4-12

$$X(f) = \frac{A}{\alpha + j2\pi f}$$

$$G(f) = \frac{A^2}{\alpha^2 + 4\pi^2 f^2}$$

$$E_B = \int_{-B}^{B} \frac{A^2}{\alpha^2 + 4\pi^2 f^2} df = \frac{A^2}{\pi\alpha} \tan^{-1}\left(\frac{2\pi B}{\alpha}\right)$$

a) $E_B = \frac{A^2}{\pi\alpha} \tan^{-1}\left(\frac{2\pi \frac{\alpha}{\pi}}{\alpha}\right) = \frac{A^2}{\pi\alpha} \tan^{-1}(2)$

b) $E_B = \frac{A^2}{\pi\alpha} \tan^{-1}\left(\frac{2\pi \frac{\alpha}{2\pi}}{\alpha}\right) = \frac{A^2}{\pi\alpha} \tan^{-1}(1)$

$$= \frac{A^2}{\pi\alpha}\left(\frac{\pi}{4}\right) = \frac{A^2}{4\alpha}$$

$$E_{total} = \frac{A^2}{\pi\alpha}\left(\frac{\pi}{2}\right) \quad \left\{ \text{Let: } B \to \infty \right\}$$

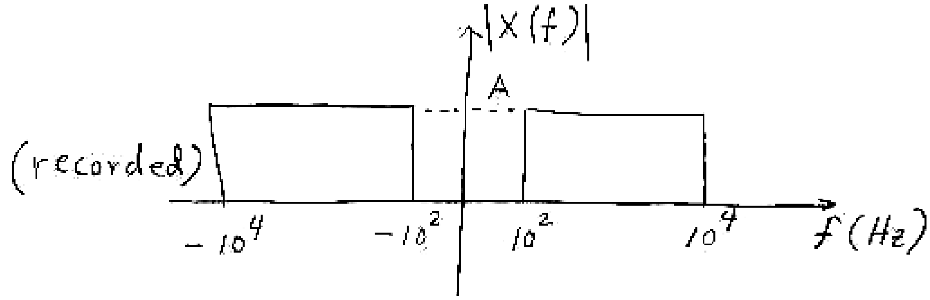
$$= \frac{A^2}{2\alpha}$$

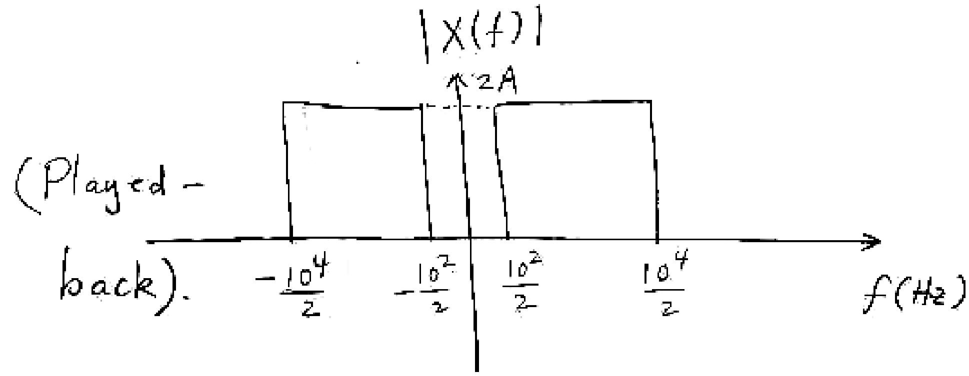
fractim $= \frac{A^2/4\alpha}{A^2/2\alpha} = \frac{1}{2}$, (50% of the energy).

4-17

The played-back signal is an expanded (in time) version of the recorded signal, by a factor of two.

Using $x\left(\frac{t}{2}\right) \Rightarrow 2X(2f)$, we can see the new spectrum is compressed.





(i.e. the maximum frequency contained in the played-back signal is reduced. This is supported by experience, because the played-back sound will sound less sharp).

4-35

$$H(f) = H_0\left[1 - \Pi\left(\frac{f}{2B}\right)\right] e^{-j2\pi f t_0}$$

$$h(t) = \mathcal{F}^{-1}\left[H(f)\right]$$

$$H(f) = \left[H_0 - H_0\Pi\left(\frac{f}{2B}\right)\right] e^{-j2\pi f t_0}$$

$$\mathcal{F}^{-1}\left[H_0 - H_0\Pi\left(\frac{f}{2B}\right)\right] = H_0\delta(t) - H_0 2B\,\mathrm{sinc}\,(2Bt)$$

$$\therefore\ h(t) = H_0\delta(t - t_0) - H_0 2B\,\mathrm{sinc}\left[2B(t - t_0)\right]$$