 + j\omega/\omega_c}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_c)^2}}$$

$$\Phi(\omega) = -\arctan(\omega/\omega_c)$$

At the frequency $\omega = \omega_c \Rightarrow \frac{\omega}{\omega_c} = 1$



$$|H(\omega_c)| = \frac{|V_o(\omega_c)|}{|V_i(\omega_c)|} = \frac{1}{\sqrt{2}}$$

The ratio of the normalized power of the **input** and **output** signals is given by

$$|H(\omega_c)|^2 = \frac{|V_o(\omega_c)|^2}{|V_i(\omega_c)|^2} = \frac{1}{2}$$

This type of filter is often called the ***half-power frequency***